

Magnetism and Electromagnetic Induction

CHAPTER 5

Inductance

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5

Inductance

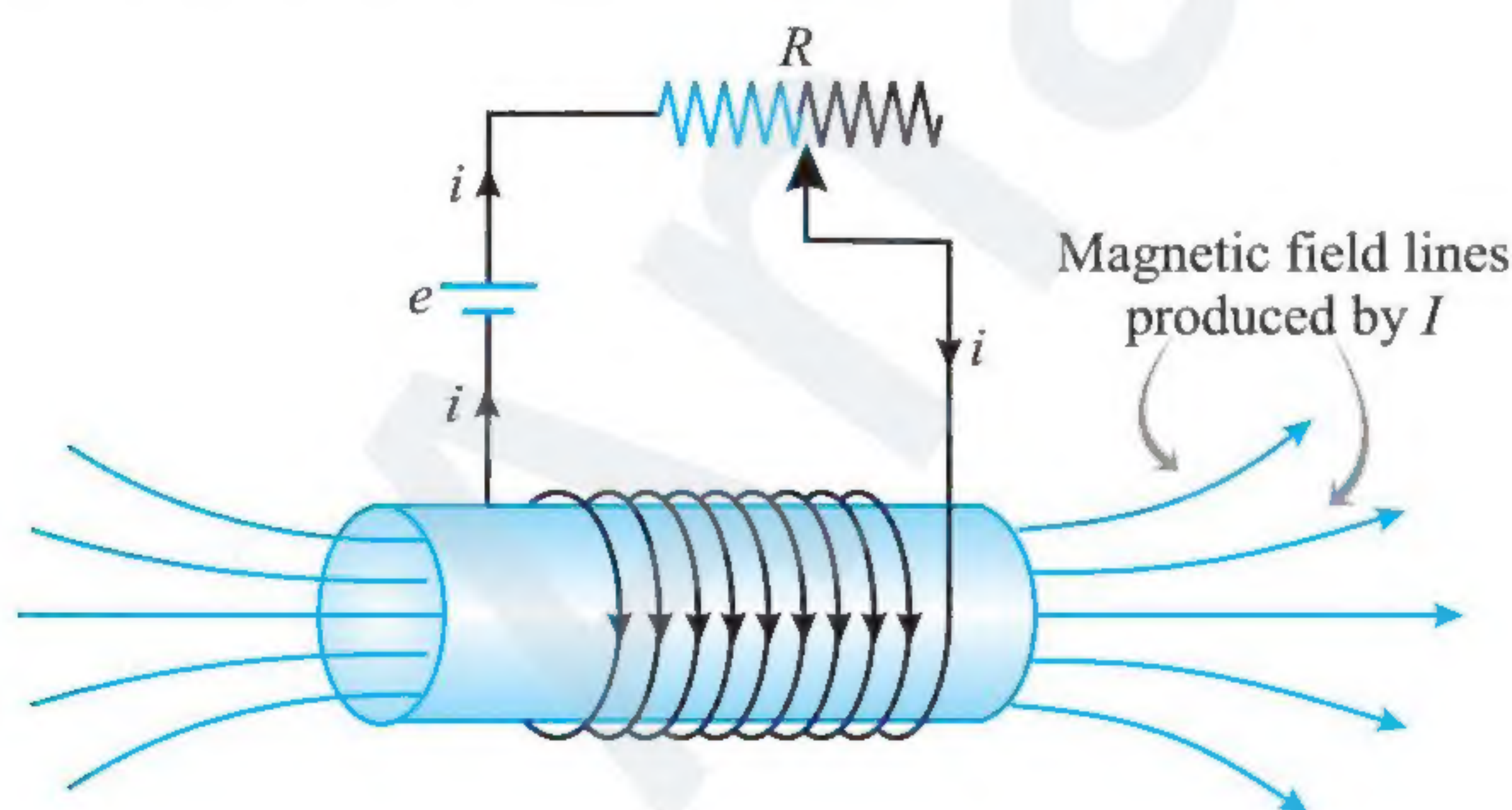
INTRODUCTION

In previous chapter we learned that an emf and a current are induced in a loop of wire when the magnetic flux through the area enclosed by the loop changes with time. In this chapter we will learn some practical applications of flux change, we first discuss two important effects of electromagnetic induction known as self-induction and mutual-induction.

Next, we will discuss the energy stored in the magnetic field of an inductor and the energy density associated with the magnetic field. Finally, we will examine the characteristics of circuits that contain inductors, resistors, and capacitors in various combinations.

SELF-INDUCTANCE

In almost all cases of induced emfs presented in previous chapter, the magnetic field has been produced by some external source, such as a permanent magnet or an electromagnet. But it is not necessary that the magnetic field always arise from an external source. The emf can be induced in a current-carrying coil by a change in the magnetic field that the current itself produces. For instance, figure shows a coil connected with a circuit where the current can be changed. The current creates magnetic field that can be changed by changing the current, which in turn creates a changing flux through the coil. The change in flux induces an induced emf in the coil, in accordance with Faraday's law. The effect in which a changing current in a circuit induces an emf in the same circuit is referred to as self-induction. The emf set up in this case is called a **self-induced emf**.



Let us consider a coil which we can now call inductor, having N turns and a current I is flowing through it, if ϕ is the magnetic flux that passes through one turn of the coil, then $N\phi$ is the net flux through a coil of N turns. Since ϕ is proportional to the magnetic field, and the magnetic field is proportional to the current I , so we have

$$N\phi \propto I \Rightarrow N\phi = LI \quad \dots(i)$$

In above Eq. (i), where L is a proportionality constant—called ‘Coefficient of Self Induction’ or simply the inductance, of the coil or inductor and its associated circuit in which current I is flowing—that depends on the geometry of the loop and other physical characteristics.

If current changes at a rate of dI/dt then EMF induced in the circuit of coil or conductor is given as

$$e = \left| N \frac{d\phi}{dt} \right| = L \left| \frac{dI}{dt} \right| \quad \dots(ii)$$

According to Lenz's law the direction of induced EMF is such that it opposes the causes of induction which is change in current thus signs of e and dI/dt are opposite in above equation which can be written without modulus as

$$e = -L \frac{dI}{dt} \quad \dots(iii)$$

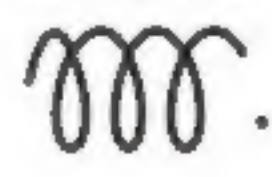
From Eq. (iii), we can also write the inductance as the ratio

$$L = -\frac{e}{di/dt} \quad \dots(iv)$$

We can write resistance as $R = \frac{\Delta V}{i} \quad \dots(v)$

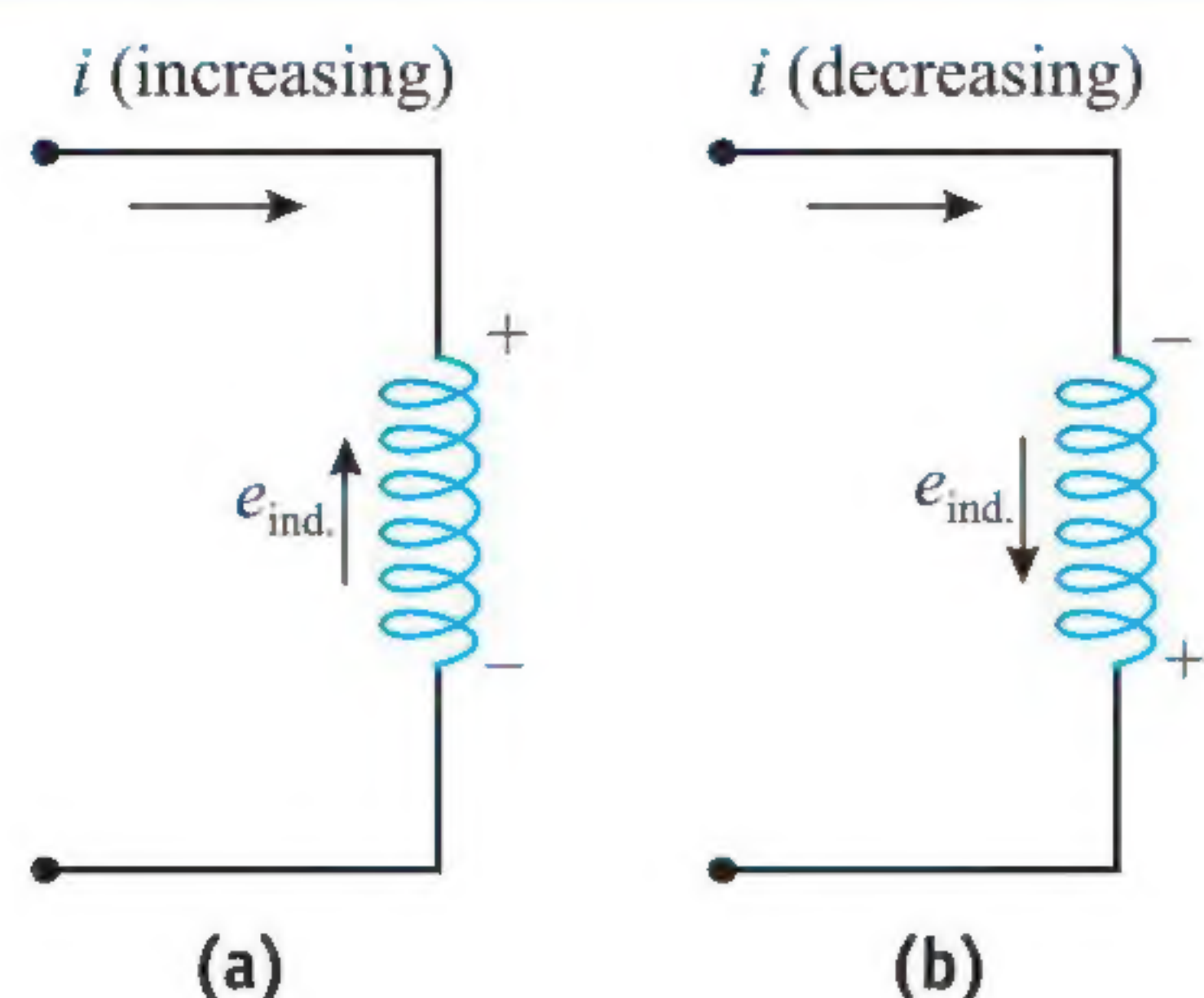
We say that resistance is a measure of the opposition to current. In comparison, Eq. (i), being of the same mathematical form as Eq. (v), shows us that inductance is a measure of the opposition to a change in current.

The SI unit of inductance is the **henry** (H), which as we can see from Eq. (iv) is 1 volt-second per ampere: $1 \text{ H} = 1 \text{ V}\cdot\text{s}/\text{A}$

A circuit device that is designed to have a particular inductance is called an inductor. The usual circuit symbol for an inductor is .

DIRECTION OF THE SELF-INDUCED EMF

Lenz's Law provides the direction of the self-induced potential difference. The negative sign in Eq. (iii) provides the clue that the induced potential difference always opposes any change in current. For example, Fig. (a) shows current flowing through an inductor and increasing with time. Thus, the self-induced potential difference will oppose the increase in current. In Fig. (b), the current flowing through an inductor is decreasing with time. Thus, a self-induced potential difference will oppose the decrease in current. We have assumed that these inductors are ideal inductors; that is, they have no resistance. All induced potential differences manifest themselves across the connections of the inductor.



INDUCTANCE OF A SOLENOID

Consider a long solenoid with N turns carrying a current, i . This current creates a magnetic field in the centre of the solenoid, resulting in a magnetic flux, ϕ_B . The same magnetic flux goes through each of the N windings of the solenoid. It is customary to define the flux **linkage** as the product of the number of windings and the magnetic flux, or $N\Phi_B$.

$$N\Phi_B = (nl)(BA) = (nl)(\mu_0 ni)(A)$$

Thus, the inductance is given by

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(\mu_0 ni)(A)}{i} = \mu_0 n^2 l A \quad \dots(i)$$

$$L = \mu_0 n^2 l A = \mu_0 n^2 V$$

Inductance per unit volume = $\mu_0 n^2$

This expression for the inductance of a solenoid is good for long solenoids because fringe field effects at the ends of such a solenoid are small. You can see from Eq. (i) that the inductance of a solenoid depends only on the geometry (length, area, and number of turns) of the device. This dependence of inductance on geometry alone holds for all coils and solenoids, just as the capacitance of any capacitor depends only on its geometry.

ILLUSTRATION 5.1

How many meters of a thin wire are required to manufacture a solenoid of length l_0 and inductance L , if the solenoid's cross-sectional diameter is considerably less than its length?

Sol. Self-induction coefficient of a solenoid,

$$L = \mu_0 n^2 l_0 A = \frac{\mu_0 N^2 \pi r^2}{l_0} \quad \dots(i)$$

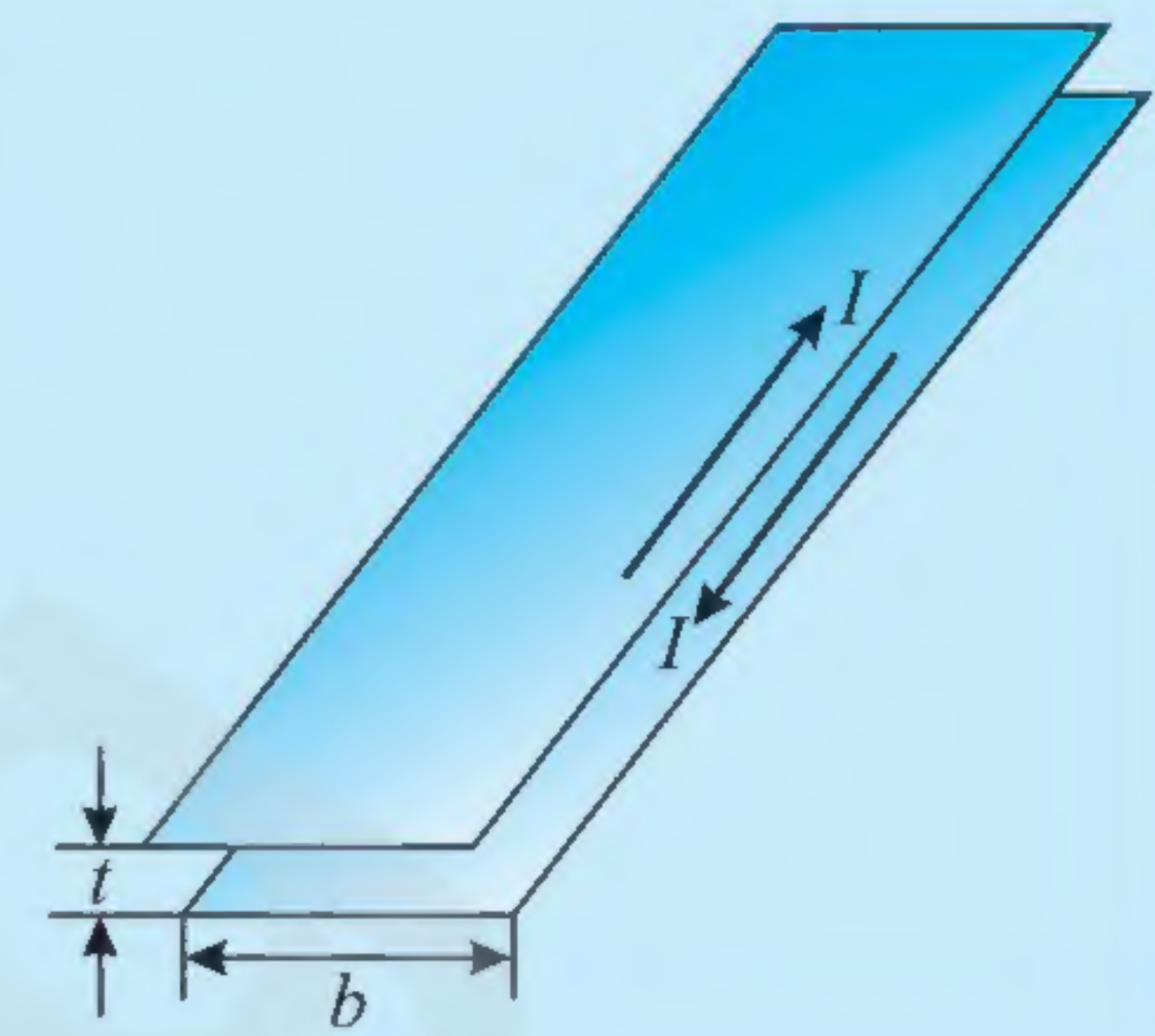
$$\text{Total length of wire, } l = N \times 2\pi r \Rightarrow N = \frac{l}{2\pi r} \quad \dots(ii)$$

$$\text{From (i) and (ii), we get } L = \frac{\mu_0 \left(\frac{l}{2\pi r}\right)^2 \pi r^2}{l_0} = \frac{\mu_0 l^2}{4\pi l_0}$$

$$\text{Hence length of wire used to make solenoid, } l = \sqrt{\frac{4\pi l_0 L}{\mu_0}}$$

ILLUSTRATION 5.2

Find the inductance per unit length of a double tape line if the tapes are separated by a distance t which is considerably less than their width b ($t \ll b$). The current is flowing in the two sheets of the tape line in opposite direction as shown in figure.



Sol. The current is uniformly distributed over the width of the tape.

The current per unit width of the sheets of tape, $j = \frac{I}{b}$

The magnetic field between the sheets is given as

$$B = \mu_0 j = \mu_0 \left(\frac{I}{b}\right)$$

The magnetic flux passing through the region between the two sheets of tape line of length l is given as

$$\phi = \frac{\mu_0 I}{b} l t \Rightarrow \frac{\phi}{I} = \frac{\mu_0 t}{b} l$$

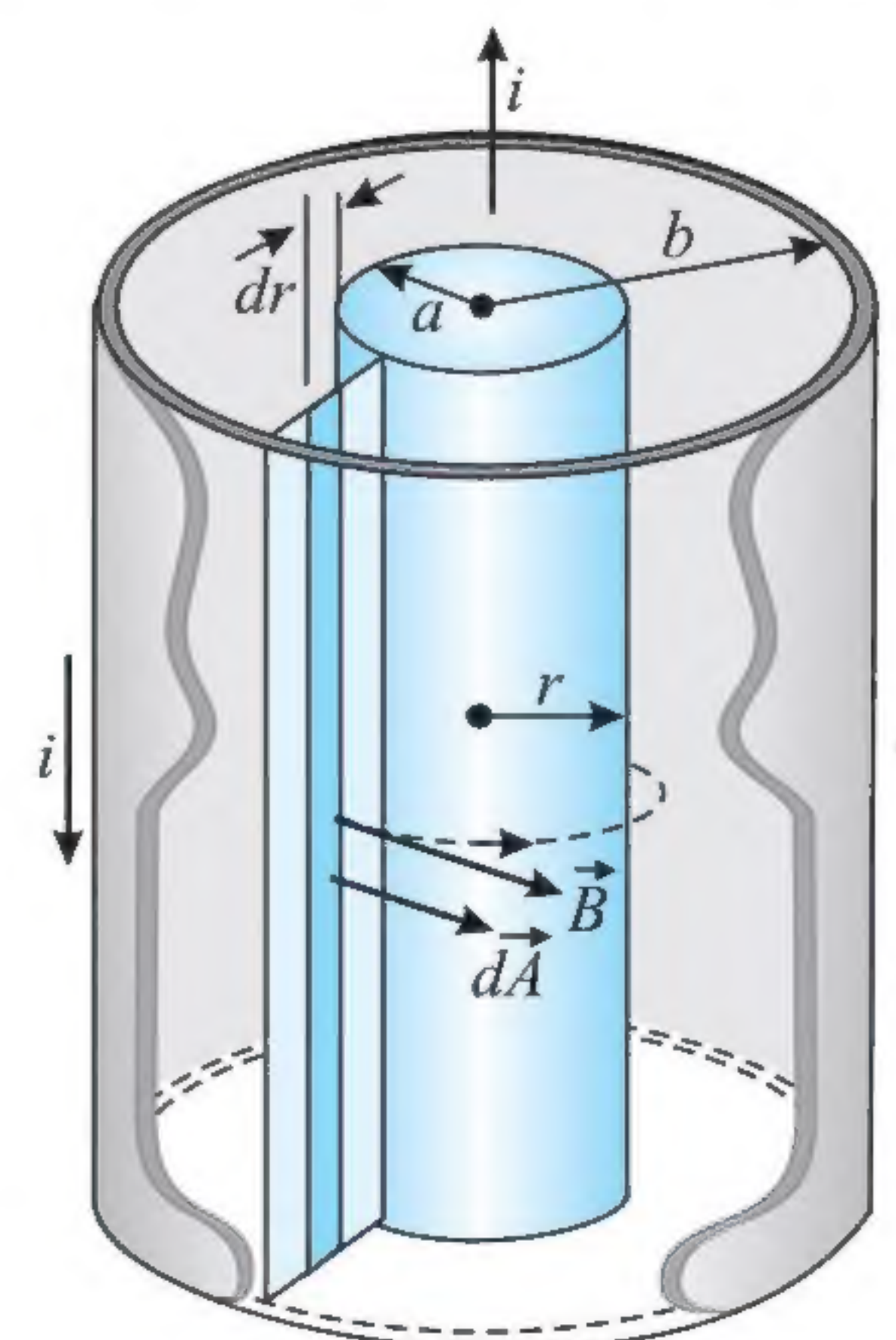
If self-inductance of the tape line, $L = \frac{\phi}{I} = \frac{\mu_0 t}{b} l$

Self-inductance per unit length of the tape, $\frac{L}{l} = \frac{\mu_0 t}{b}$

ILLUSTRATION 5.3

Coaxial cables are often used to connect electrical devices, such as your video system, and in receiving signals in television cable systems. Model a long coaxial cable as a thin, cylindrical conducting shell of radius b concentric with a solid cylinder of radius a as in figure. The conductors carry the same current I in opposite directions. Calculate the inductance L of a length l of this cable.

Sol. The current in the loop sets up a magnetic field between the inner and outer conductors that passes through this loop. If the current changes, the magnetic field changes and the induced emf opposes the original change in the current in the conductors.



The magnetic field is perpendicular to the light blue rectangle of length l and width $b - a$, the cross section of interest. Because the magnetic field varies with radial position across this rectangle, we must use calculus to find the total magnetic flux.

Divide the light blue rectangle into strips of width dr such as the darker strip in figure. Evaluate the magnetic flux through such a strip:

$$d\Phi_B = B dA = B l dr$$

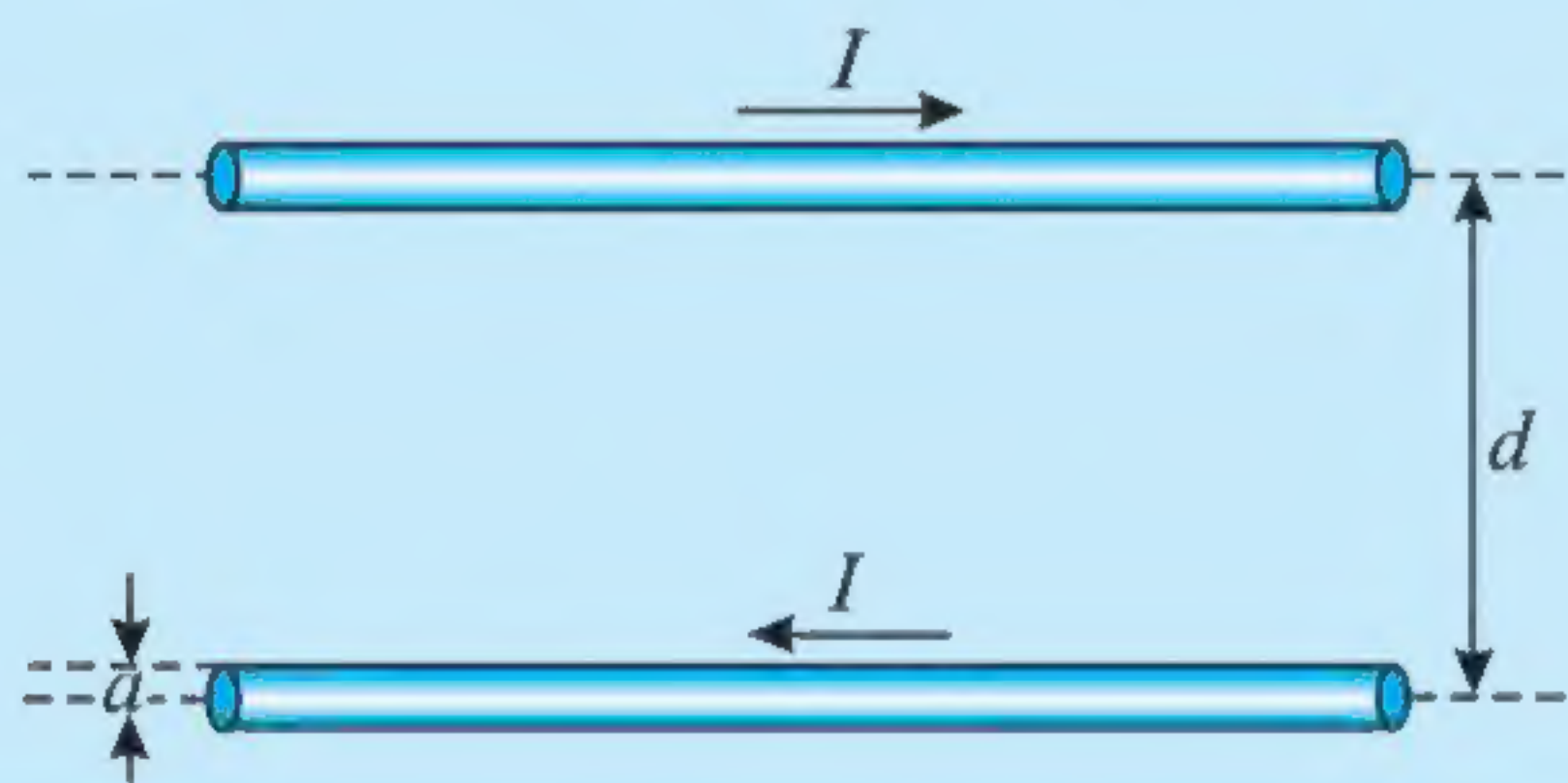
Substitute for the magnetic field and integrate over the entire light gold rectangle:

$$\Phi_B = \int_a^b \frac{\mu_0 i}{2\pi r} l dr = \frac{\mu_0 i l}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 i l}{2\pi} \ln\left(\frac{b}{a}\right)$$

Hence the inductance of the cable $L = \frac{\Phi_B}{i} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right)$

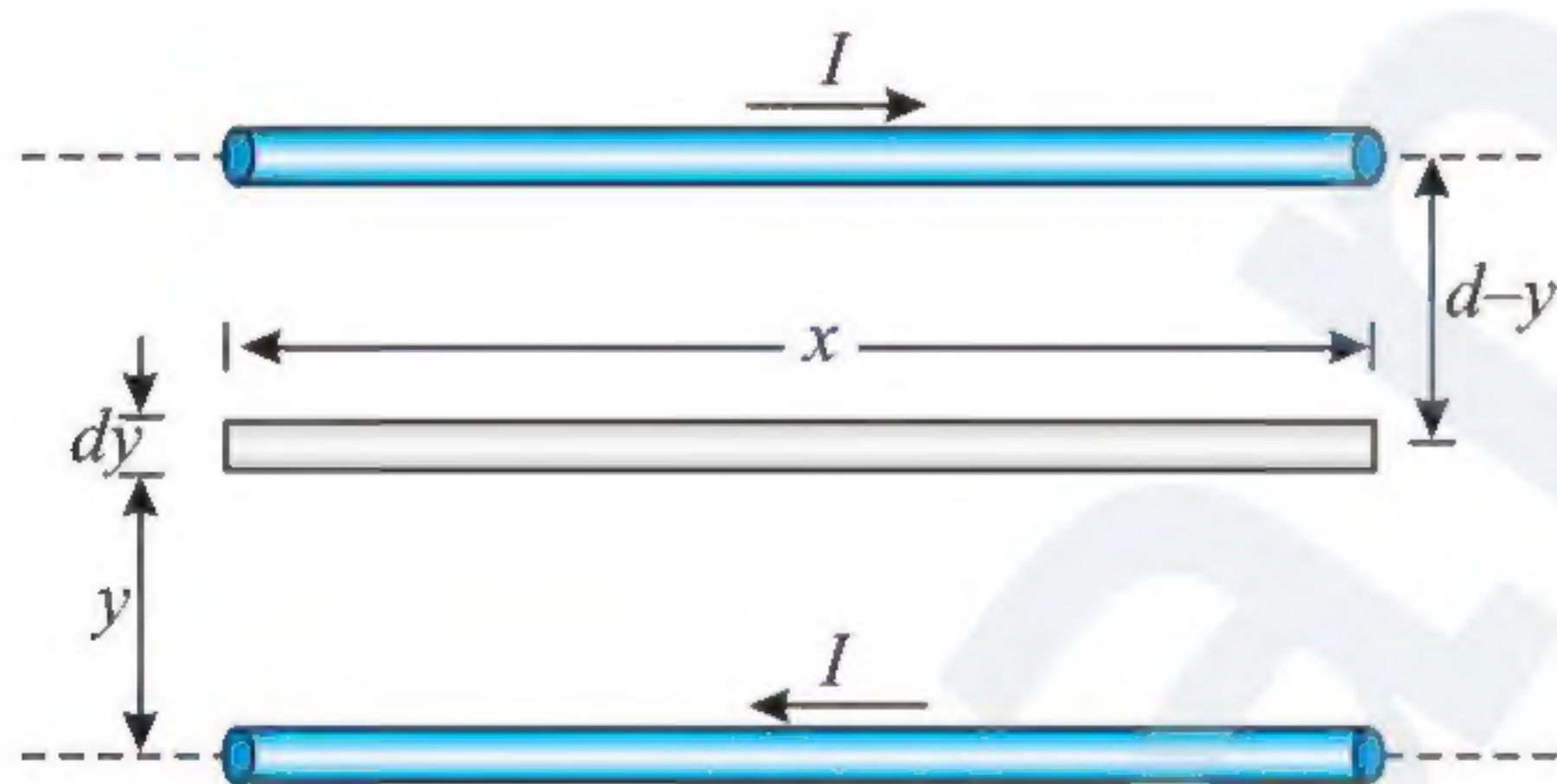
ILLUSTRATION 5.4

Two long straight cylindrical wires each of radius a are placed at a separation d . Both the wires carry equal and opposite current of magnitude I . Find the inductance (L) per unit length of this arrangement. Neglect magnetic flux inside the wires.



Sol. Let us calculate the coefficient of self-induction for this arrangement, we need to find the flux between the wires.

Let us consider a strip of width dy and length x as shown in figure.



Magnetic field at the strip due to current in two wires,

$$B = \frac{\mu_0 I}{2\pi} \left(\frac{1}{y} + \frac{1}{d-y} \right) \otimes$$

Area of the strip, $dA = x dy$

Hence the flux through the strip, $d\phi = \frac{\mu_0 I}{2\pi} \left(\frac{1}{y} + \frac{1}{d-y} \right) x dy$

Flux linked with the area between two wires in a length x will be

$$\begin{aligned} \phi &= \frac{\mu_0 I x}{2\pi} \left[\int_a^{d-a} \frac{dy}{y} + \int_a^{d-a} \frac{dy}{d-y} \right] \\ &= \frac{\mu_0 I x}{2\pi} \left[\ln\left(\frac{d-a}{a}\right) - \ln\left(\frac{d-d+a}{d-a}\right) \right] \\ &= \frac{\mu_0 I x}{\pi} \ln\left(\frac{d-a}{a}\right) \end{aligned}$$

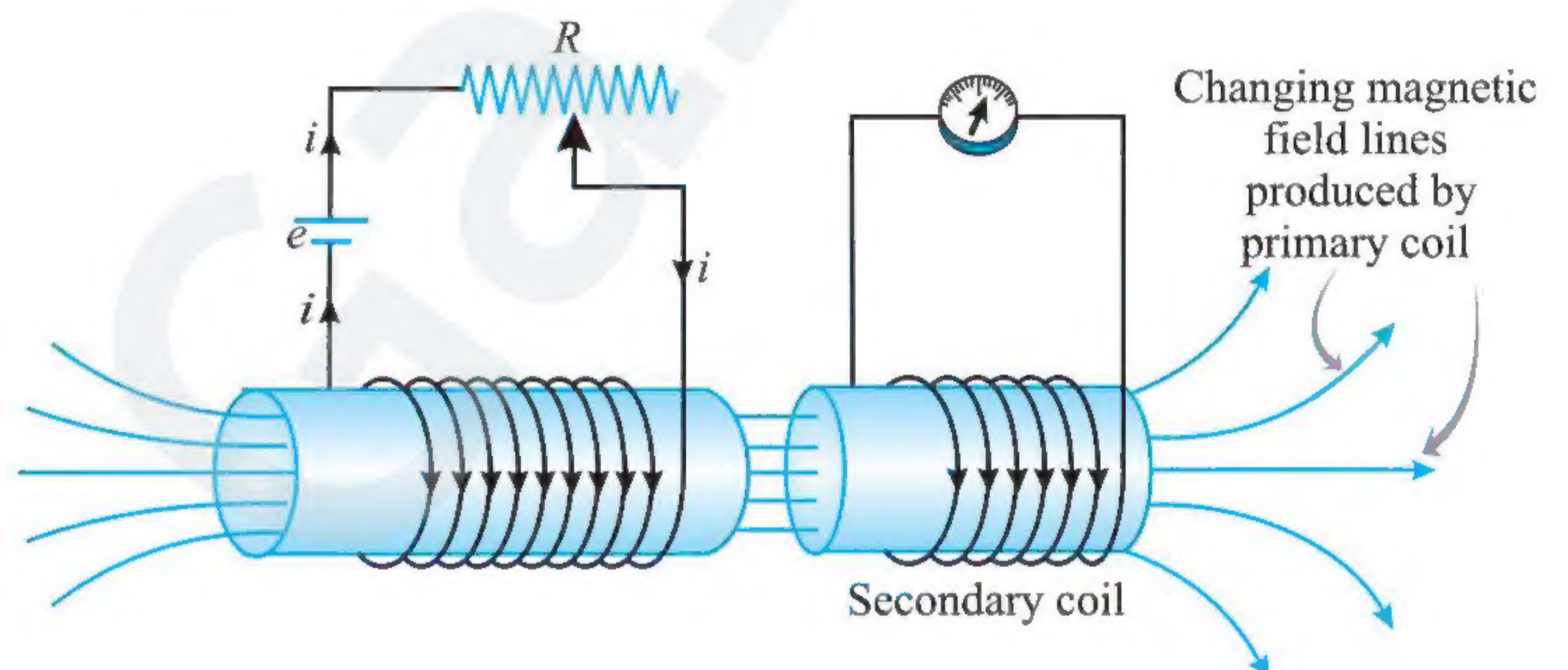
$$\therefore L = \frac{\phi}{I} = \frac{\mu_0 x}{\pi} \ln\left(\frac{d-a}{a}\right)$$

Hence the inductance (L) per unit length of this arrangement,

$$\frac{L}{x} = \frac{\mu_0}{\pi} \ln\left(\frac{d-a}{a}\right)$$

MUTUAL INDUCTANCE

In this section we will analyze the case of two interacting coils, if two coils are close together, a steady current i in one coil will set up a magnetic flux ϕ through the other coil (linking the other coil). If we change i with time, this condition induces an emf through a process known as mutual induction, so named because it depends on the interaction of two circuits.



Consider the two closely wound coils of wire, called the primary coil, and the secondary coil, are placed close to each other. The current i_1 in primary coil, which has N_1 turns, creates a magnetic field. Some of the magnetic field lines pass through secondary coil, which has N_2 turns. The magnetic flux caused by the current in primary coil and passing through secondary coil is represented by Φ_{12} . In analogy to Coefficient of Self Induction, we can identify the 'Coefficient of mutual inductance' M_{12} of coil 2 with respect to coil 1:

$$M_{12} = \frac{N_2 \Phi_{12}}{i_1} \quad \dots(i)$$

Mutual inductance depends on the geometry of both circuits and on their orientation with respect to each other. As the circuit separation distance increases, the mutual inductance decreases because the flux linking the circuits decreases.

If the current i_1 varies with time, we see from Faraday's law and Eq. (i) that the emf induced by primary coil in secondary coil is

$$e_{12} = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt} \left(\frac{M_{12} i_1}{N_2} \right) = -M_{12} \frac{di_1}{dt} \quad \dots(ii)$$

In the preceding discussion, it was assumed the current is in primary coil. Let's also imagine a current i_2 in secondary coil. The preceding discussion can be repeated to show that there is a mutual inductance M_{21} . If the current i_2 varies with time, the emf induced by secondary coil in primary coil is

$$e_{21} = -M_{21} \frac{di_2}{dt} \quad \dots(iii)$$

In mutual induction, the emf induced in one coil is always proportional to the rate at which the current in the other coil is changing. Although the proportionality constants M_{12} and M_{21} have been treated separately, it can be shown that they are equal.

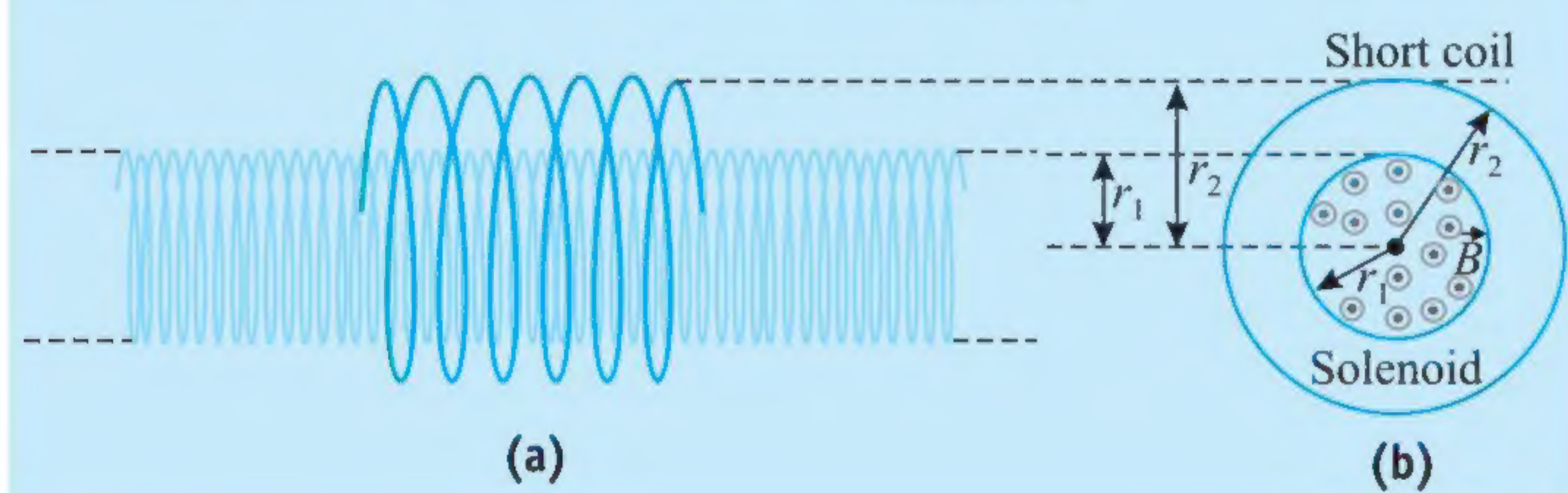
Therefore, with $M_{12} = M_{21} = M$, Eqs. (ii) and (iii) become

$$e_2 = -M \frac{di_1}{dt} \quad \text{and} \quad e_1 = -M \frac{di_2}{dt} \quad \dots(\text{iv})$$

These two equations are similar in form to equation for the self-induced emf $e = -L \left(\frac{di}{dt} \right)$. The direction of mutually induced EMF in either coil will be such that it opposes the change in current in the other coil. The SI unit of mutual inductance is the henry. The unit of mutual inductance is the henry.

ILLUSTRATION 5.5

A long solenoid with circular cross section of radius r_1 and n turns per unit length is inside and coaxial with a short coil with circular cross section of radius r_2 and N turns (figure). The current in the solenoid is increased at a constant rate from zero to i over a time interval of ' T '. Find the mutual inductance between the two coils and the potential difference induced in the short coil while the current is changing.



Sol. The potential difference induced in the short coil is due to the changing current flowing in the solenoid.

The mutual inductance between the coil and the solenoid,

$$M = \frac{N_c \Phi_{s \rightarrow c}}{i}, \quad \dots(\text{i})$$

where N_c is the number of turns in the short coil, $N_c \Phi_{s \rightarrow c}$ is the flux linkage in the coil resulting from the magnetic field of the solenoid, and i is the current in the solenoid. The flux can be expressed as

$$\Phi_{s \rightarrow c} = BA = (\mu_0 ni) (\pi r_1^2) \quad \dots(\text{ii})$$

where B is the magnitude of the magnetic field inside the solenoid and A is its cross-sectional area.

For a solenoid, the magnitude of the magnetic field is $B = \mu_0 ni$, where n is the number of turns per unit length. The cross-sectional area of the solenoid is $A = \pi r_1^2$.

We can combine Eqs. (i) and (ii), to obtain the mutual inductance between the two coils:

$$M = \frac{NBA}{i} = \frac{N(\mu_0 ni)(\pi r_1^2)}{i} = N\pi\mu_0 nr_1^2.$$

The potential difference induced in the short coil is ,

$$e_c = -M \left(\frac{di}{dt} \right) = -(N\pi\mu_0 nr_1^2) \frac{di}{dt} \quad \dots(\text{iii})$$

The rate of change in the current is constant,

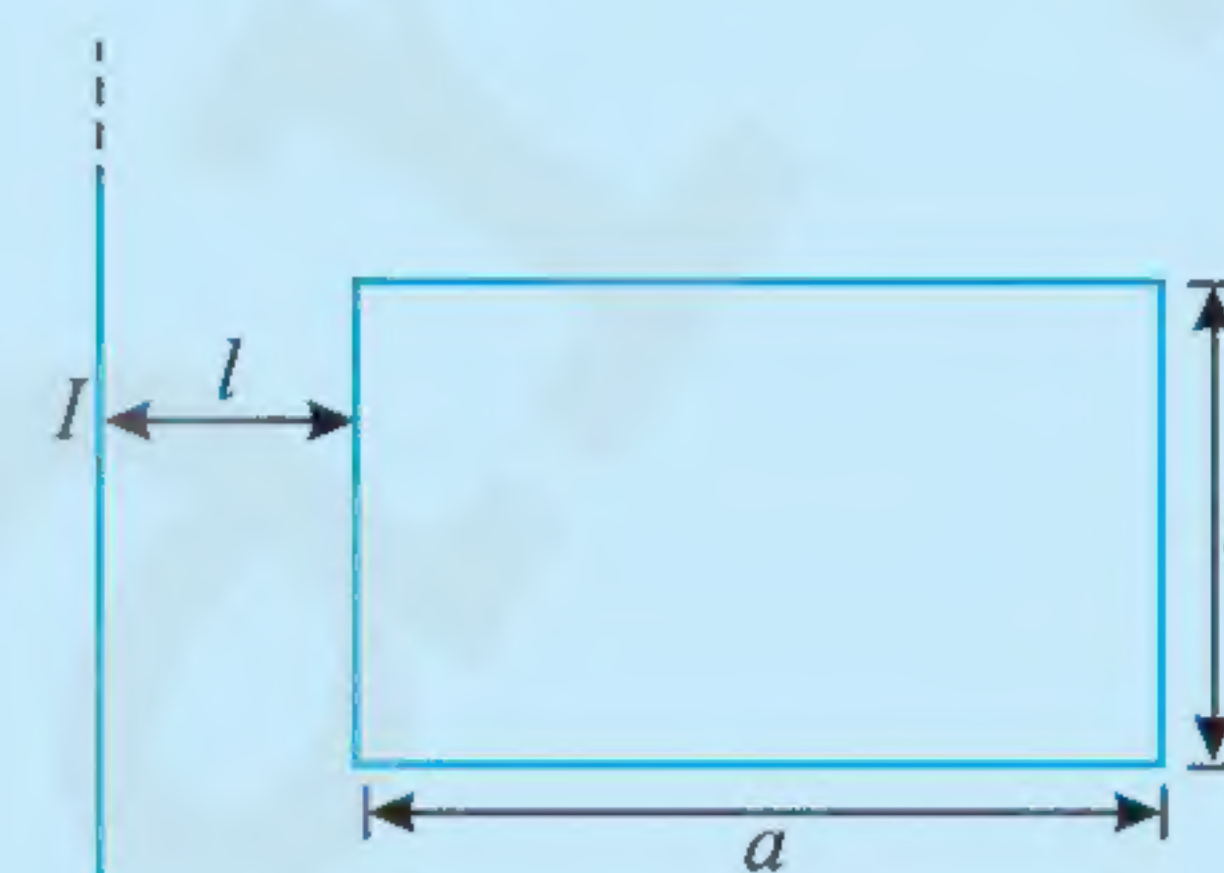
$$\frac{di}{dt} = \frac{(i-0)}{T} = \frac{i}{T}. \quad \dots(\text{iv})$$

From (iii) and (iv), the magnitude of the potential difference induced in the short coil,

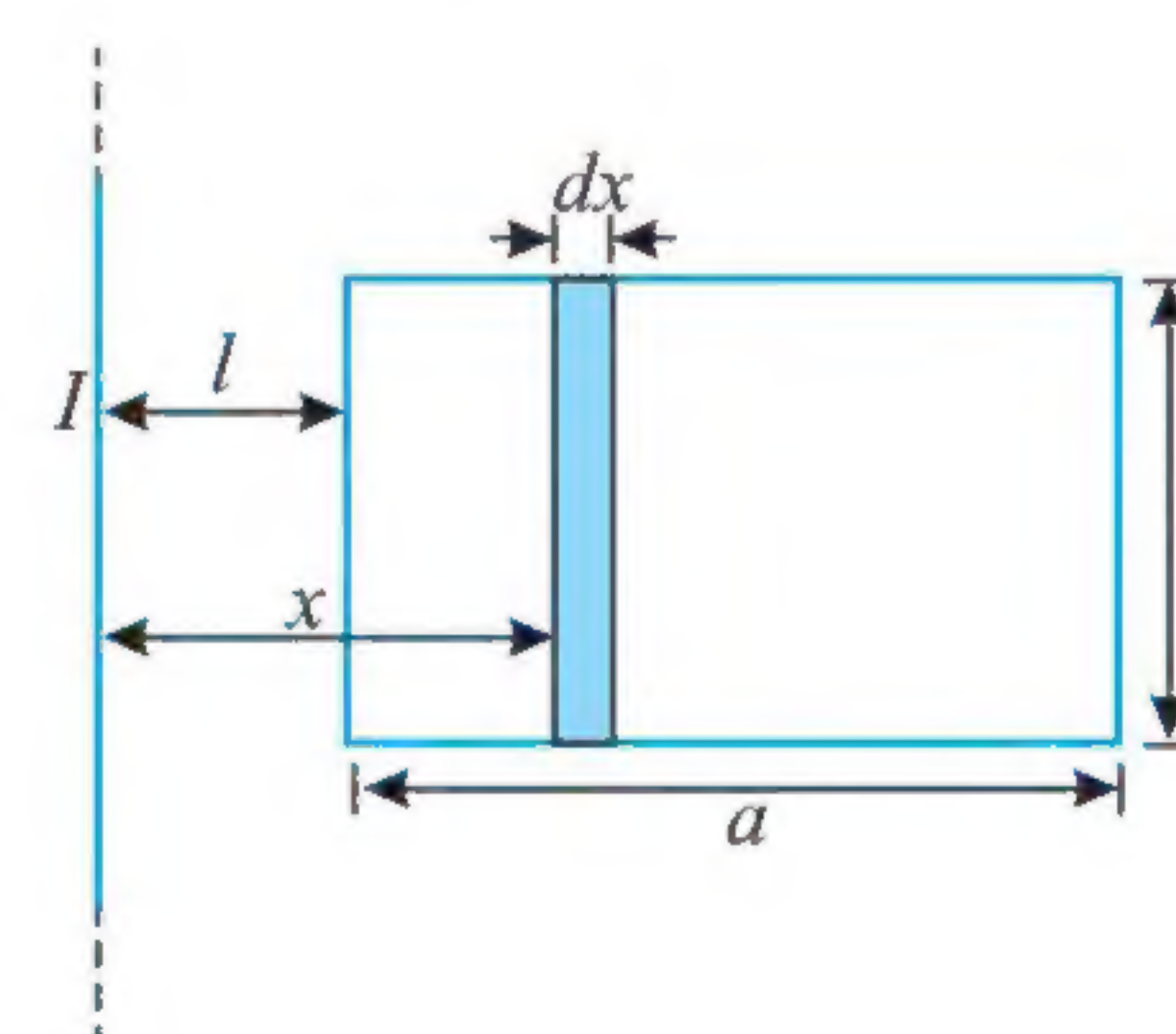
$$|e_c| = (N\pi\mu_0 nr_1^2) \frac{i}{T}$$

ILLUSTRATION 5.6

A rectangular wire frame with side a and b is placed near a long straight wire and a rectangular frame with side a and b as shown in figure. The straight wire and the wire frame lie in the same plane. Calculate the mutual inductance of a long straight wire and a rectangular frame.



Sol. For finding the mutual inductance between the wire and the frame, let us supply a current I through the wire and calculate the magnetic flux through the frame. For flux calculation, we consider an elemental strip in the frame of width dx at a distance x from wire as shown in figure.



The magnetic flux through the elemental strip is given as

$$d\phi = \left(\frac{\mu_0 I}{2\pi x} \right) b dx$$

$$\Rightarrow \phi = \int_l^{a+l} \left(\frac{\mu_0 I}{2\pi x} \right) b dx = \frac{\mu_0 Ib}{2\pi} \int_l^{a+l} \frac{dx}{x}$$

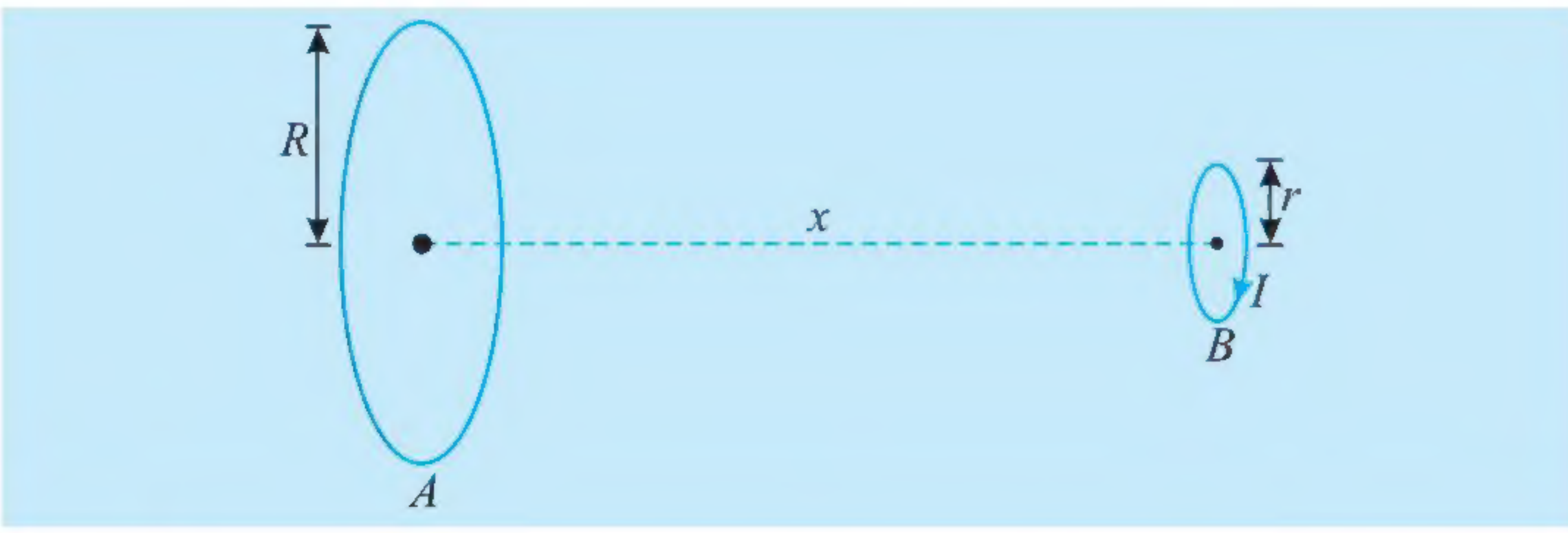
$$\Rightarrow \phi = \frac{\mu_0 Ib}{2\pi} [\ln x]_l^{a+l} = \frac{\mu_0 Ib}{2\pi} \ln \left(1 + \frac{a}{l} \right)$$

Hence the mutual inductance between the two is given as

$$M = \frac{\phi}{I} = \frac{\mu_0 b}{2\pi} \ln \left(1 + \frac{a}{l} \right)$$

ILLUSTRATION 5.7

Two coaxial coils of radii r and R ($R \gg r$) placed at large separation x ($x \gg R$). Calculate the magnetic flux passing through coil L_1 due to a current I in coil L_2 .



Sol. Let M be the coefficient of mutual induction between the two coils then magnetic flux through coil L_1 due to current in coil L_2 is given as

$$\phi_1 = MI \quad \dots(i)$$

Let us calculate the coefficient of mutual induction between the two coils and for this we find the flux through coil L_2 if a current I_1 flow in coil L_1 as shown in figure.

The magnetic field at the center of coil L_2 due to current in coil L_1 is given as

$$B_2 = \frac{\mu_0 I_1 R^2}{2(x^2 + R^2)^{3/2}}$$

As the size of the coil is small, we can assume uniform magnetic field inside the loop. The magnetic flux through coil L_2 due to above magnetic induction is given as

$$\phi_2 = B_2 \cdot \pi r^2 \Rightarrow \phi_2 = \frac{\mu_0 \pi r^2 R^2}{2(x^2 + R^2)^{3/2}} \cdot I_1$$

The coefficient of mutual inductance, $M = \frac{\phi_2}{I_1} = \frac{\mu_0 \pi r^2 R^2}{2(x^2 + R^2)^{3/2}}$

Above is the mutual inductance coefficient for the pair of these coil which can be used in calculation of flux from either coil due to current in other coil so from reciprocity property, we have

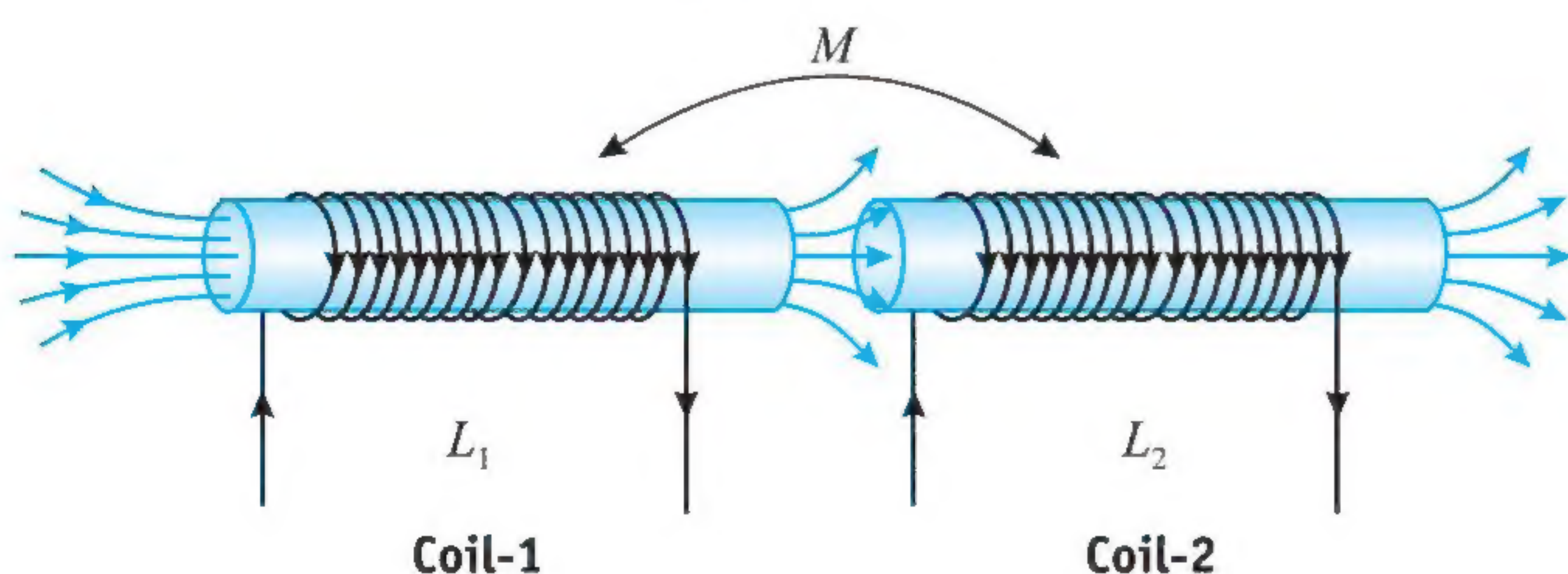
$$\phi_1 = MI_2 = \frac{\mu_0 \pi r^2 R^2 I}{2(x^2 + R^2)^{3/2}}$$

TOTAL FLUX LINKED TO A PAIR OF COILS

If two current carrying coils are placed near to each other, for calculation of total flux, we need to consider self-induction of individual coils and mutual induction between the coils. Let two coils having their self-induction coefficients L_1 and L_2 and mutual induction coefficient for the pair M are kept close to each other, two coils carry currents i_1 and i_2 respectively.

The total magnetic flux linked with either coil will be the sum of magnetic flux linked with the coil due to both self-induction and mutual induction. Here we consider two cases

(i) **When flux of the two inductors are in same direction**



If the magnetic flux due to both coils are in same direction as shown in figure, then the net flux linked with coil-1 and coil-2 are given as

$$\phi_1 = L_1 i_1 + M i_2 \quad \dots(i)$$

$$\phi_2 = L_2 i_2 + M i_1 \quad \dots(ii)$$

If currents in the coils start changing, then EMF induced in the two coils are given as

$$e_1 = \frac{d\phi_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad \dots(iii)$$

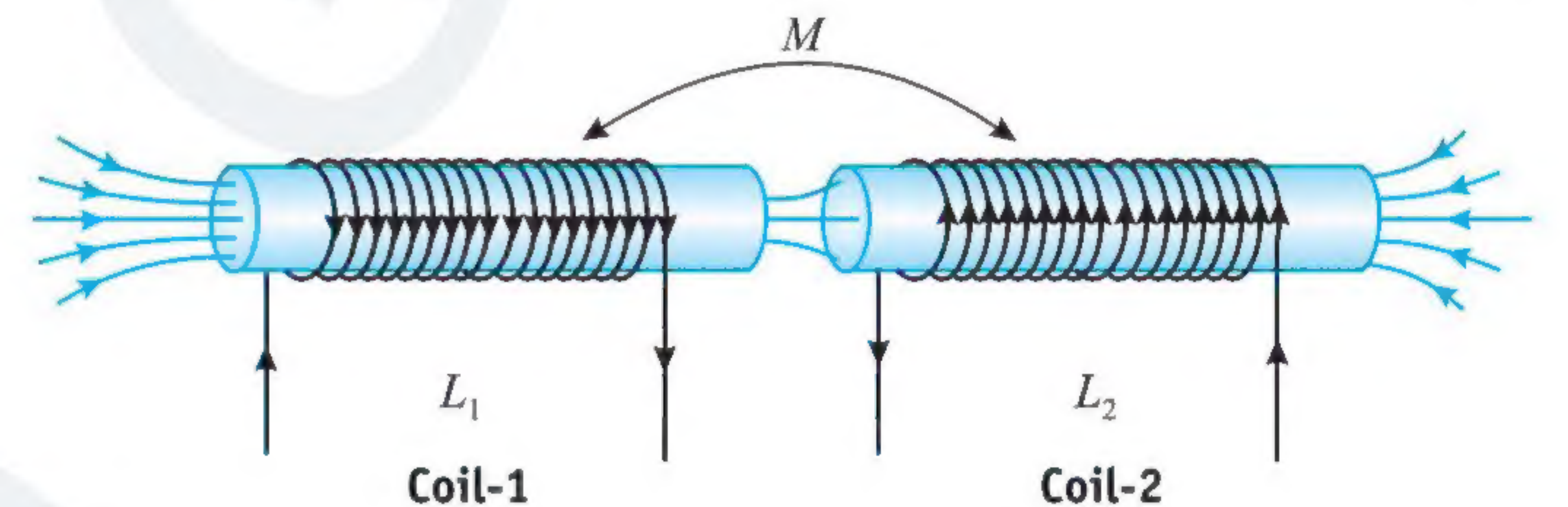
$$\text{and } e_2 = \frac{d\phi_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad \dots(iv)$$

(ii) **When the flux of the two inductors are in opposite direction**

In this case the net magnetic flux linked with the two coils are given as

$$\phi_1 = L_1 i_1 - M i_2 \quad \dots(v)$$

$$\phi_2 = L_2 i_2 - M i_1 \quad \dots(vi)$$



If current in above coils starts changing then EMF induced in the two coils are given as

$$e_1 = \frac{d\phi_1}{dt} = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \quad \dots(vii)$$

$$\text{and } e_2 = \frac{d\phi_2}{dt} = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \quad \dots(viii)$$

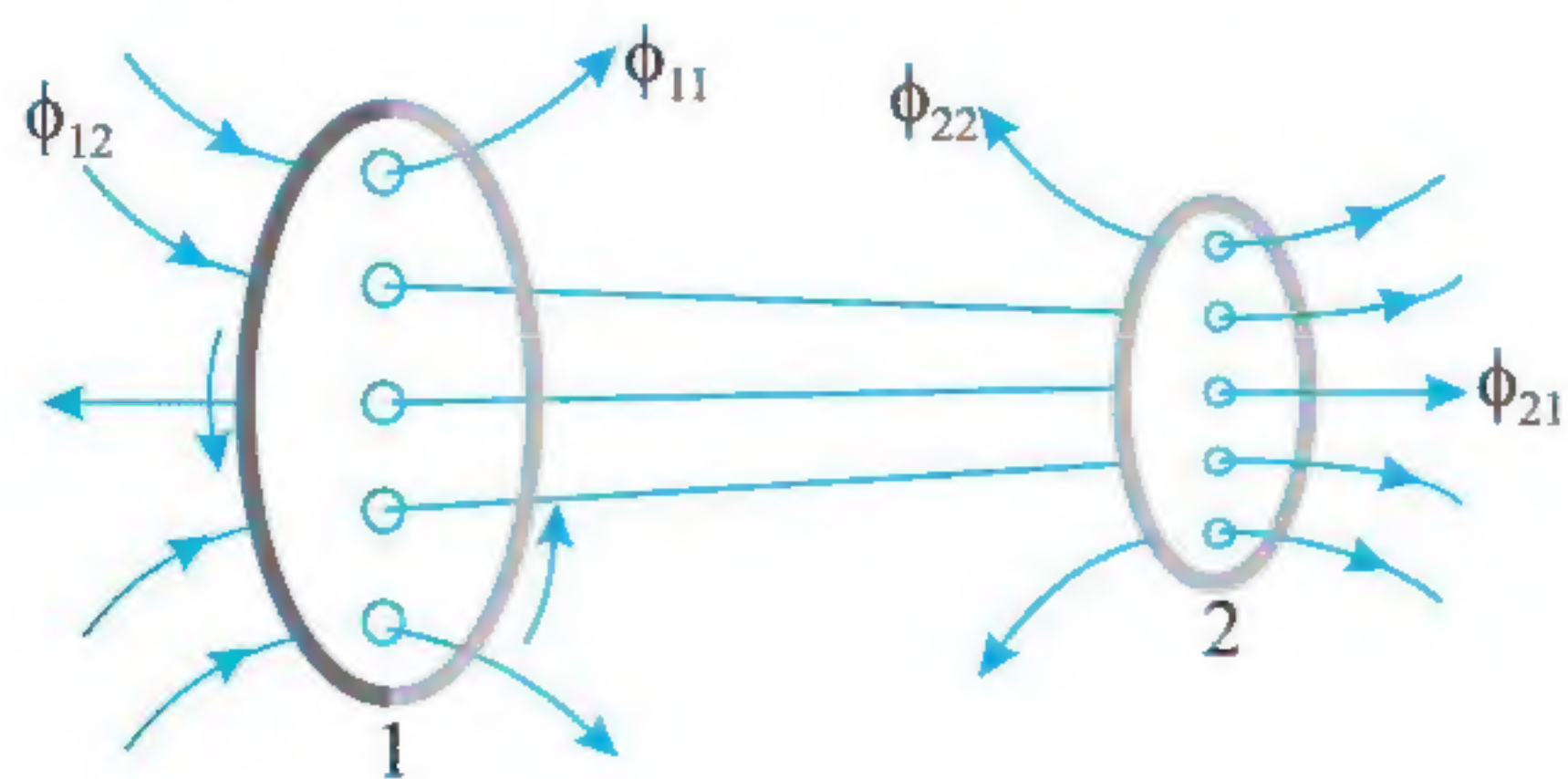
CALCULATION OF MUTUAL INDUCTANCE BETWEEN TWO COILS FROM THEIR SELF-INDUCTANCE

Suppose, two coils carry currents i_1 and i_2 respectively. Then, their self-inductances can be given as $L_1 = \frac{\phi_{11}}{i_1}$ and $L_2 = \frac{\phi_{22}}{i_2}$

respectively. Similarly, their mutual inductance $M = \frac{\phi_{12}}{i_2} = \frac{\phi_{21}}{i_1}$.

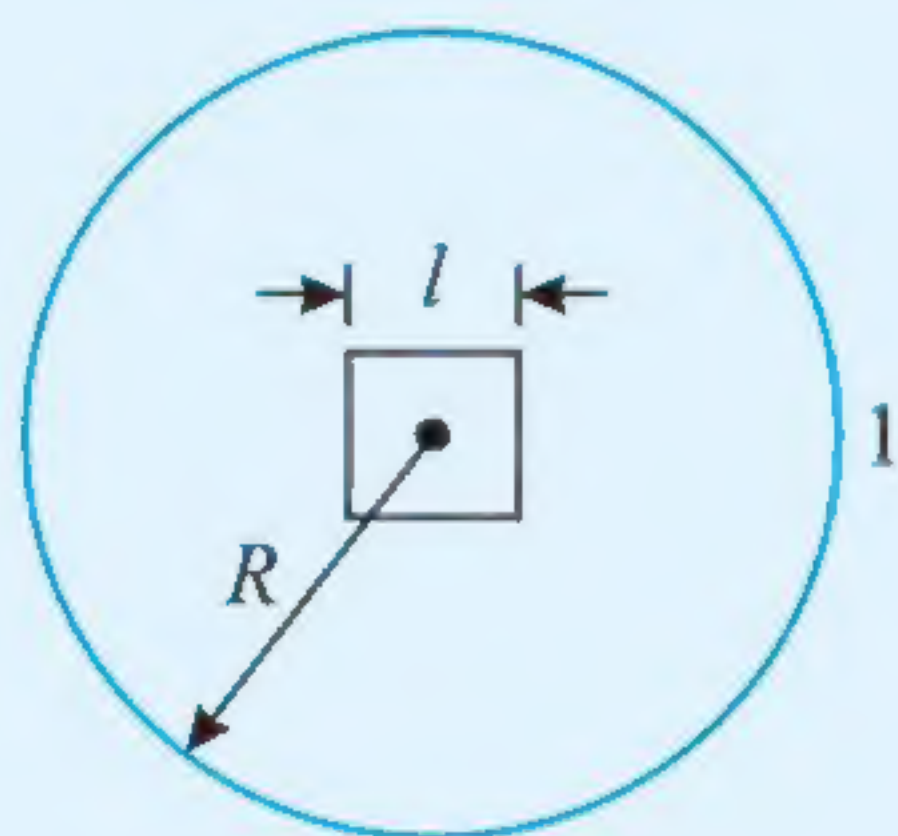
When the coils are placed very close side by side, they can interlink a maximum flux. In that case, there is no difference between self-flux and mutual flux; then $\phi_{11} = \phi_{12} = \phi_{21} = \phi_{22}$. Hence, $L_1 L_2$ will be equal to M^2 . In other words, maximum mutual inductance is equal to the geometric mean of the self-inductances of the coils, that is, $M = \sqrt{L_1 L_2}$ more generally, $M \leq \sqrt{L_1 L_2}$ in most practical cases because a fraction of the (self) flux produced by one coil is linked with the other. We call this fraction K "coefficient of flux linkage," which is given as $K = \frac{\text{mutual flux}}{\text{self flux}}$; $0 \leq K \leq 1$.

Then we have $M = K \sqrt{L_1 L_2}$, where K can also be termed as "coefficient of coupling" of magnetic flux between any two circuits.

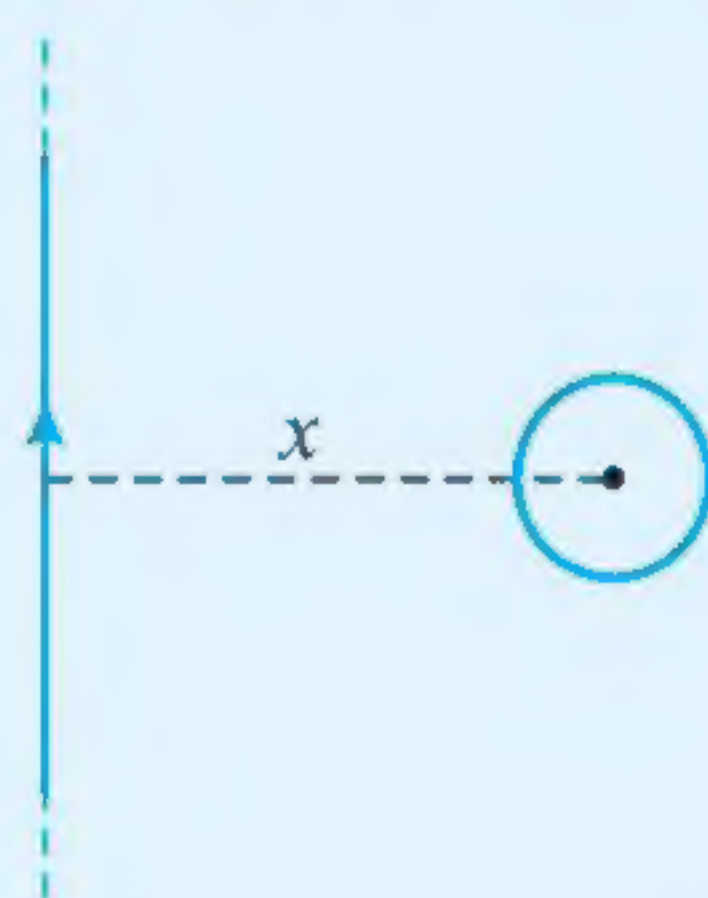


CONCEPT APPLICATION EXERCISE 5.1

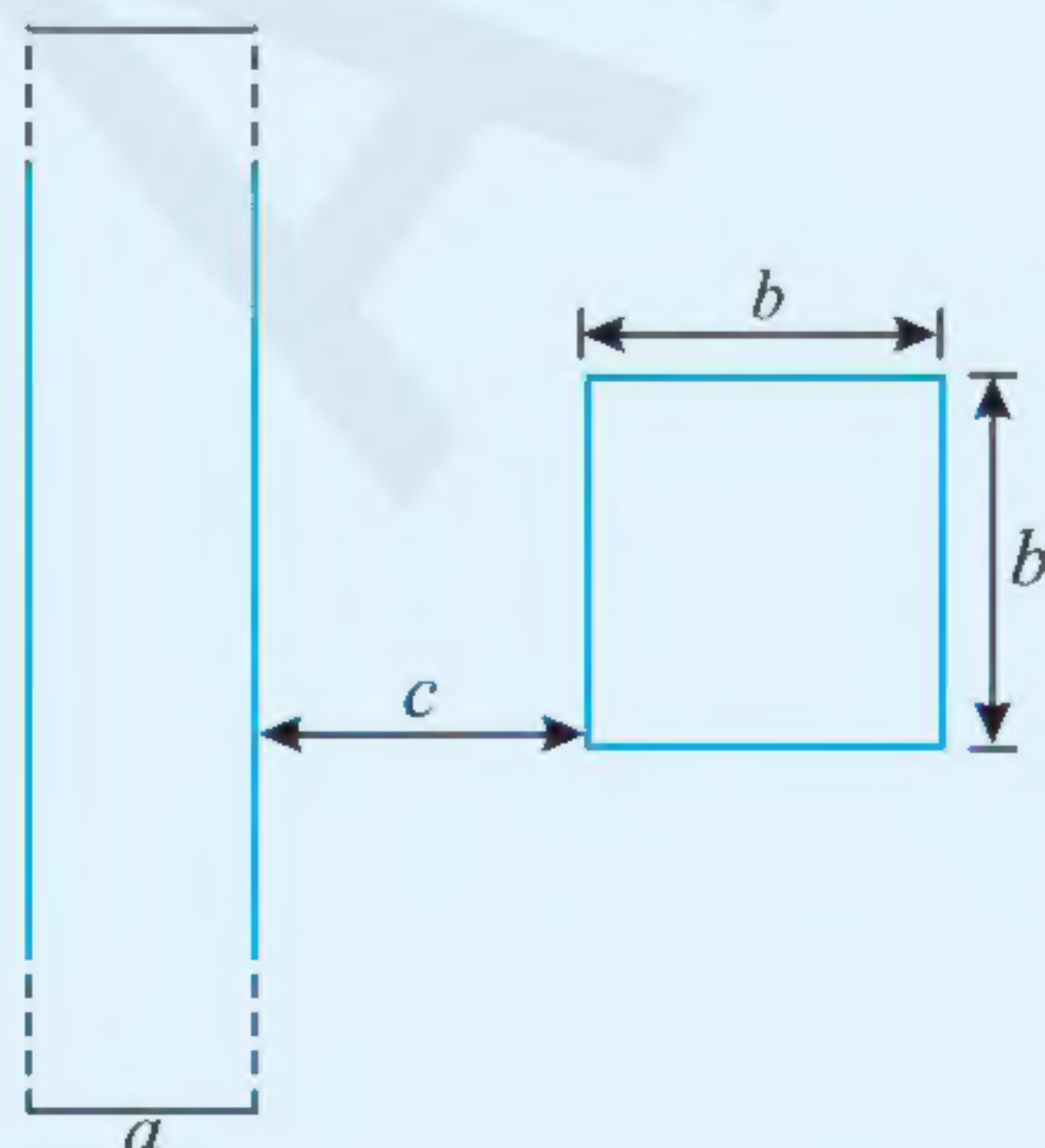
1. An inductor coil of negligible resistance drops a voltage of 6 V when the current increases at a rate of 600 A/s. Find the inductance of the coil.
2. A coil has inductance of 4 mH. If the current through the coil changes from 0.25 amp to 2 amp during 0.5 s, find the average voltage dropped across the inductor.
3. A square loop of side $l = 1$ cm is placed at the center of a circular loop of radius $R = 0.25$ m as shown in figure. Find the mutual inductance between the circular loop and square loop.



4. Find the mutual inductance between the long straight conductor and a small loop of area 10^{-8} m^2 placed at a distance of separation $x = 0.5$ m as shown in figure.



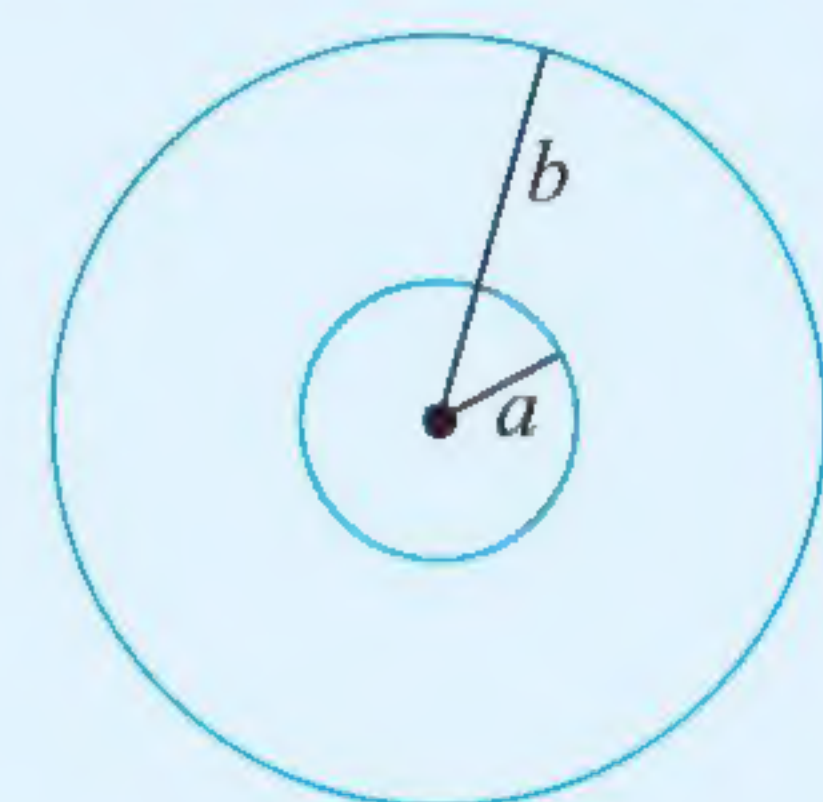
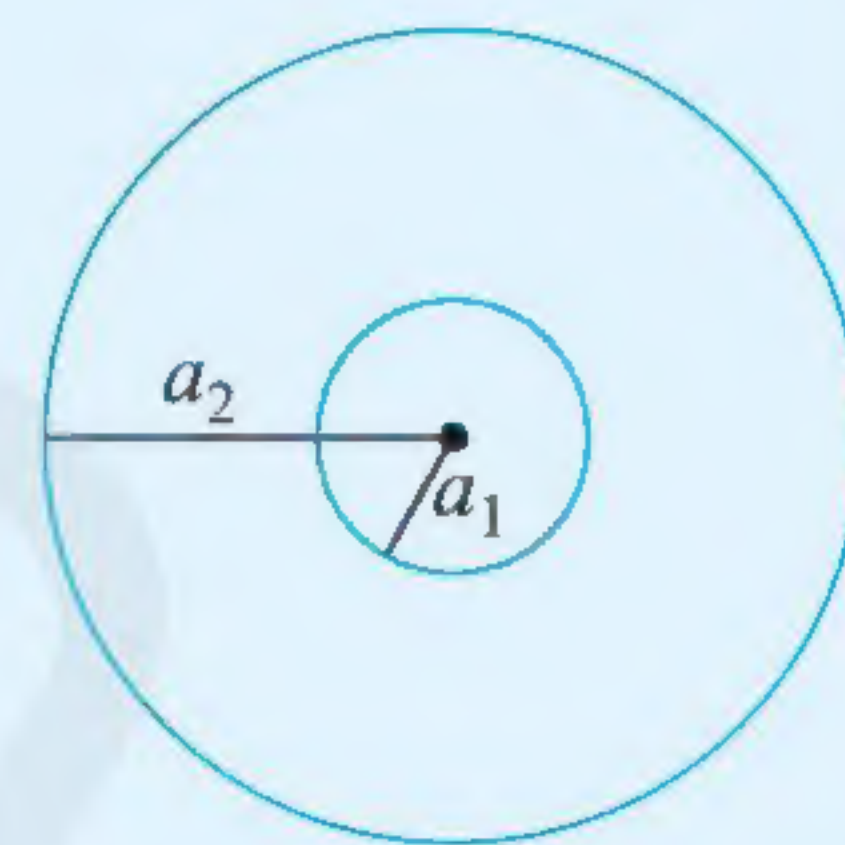
5. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($\gg l$). The loops are coplanar and their centers coincide. What is the mutual inductance of the system?
6. A finite (square) loop is placed near an infinite loop as shown in figure. Find the mutual inductance between two rectangular loops.



7. Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common

air-filled core. Let l be the length of the core, A the cross-sectional area of the core, N_1 the number of times the first wire is wound around the core, and N_2 the number of times the second wire is wound around the core. Find the mutual inductance of the two solenoids, neglecting the end effects.

8. Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 \ll a_2$) if the planes of the coils are same.
9. Solve problem 8 if the planes of the coils are perpendicular.
10. Solve problem 8 if the planes of the coils make an angle θ with each other.
11. Figure shows two concentric coplanar coils with radii a and b ($a \ll b$). A current $i = 2t$ flows in the smaller loop. Neglecting self-inductance of the larger loop,
 - (a) find the mutual inductance of the two coils;
 - (b) find the emf induced in the larger coil;
 - (c) if the resistance of the larger loop is R , find the current in it as a function of time.



ANSWERS

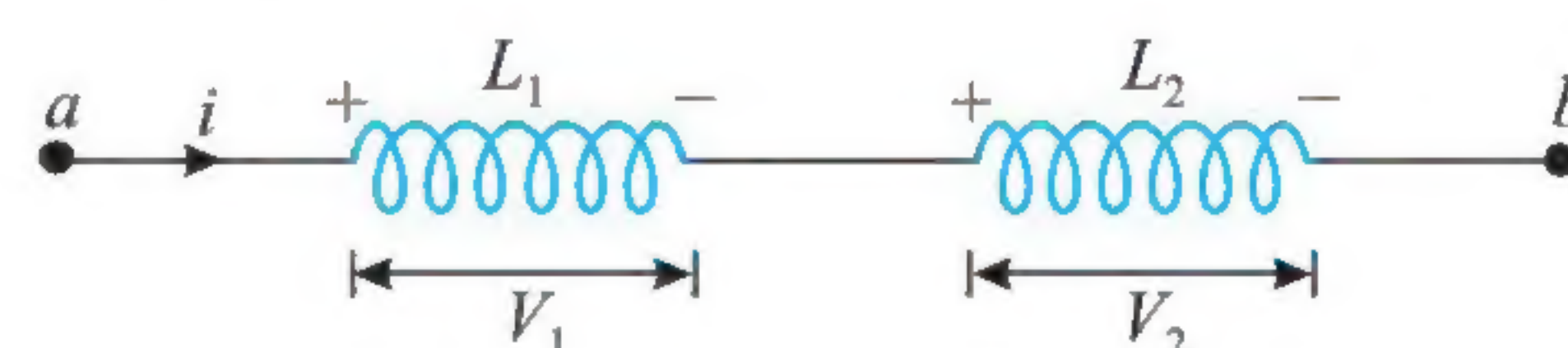
1. 10^{-2} H
2. 14 mV
3. $8\pi \times 10^{-11} \text{ H}$
4. $4 \times 10^{-15} \text{ H}$
5. $\frac{2\sqrt{2} \mu_0 l^2}{\pi l}$
6. $\frac{\mu_0 b}{2\pi} \ln \frac{(c+b)(a+c)}{c(a+b+c)}$
7. $\frac{\mu_0 N_1 N_2 A}{l}$
8. $\frac{\mu_0 \pi a_1^2}{2a_2}$
9. 0
10. $\frac{\mu_0 \pi a_1^2 \cos \theta}{2a_2}$
11. (a) $\frac{\mu_0}{2b} \pi a^2$ (b) $\frac{\mu_0 \pi a^2}{b}$ (c) $\frac{\mu_0 \pi a^2}{bR}$

COMBINATION OF INDUCTORS

SERIES COMBINATION

Non-Interacting Coils

When the coils are electromagnetically isolated and connected in series, only the self-induced voltages are taken into account. For an increasing current, a voltage is induced across each coil. Then the total voltage drop V_{ab} is equal to the summation of induced voltages due to each coil.



$$V_{ab} = V_1 + V_2 + \dots + V_n \quad \dots(i)$$

If we choose a coil of inductance L so as to induce same voltage V_{ab} for same variation of current, we can call it as equivalent inductance.

Substituting $V_{ab} = L \frac{di}{dt}$, $V_1 = L_1 \frac{di}{dt}$, $V_2 = L_2 \frac{di}{dt}$, ..., $V_n = L_n \frac{di}{dt}$,

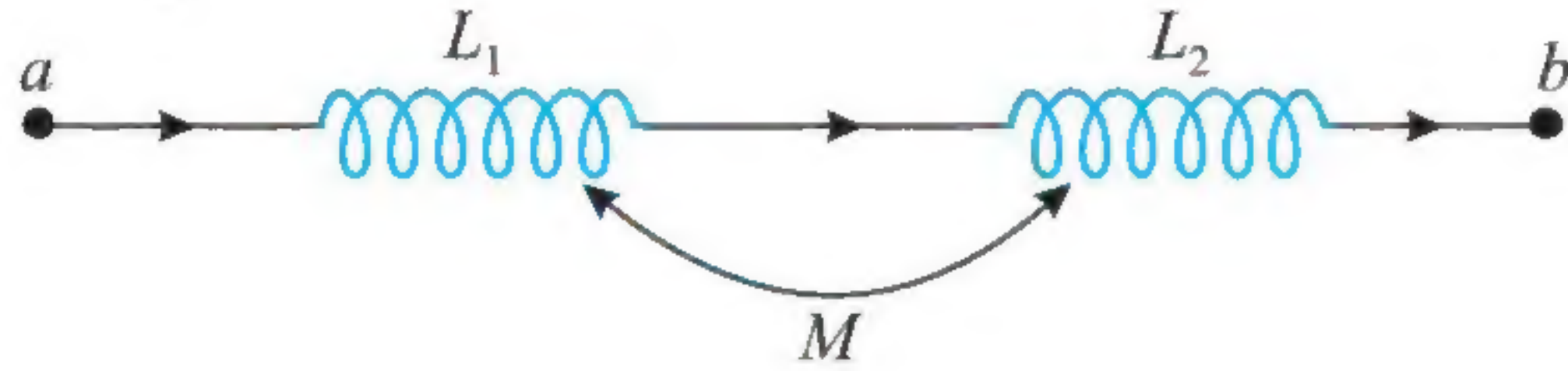
in Eq. (i) etc., we get

$$L \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \dots + L_n \frac{di}{dt}$$

$$L = L_1 + L_2 + \dots + L_n$$

Interacting Coils

If inductor coils in series with flux of one coil favors the other. We have to take the mutually induced voltage in addition to self-induced voltage.



The total induced voltage across the coils of inductances L_1 and L_2 are given as

$$V_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}, \quad \dots(i)$$

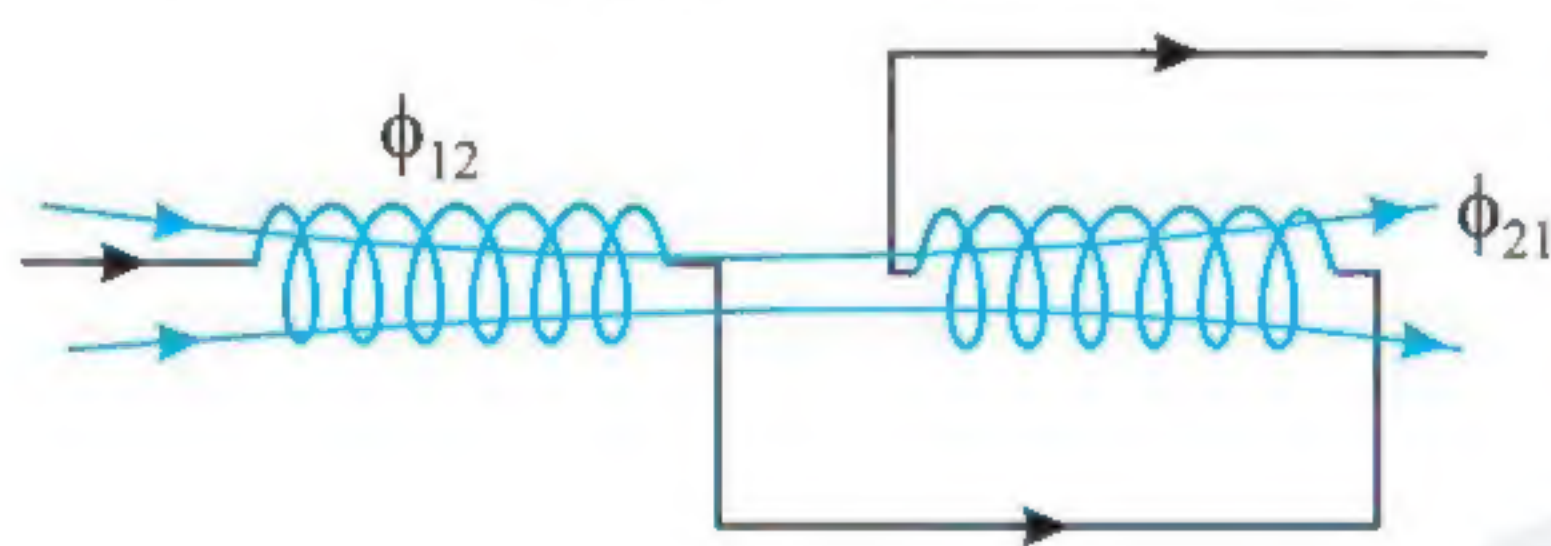
$$V_2 = L_2 \frac{di}{dt} + M \frac{di}{dt} \quad \dots(ii)$$

Then, for getting total voltage drop across the combination, we need to add Eqs. (i) and (ii)

$$V_{ab} = V_1 + V_2 = (L_1 + L_2 + 2M) \frac{di}{dt}$$

It gives the equivalent inductance $L = \frac{V_{ab}}{di/dt} = L_1 + L_2 + 2M$

When the flux of one coil opposes that of the other,



$$V_1 = L_1 \frac{di}{dt} - M \frac{di}{dt} \quad \text{and} \quad V_2 = L_2 \frac{di}{dt} - M \frac{di}{dt}$$

Then following the above argument, the equivalent inductance can be given as $L = L_1 + L_2 - 2M$. Generalizing the above results we can write the equivalent inductance of two interacting coils connected in series as,

$$L = L_1 + L_2 \pm 2M$$

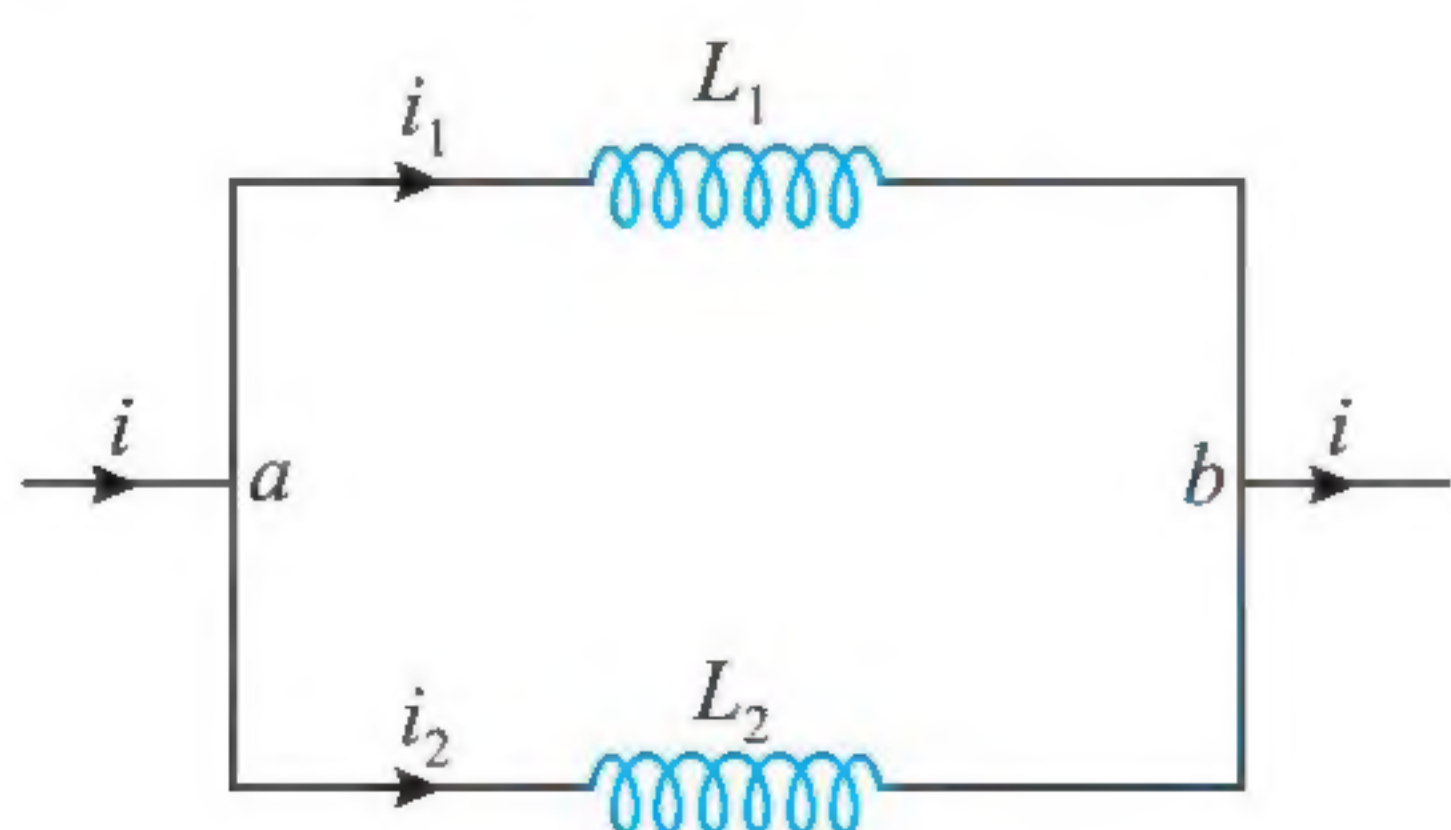
When there is no interaction between the coils, M is equal to zero. Then the equivalent inductance is given as,

$$L = L_1 + L_2$$

PARALLEL COMBINATION

For parallel combination of the inductors, voltage between the common terminals a and b of the inductors is given as

$$V_{ab} = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$$



Application of Kirchhoff's first law yields $i = i_1 + i_2$... (i)

Differentiating Eq. (i) both sides with respect to time,

$$\text{we obtain } \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}; \quad \dots(ii)$$

Substituting $\frac{di_1}{dt} = \frac{V_{ab}}{L_1}$ and $\frac{di_2}{dt} = \frac{V_{ab}}{L_2}$ in Eq. (ii) we have

$$\frac{di}{dt} = \frac{V_{ab}}{L_1} + \frac{V_{ab}}{L_2} = V_{ab} \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \quad \dots(iii)$$

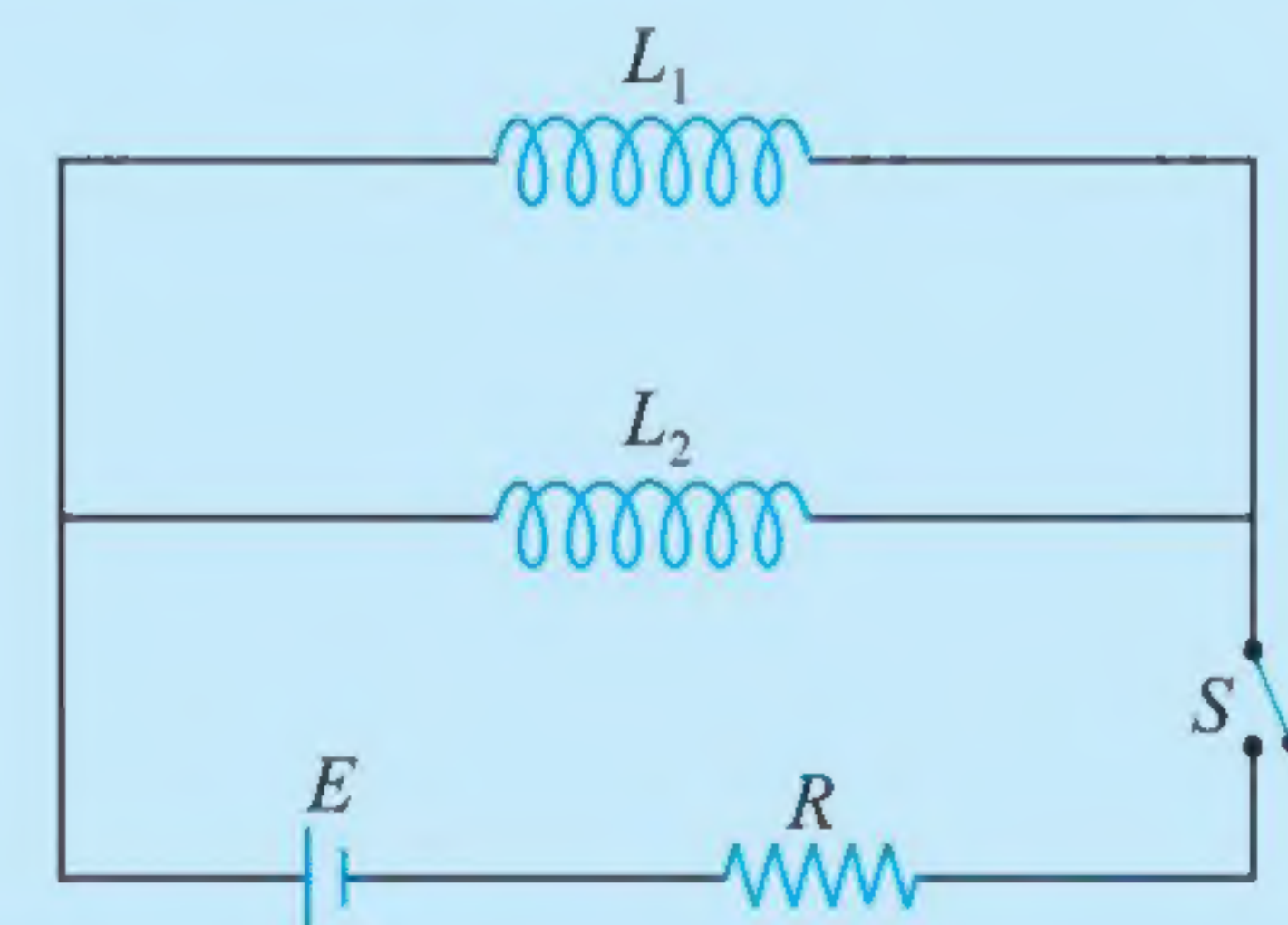
If L is the equivalent inductance connected between a and b , for same variation $\frac{di}{dt}$, it induces same voltage V_{ab} . Then

$$V_{ab} = L \left(\frac{di}{dt} \right) \Rightarrow \frac{1}{L} = \frac{di/dt}{V_{ab}} \quad \dots(iv)$$

From (iii) and (iv), we get $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ hence, $\frac{1}{L} = \sum_{i=1}^n \frac{1}{L_i}$

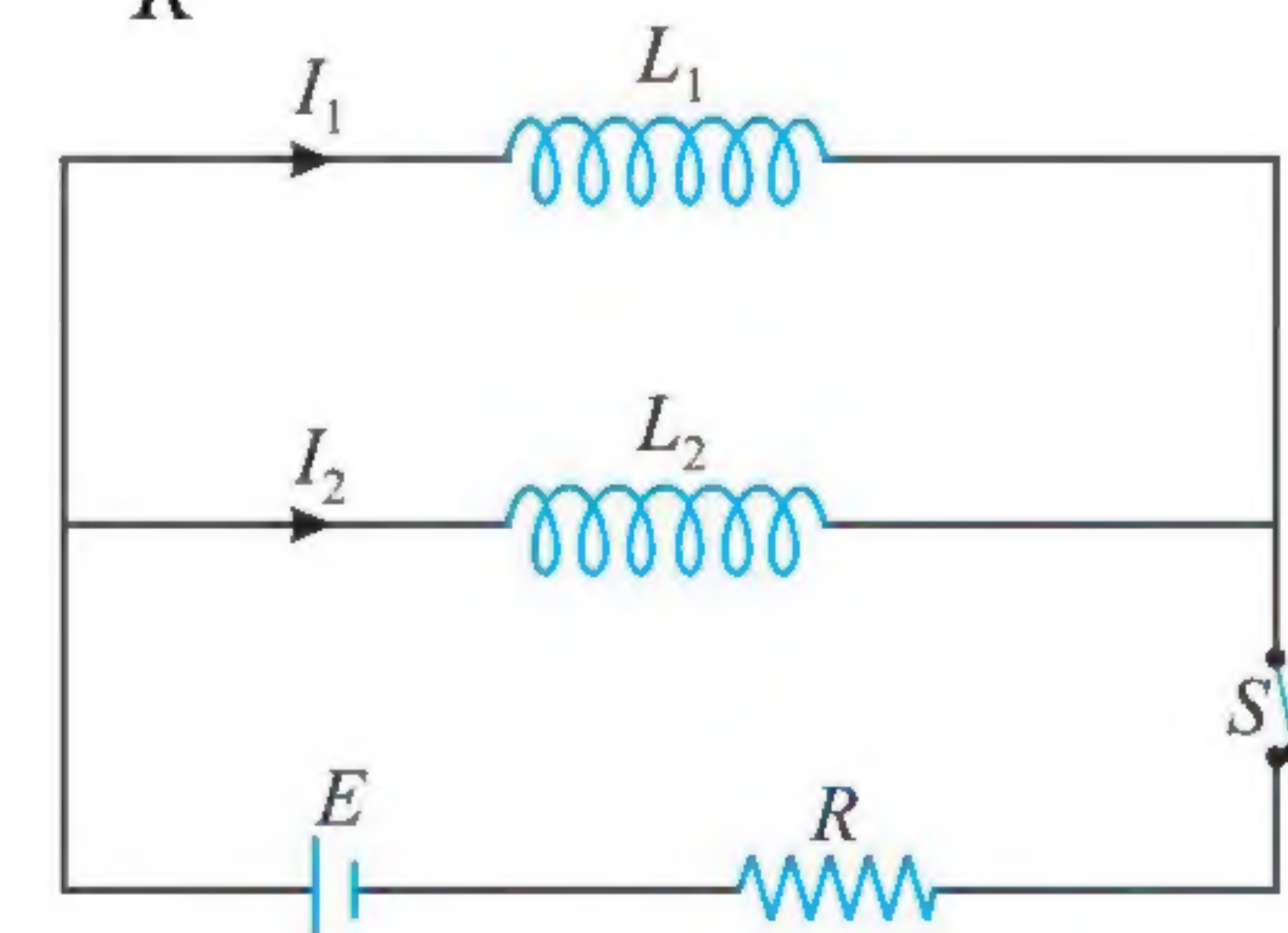
ILLUSTRATION 5.8

Two inductor coils of inductances L_1 and L_2 are connected in parallel with a battery of EMF E , and a resistance R . The internal resistance of the source and the coil resistances are negligible. Find the steady state currents in the coils long time after the switch S was shorted.



Sol. At steady state constant current flows through inductor coil. It means the flux associated with the coil remains constant. In this case no emf will be induced in the coil. It means in steady state inductors behave like conducting wire or short circuit so the current through battery is given as

$$i = i_1 + i_2 = \frac{E}{R} \quad \dots(i)$$



For two inductors connected in parallel, the potential difference across the coils should be same.

$$\text{Hence, } L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} \quad \dots(ii)$$

$$\text{Integrating Eq. (ii) both sides we obtain } L_1 i_1 = L_2 i_2 \quad \dots(iii)$$

From Eqs. (i) and (iii), we get

$$i_1 = \frac{EL_2}{R(L_1 + L_2)} \text{ and } i_2 = \frac{EL_1}{R(L_1 + L_2)}$$

It means the current in inductors connected in parallel combination is divided in inverse ratio of their inductances.

ILLUSTRATION 5.9

The equivalent inductance of two inductors is 2.4 H when connected in parallel and 10 H when connected in series. What is the value of inductance of the individual inductors?

Sol. As inductances obey laws similar to the “grouping of resistances,”

$$L_1 + L_2 = 10 \text{ H and } \frac{L_1 L_2}{L_1 + L_2} = 2.4 \text{ H}$$

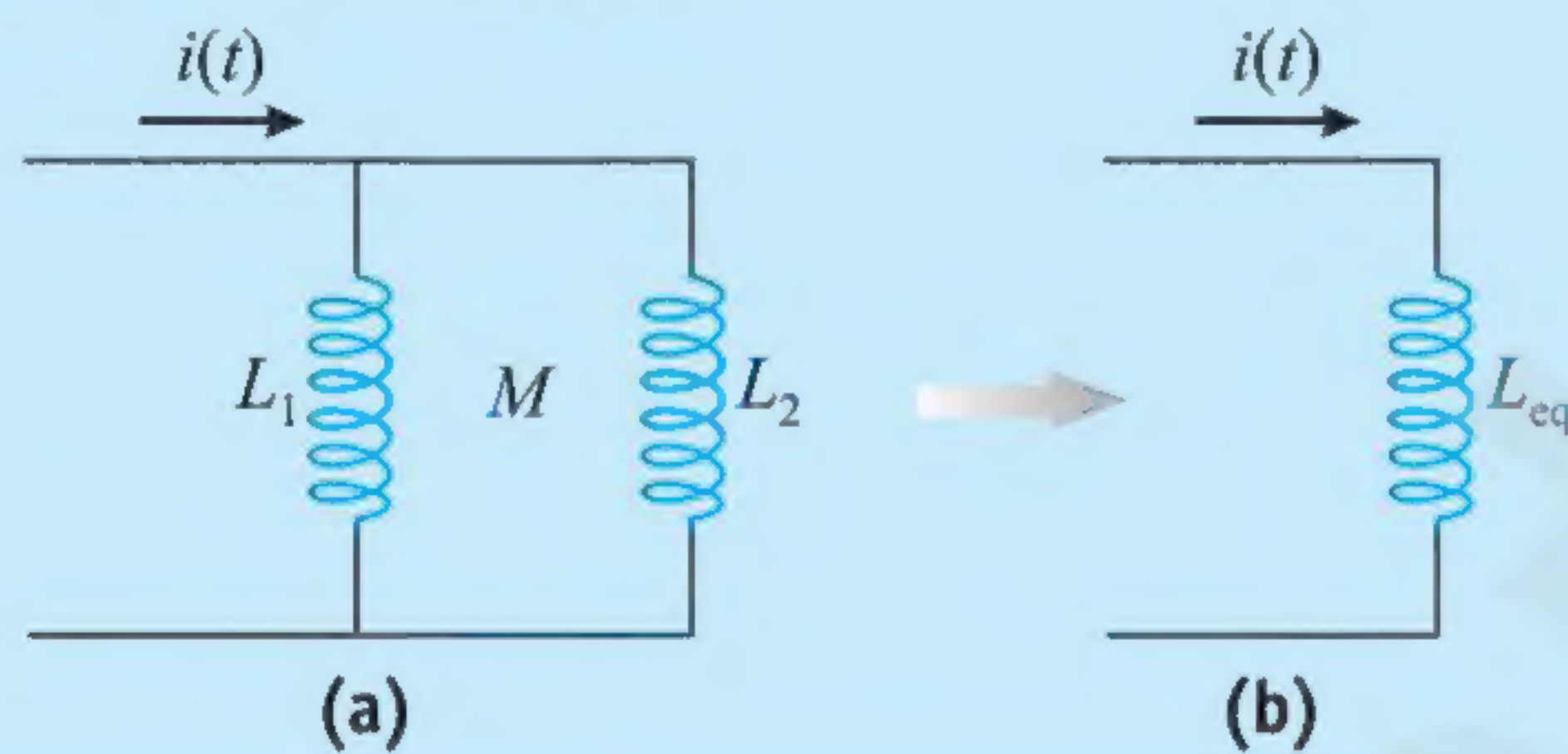
Substituting the value of $(L_1 + L_2)$ from first expression into the second, $L_1 L_2 = (2.4)(L_1 + L_2) = 2.4 \times 10 = 24$

so that $(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1 L_2$, i.e., $L_1 - L_2 = 2 \text{ H}$

and as $L_1 + L_2 = 10 \text{ H}$, so $L_1 = 6 \text{ H}$ and $L_2 = 4 \text{ H}$

ILLUSTRATION 5.10

Two inductors having inductances L_1 and L_2 are connected in parallel as shown in [Fig. (a)]. The mutual inductance between the two inductors is M . Determine the equivalent inductance L_{eq} for the system [Fig. (b)].



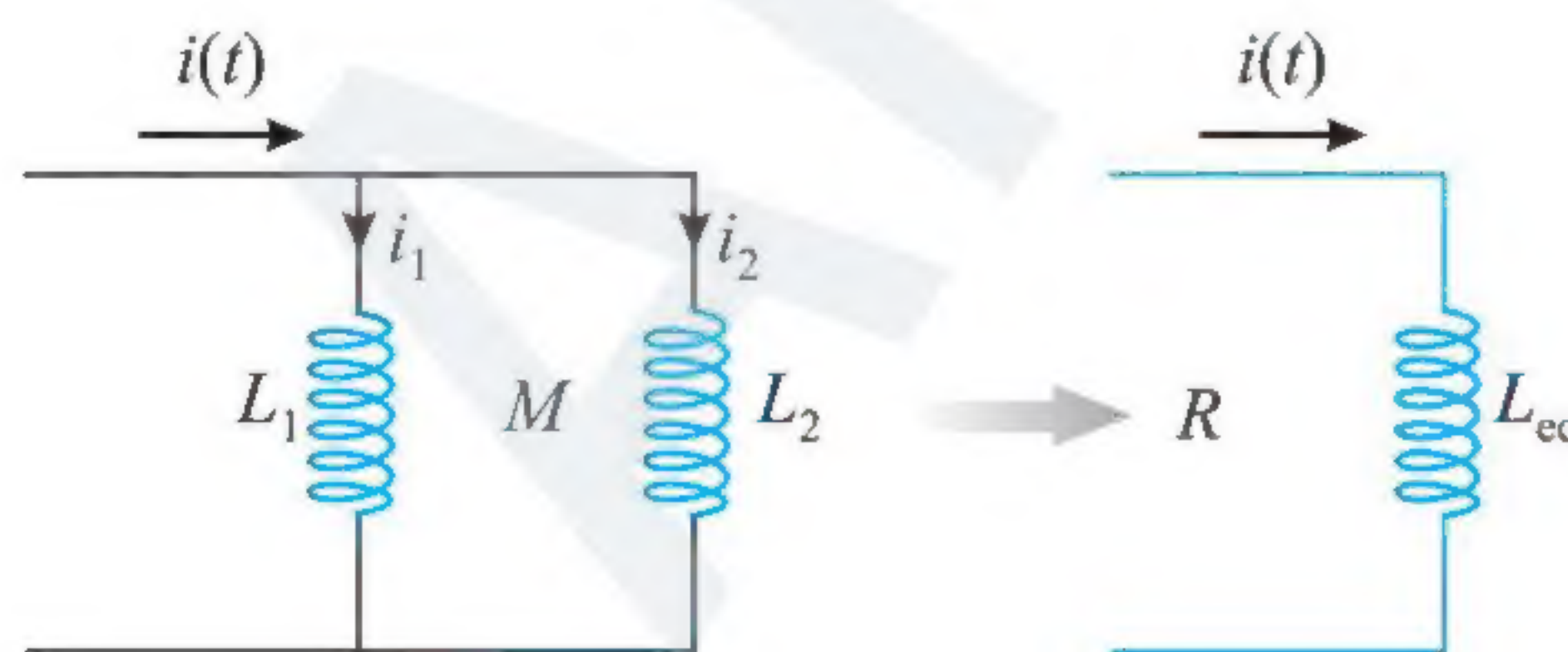
Sol. Let the currents in inductors L_1 and L_2 at any instant be i_1 and i_2 respectively.

We have $i = i_1 + i_2$, ... (i)

the voltage across the pair is:

$$\Delta V = -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = -L_{eq} \frac{di}{dt} \quad \dots (ii)$$

$$\text{and } \Delta V = -L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = -L_{eq} \frac{di}{dt} \quad \dots (iii)$$



$$\text{So, from (ii), we have, } -\frac{di_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1} \frac{di_2}{dt} \quad \dots (iv)$$

$$\text{From (iii) and (iv), } \Delta V = -L_2 \frac{di_2}{dt} + M \left(\frac{\Delta V}{L_1} + \frac{M}{L_1} \frac{di_2}{dt} \right)$$

$$(-L_1 L_2 + M^2) \frac{di_2}{dt} = \Delta V (L_1 - M) \quad \dots (v)$$

$$\text{From (iii), } -\frac{di_2}{dt} = \frac{\Delta V}{L_2} + \frac{M}{L_2} \frac{di_1}{dt}, \quad \dots (vi)$$

$$\text{From (ii) and (vi), } \Delta V = -L_1 \frac{di_1}{dt} + M \left(\frac{\Delta V}{L_2} + \frac{M}{L_2} \frac{di_1}{dt} \right)$$

$$(-L_1 L_2 + M^2) \frac{di_1}{dt} = \Delta V (L_2 - M) \quad \dots (vii)$$

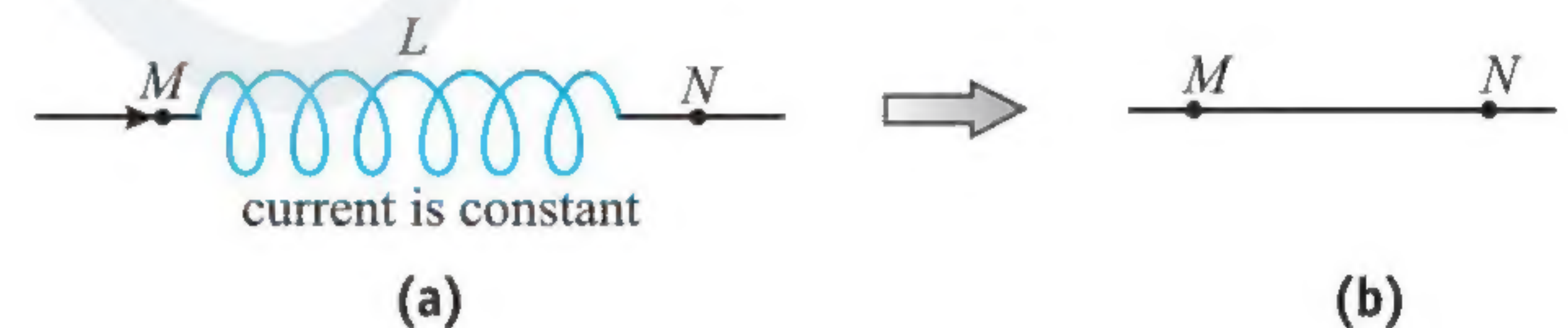
Adding (v) to (vii), we have

$$(-L_1 L_2 + M^2) \frac{di}{dt} = \Delta V (L_1 + L_2 - 2M)$$

$$\text{Hence, } L_{eq} = -\frac{\Delta V}{di/dt} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

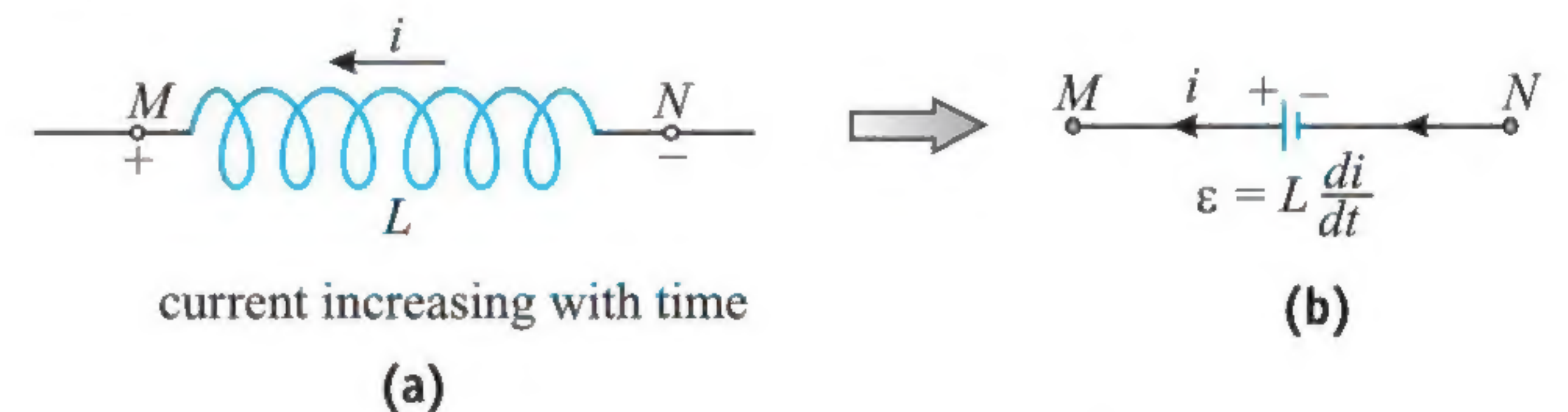
INDUCTOR IN ELECTRICAL CIRCUITS

In an inductor coil a steady current flow, the magnetic flux due to the current remains constant. In this case no EMF is induced in the coil. It means in case when the current is steady, the inductor behaves like a conducting wire as shown in figure.



Now consider the case when current flowing through the inductor changes with time. In this case due to self-induction an EMF is induced in the inductor so as to oppose the change in current. Let us consider two cases,

Case (i): In the case when current is increasing and flowing from left to right then left end of inductor will have high potential and right end of inductor will be at low potential. In this way the EMF induced in the inductor coil tend to supply its current in direction opposite to the external current so that increasing current may be opposed. In this situation then we may consider the inductor as a battery or equivalent EMF across its terminals M and N as shown in figure.



Case (ii): In the case when current is decreasing and flowing from left to right as shown in figure. In this case due to self-induction an EMF is induced so as to oppose the decrease in current. So in this case right end of inductor will be at higher potential and left end at lower potential so that the self-induced EMF in the coil will tend to supply its current in the same direction as that of the external current. In this case the inductor behaves like a battery or equivalent EMF across its terminals M and N as shown in figure.

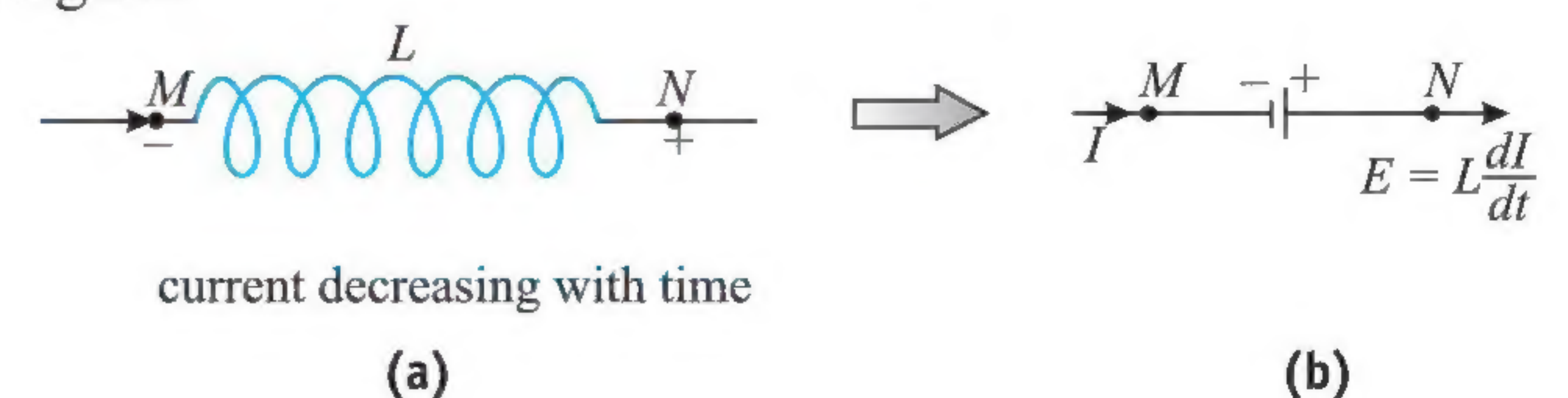
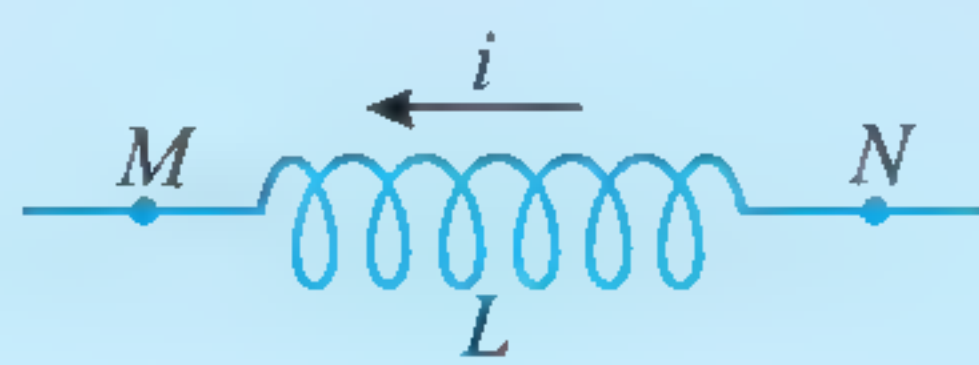


ILLUSTRATION 5.11

The current in inductor shown in figure is flowing from right to left. The inductor has inductance 0.75 H and current in it is decreasing at a uniform rate, $di/dt = -0.040 \text{ A/s}$.



- (a) Find the magnitude of the self-induced emf.
 (b) Which end of the inductor, M or N , is at a higher potential?

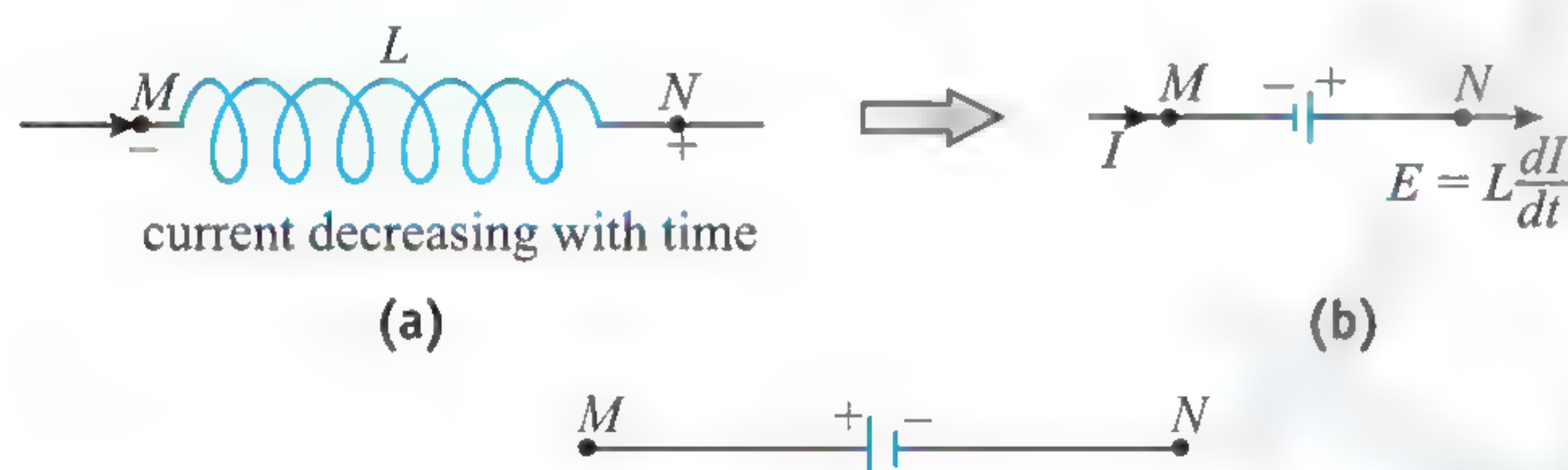
Sol.

- (a) The magnitude of the EMF induced in coil due to self-induction is given as

$$e = L \left| \frac{di}{dt} \right| = 0.75 \times 0.04 \text{ V}$$

$$\Rightarrow e = 3.0 \times 10^{-2} \text{ V}$$

- (b) Current in the shown direction is decreasing. In this case due to self-induction an EMF is induced so as to oppose the decrease in current. So, in this case left end of inductor will be at higher potential and right end at lower potential so that the self-induced EMF in the coil will tend to supply its current in the left direction as that of the external current. Hence induced battery polarity will be as shown in figure. Hence end M will be at higher potential.

**ILLUSTRATION 5.12**

A section of an electrical circuit MN which contains a resistance $R = 2 \Omega$, a battery of emf $E = 24 \text{ V}$ and an inductor coil of inductance $L = 4.0 \text{ H}$ as shown in figure. Then which carries a current of 10 A which is decreasing at the rate of 0.5 A/s . Find the potential difference $V_N - V_M$.



Sol. In the above circuit section as current is decreasing with time, the polarity of self-induced EMF is shown in figure.



In this branch of circuit writing, Kirchhoff's path equation from point M to N which is given as

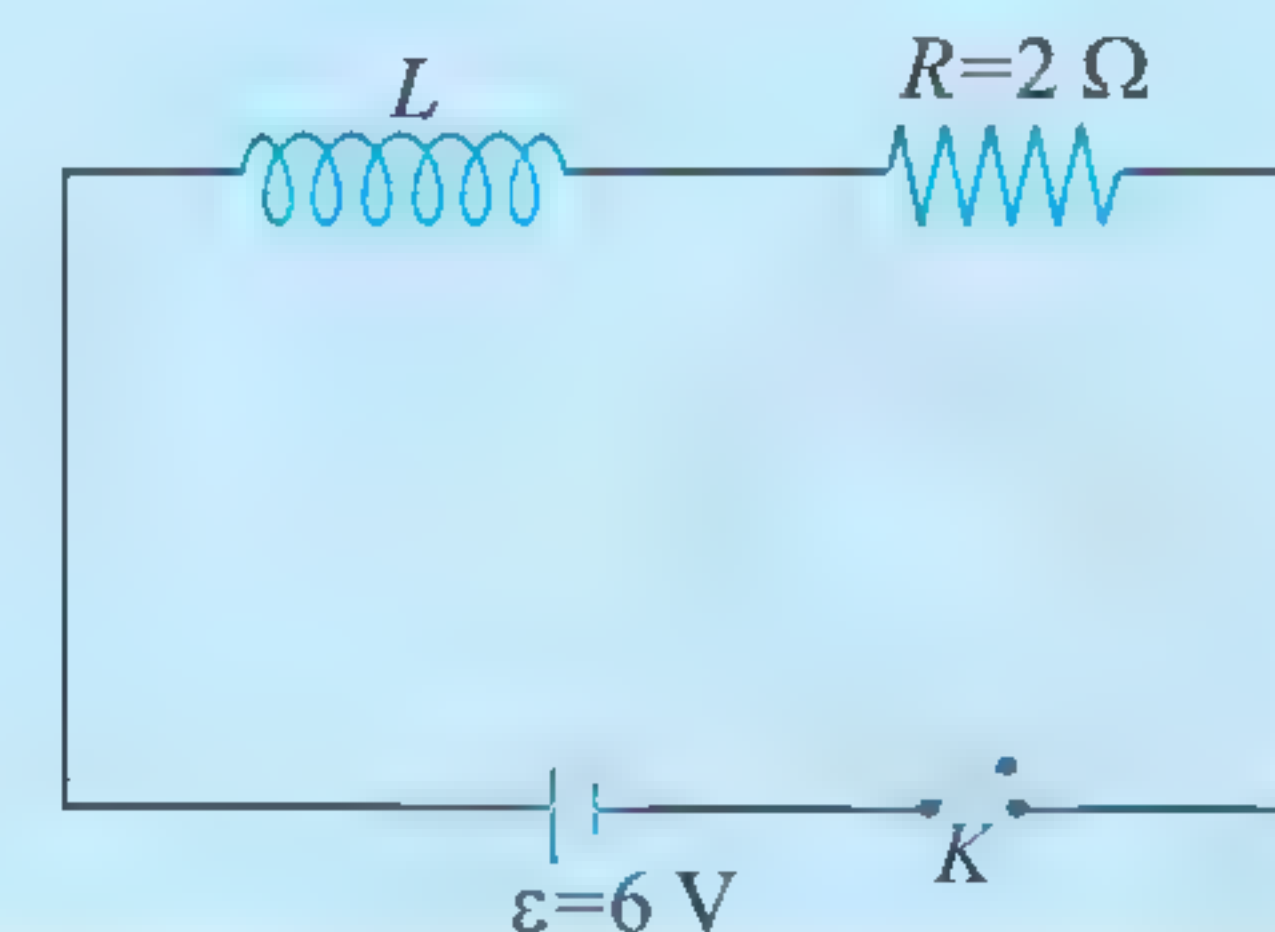
$$V_M - iR + E + L \frac{di}{dt} = V_N$$

$$\Rightarrow V_M - 10 \times 2 + 24 + 4.0 \times 0.5 = V_N$$

$$\Rightarrow V_N - V_M = 6.0 \text{ V}$$

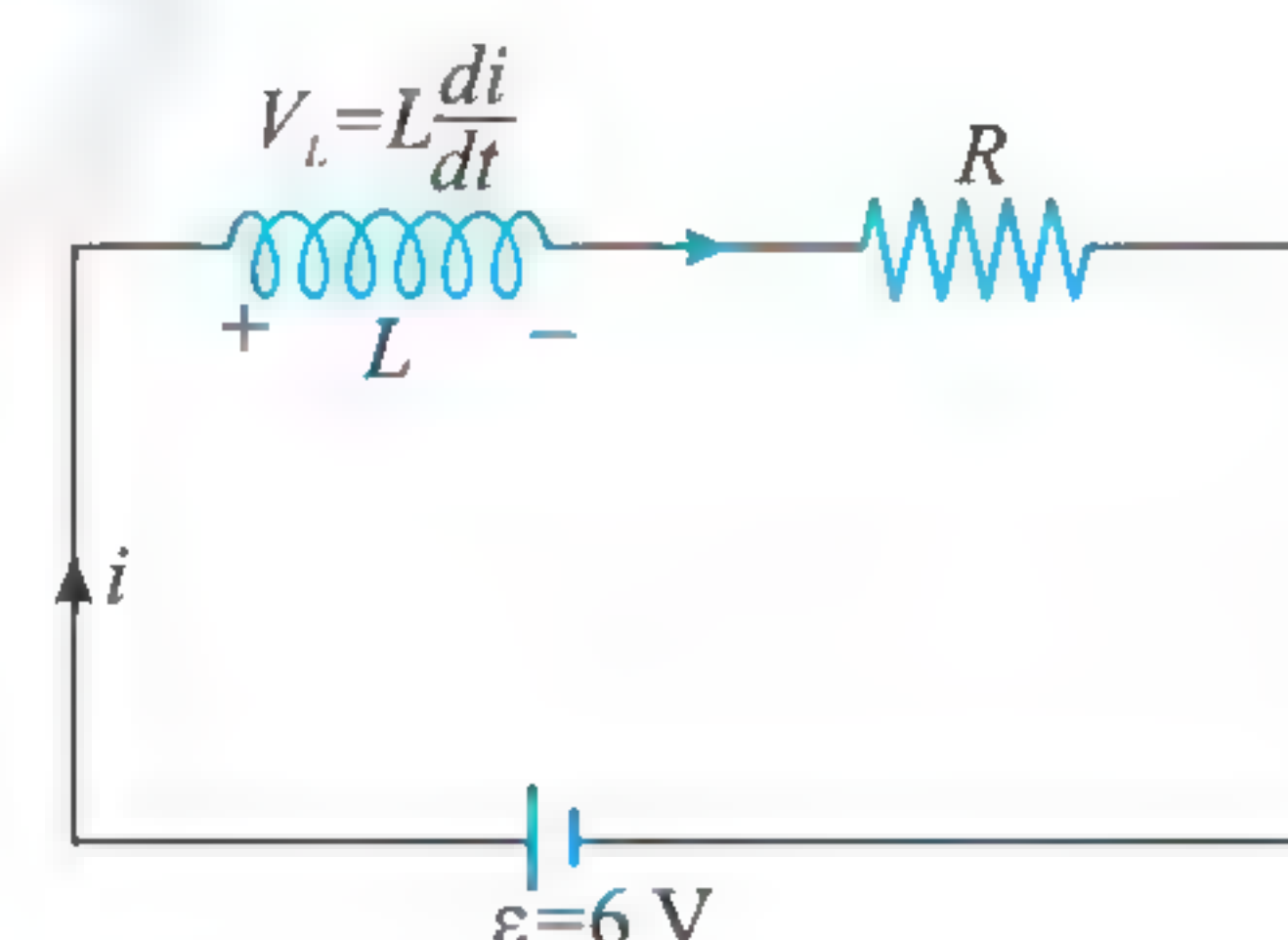
ILLUSTRATION 5.13

Find the self-inductance L of the coil whose resistance $R = 2 \text{ ohm}$, a current $i = 1.5 \text{ amp}$, which is increasing at a rate of 10^3 amp/s .



Sol. The current is increasing and flowing left to right in the inductor then left end of inductor will have high potential and right end of inductor will be at low potential. Applying Kirchhoff's loop law

$$\epsilon - V_L - iR = 0 \Rightarrow \epsilon - L \frac{di}{dt} - iR = 0$$

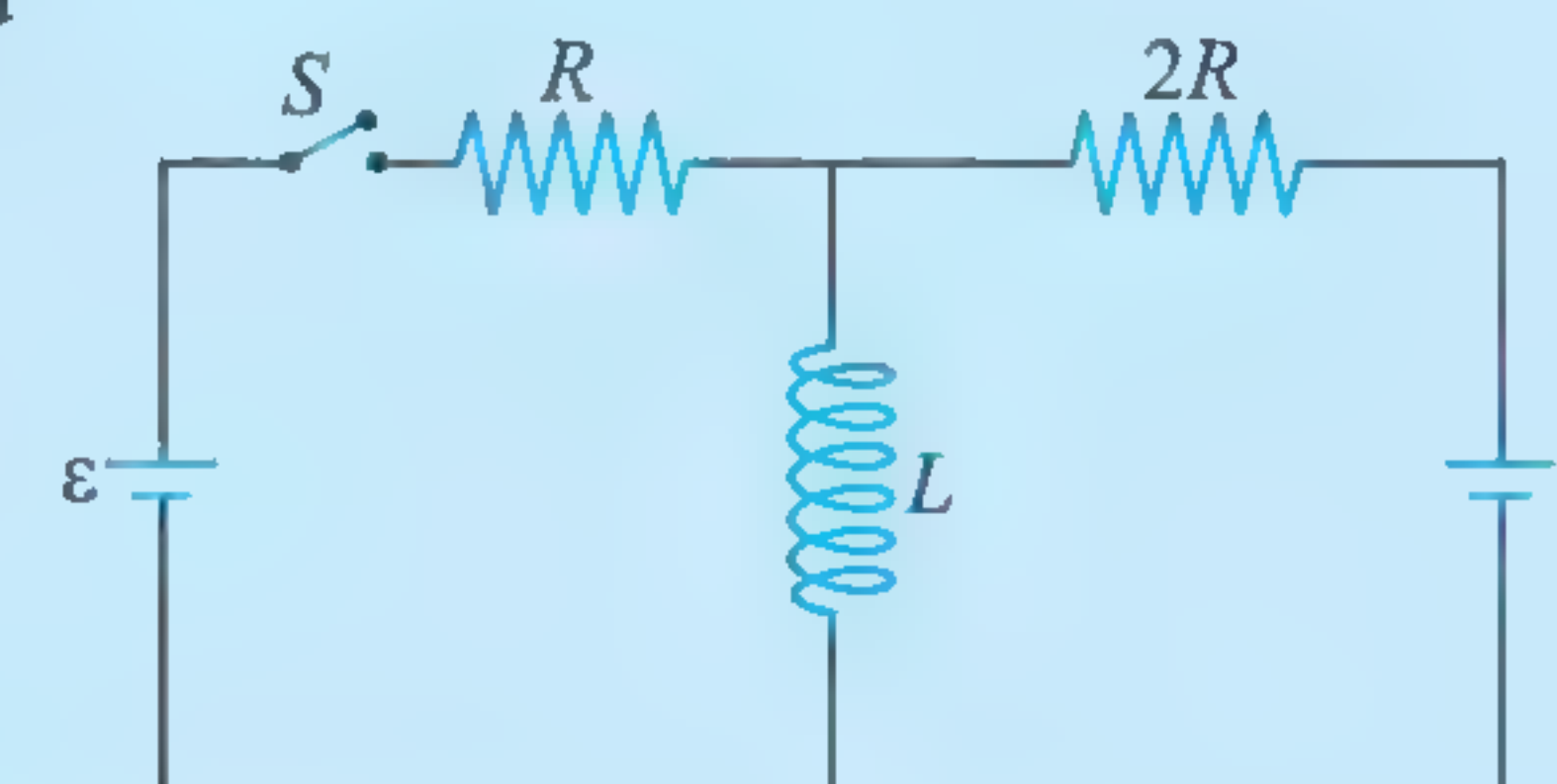


$$6.0 - L \times 10^3 - 1.5 \times 2 = 0$$

$$\Rightarrow L = \frac{6.0 - 3.0}{10^3} = 3.0 \times 10^{-3} \text{ H. Hence } L = 3 \text{ mH.}$$

ILLUSTRATION 5.14

The circuit shown in the figure is in steady state. At certain instant the switch is closed, immediately after the switch S is closed. Find

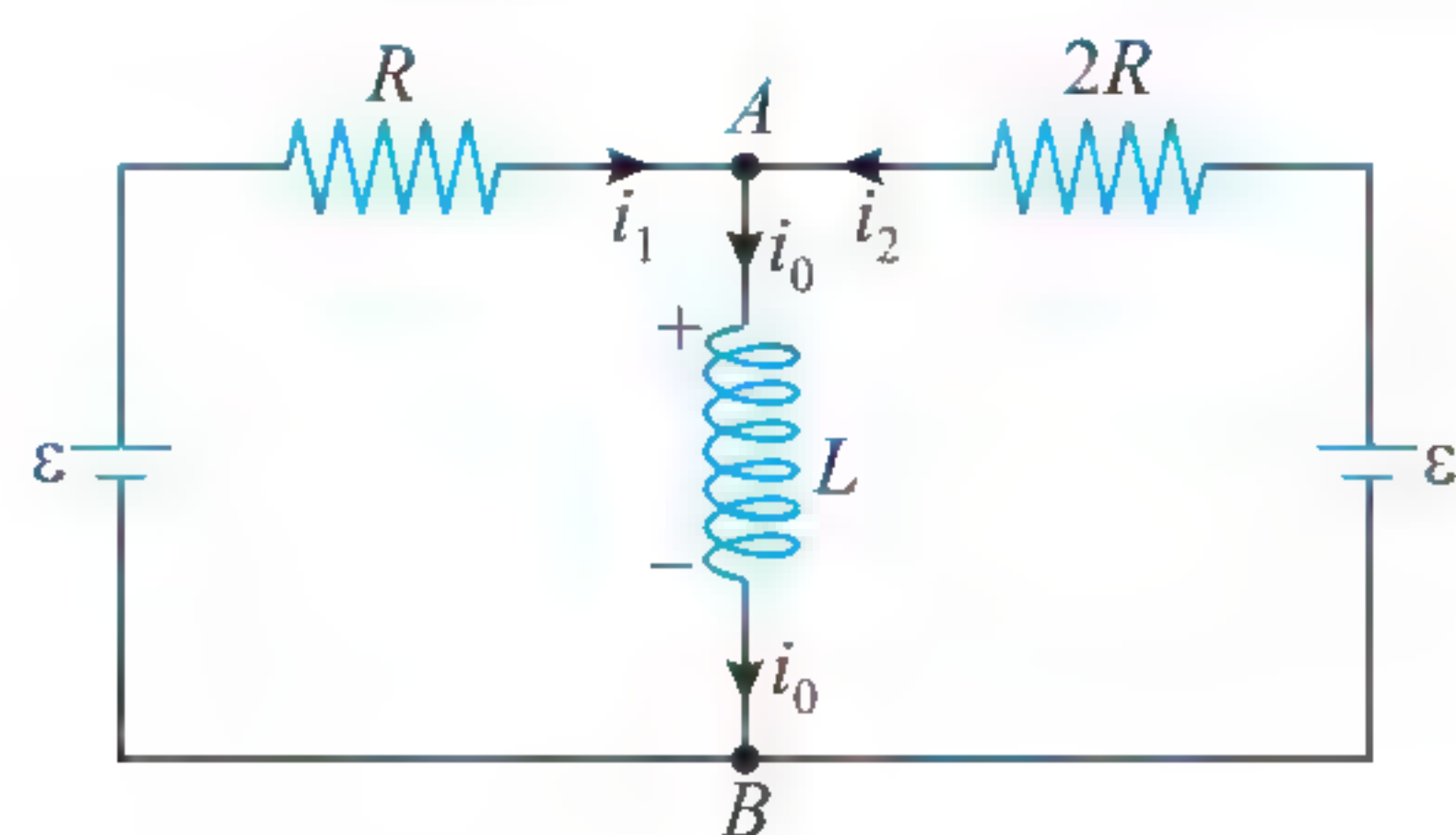


- (i) potential difference across the inductor coil
 (ii) the rate of change of current through the inductor coil
 (iii) potential difference across the resistance

Sol.

- (i) In steady state the current through the inductor is $i = \frac{\epsilon}{2R}$.

The current through the inductor will not change immediately after the switch is closed. Let current in the inductor branch is increasing, hence end A of the coil should be positive and end B should be negative and $V_{AB} = V$.



In left loop: $Ri_1 + V = \varepsilon$... (i)

In right loop: $2Ri_2 + V = \varepsilon$... (ii)

From (i) and (ii)

$$i_1 + i_2 = \frac{\varepsilon - V}{R} + \frac{\varepsilon - V}{2R}$$

Since $i_1 + i_2 = i_0 = \frac{\varepsilon}{2R}$ [Because $i_0 = \frac{\varepsilon}{2R}$]

$$\therefore \frac{\varepsilon - V}{R} + \frac{\varepsilon - V}{2R} = \frac{\varepsilon}{2R} \Rightarrow V = \frac{2\varepsilon}{3}$$

(ii) The potential difference across the inductor coil,

$$V = L \frac{di}{dt} = \frac{2\varepsilon}{3} \Rightarrow \frac{di}{dt} = \frac{2\varepsilon}{3L}$$

As $\frac{di}{dt}$ is positive. It means current is increasing.

(iii) At this instant the potential difference across R is

$$\varepsilon - \frac{2\varepsilon}{3} = \frac{\varepsilon}{3}$$

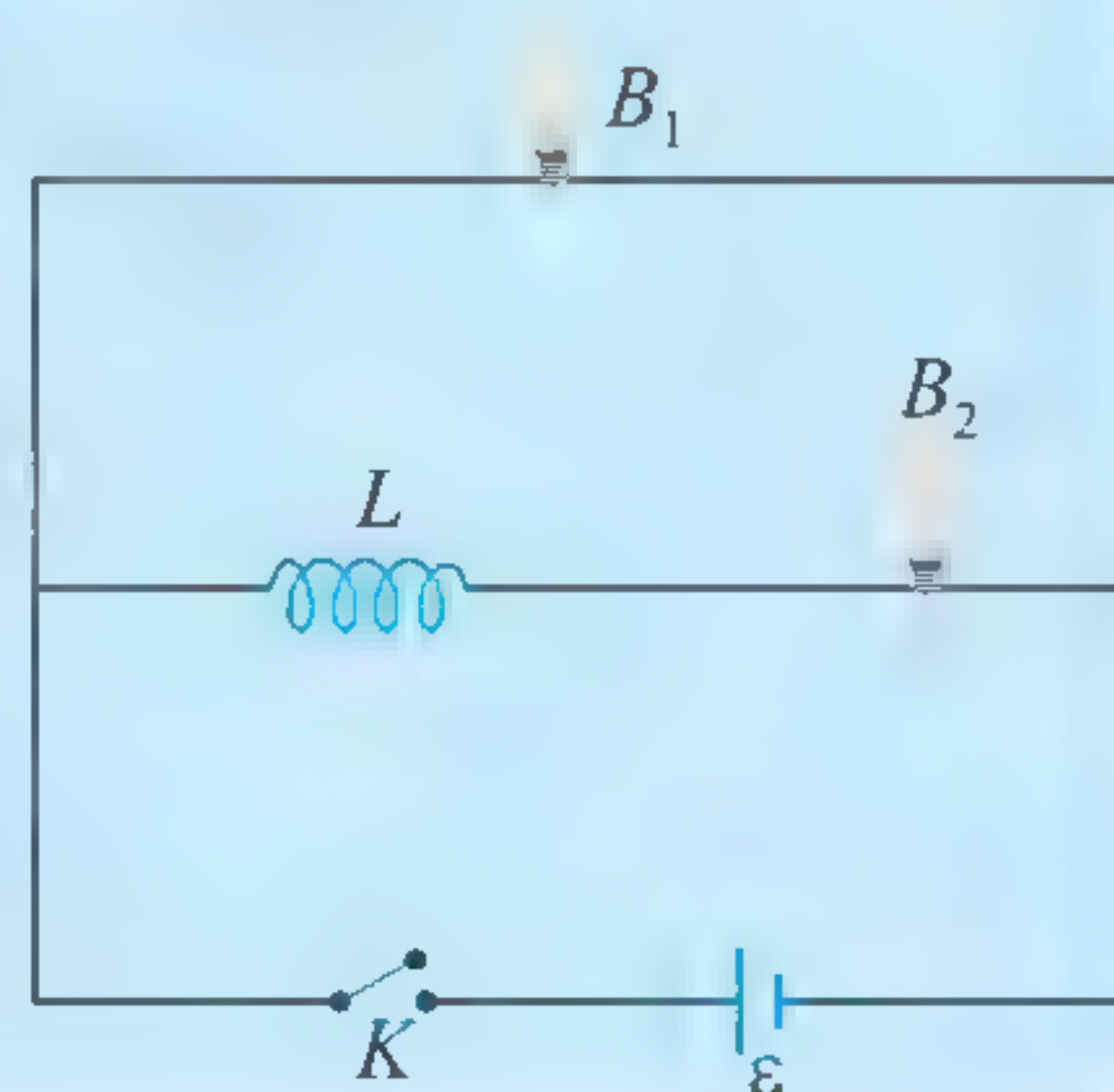
ANALOGY BETWEEN SELF-INDUCTION AND INERTIA

Newton's first law of motion defines inertia. It is the property of matter by the virtue of which it opposes the change in its state. "Mass" of a matter is the measure of its inertia. Greater the mass, it is more difficult to set it in motion (or to stop it).

In the same way, the circuit containing an inductor opposes the change in its own current by inducing an emf in it owing to the induced electric field. This is known as self-induction and this opposing (inertial) nature of the electric circuit to any change in its current is termed as electrical inertia. In the coming illustration, we will demonstrate the inertial nature of inductors.

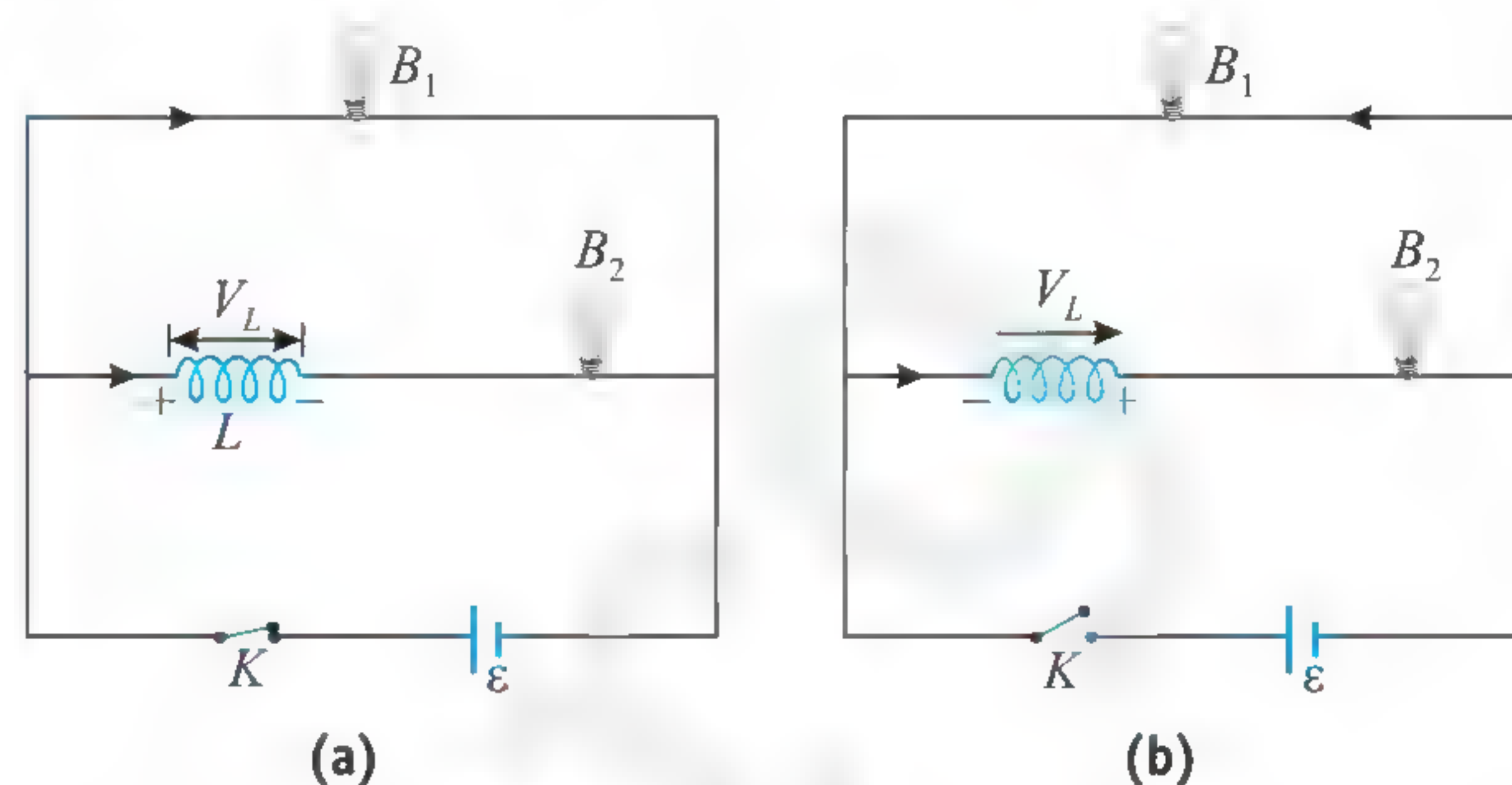
ILLUSTRATION 5.15

Two identical bulbs B_1 and B_2 connected in parallel with a battery of emf ε . The bulb B_2 is connected in series with an ideal inductor L . How the brightness of the bulbs will change after closing the key K . What will happen when we open the key K after a long time?



Sol. We can consider the bulbs as simple resistors of resistance say R_1 and R_2 . Without any inductive effect, as the bulbs are joined in parallel with the battery, we should expect them to glow with equal brightness. But after closing the key K as shown in Fig. (a), the presence of the inductor induces a voltage V_L across it. As the current increases, the induced voltage opposes the applied voltage ε . Then the bulb B_2 receives a voltage $\varepsilon - V_L$

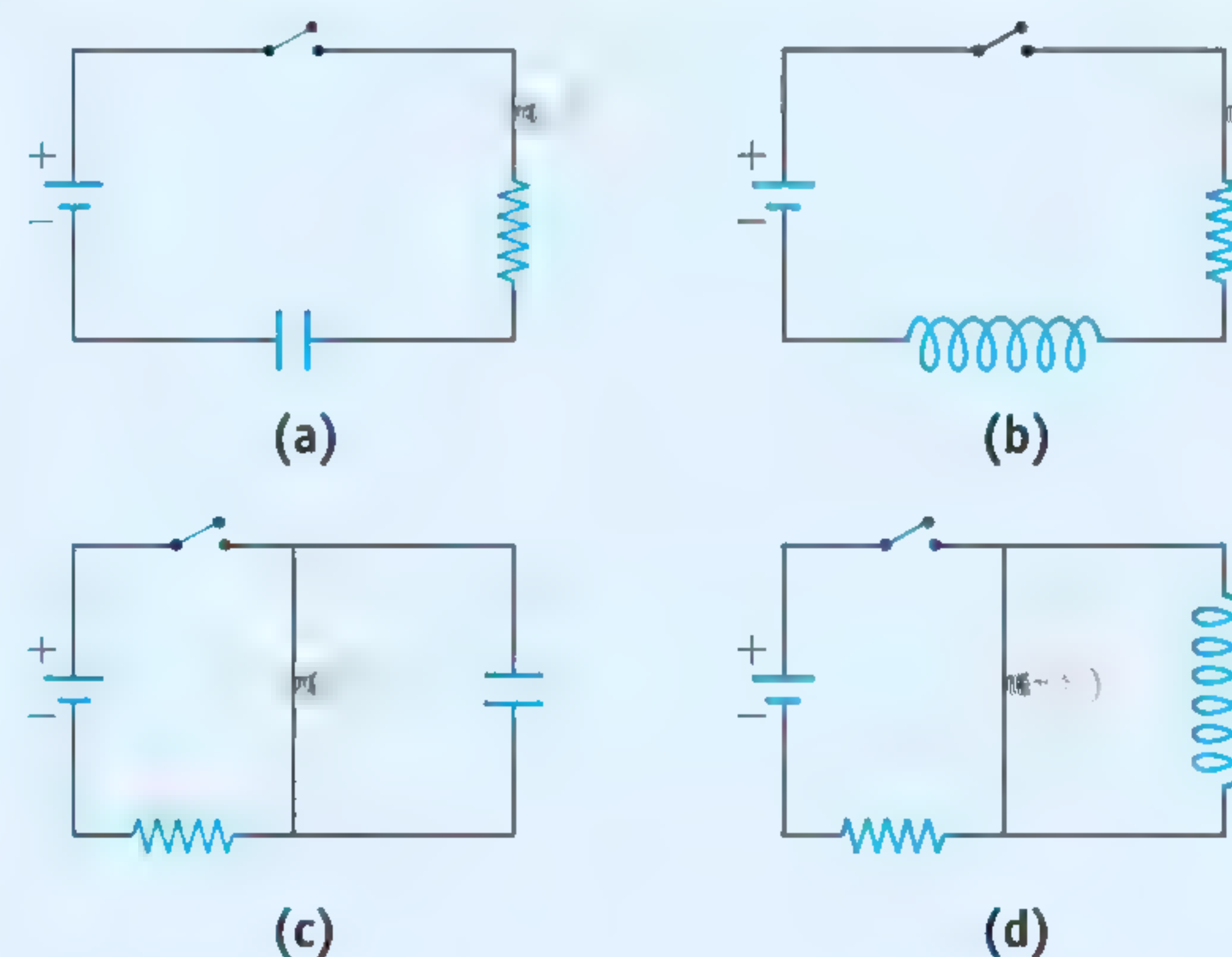
to use, whereas B_1 utilizes full voltage ε for all the times. This makes B_1 glowing brighter than B_2 till B_2 attains a steady state current. That means, after sometime both the bulbs glow with equal brightness.



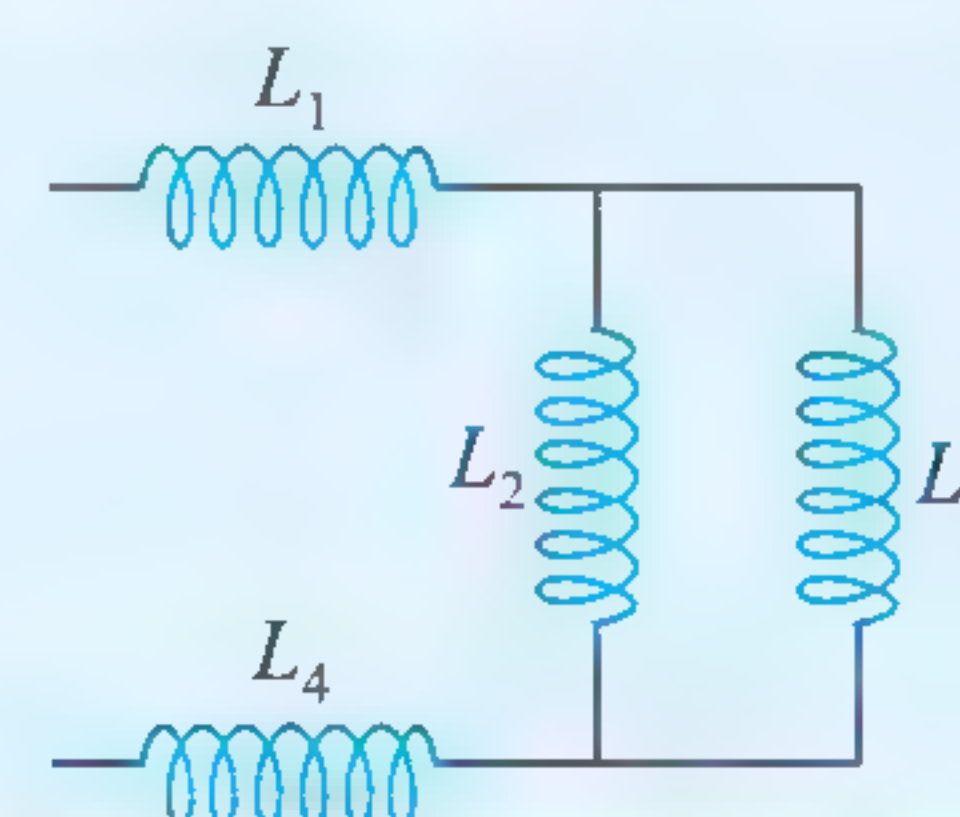
After a long time, the key is reopened. This tries to destroy the magnetic flux in the inductor rapidly. In consequence, a voltage V'_L is induced so as to retain the current. Hence, the polarity of the inductor reverses and the same current flows through both B_1 and B_2 . However, current flows in a direction opposite to the initial current of B_1 . The currents in both the bulbs that measure their brightness falls gradually to zero after sometime.

CONCEPT APPLICATION EXERCISE 5.2

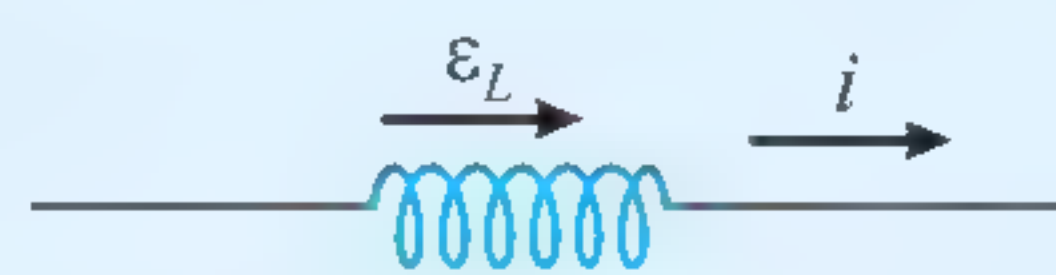
- Consider the four circuits shown in figure, each consisting of a battery, a switch, a lightbulb, a resistor, and either a capacitor or an inductor. Assume the capacitor has a large capacitance and the inductor has a large inductance but no resistance. The light bulb has high efficiency, glowing whenever it carries electric current. (i) Describe what the lightbulb does in each of circuits (a) through (d) after the switch is thrown closed. (ii) Describe what the lightbulb does in each of circuits (a) through (d) when, having been closed for a long time interval, the switch is opened.



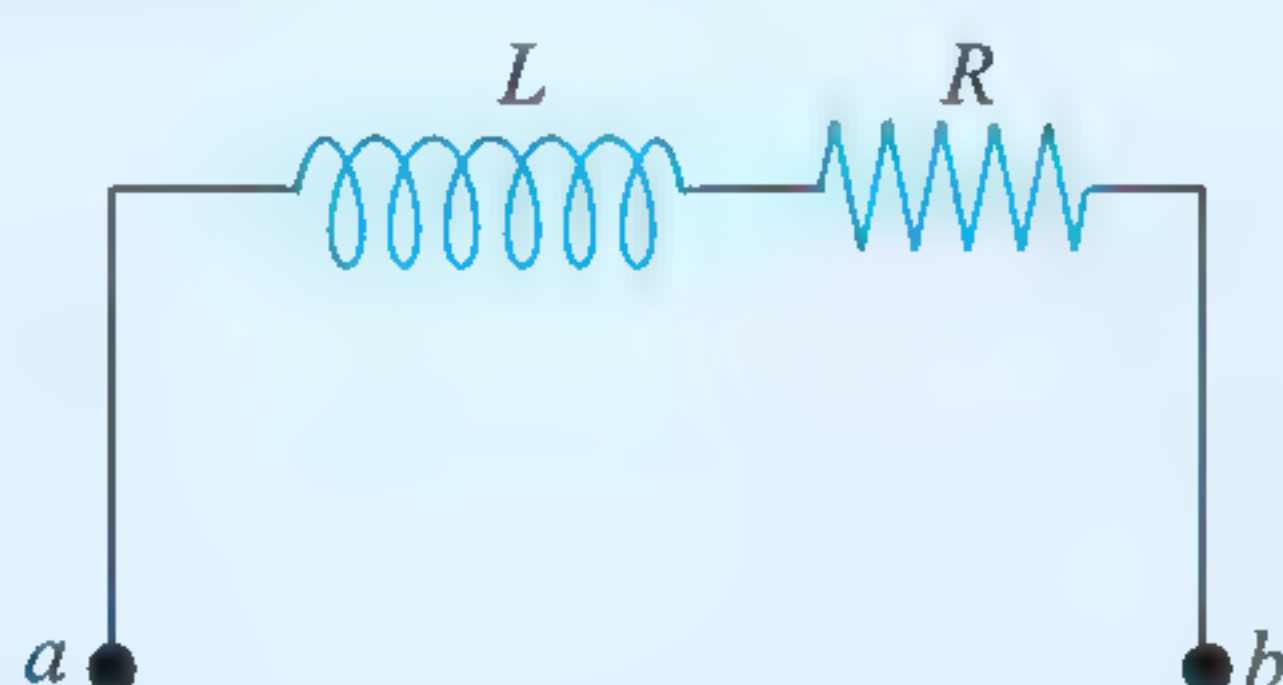
- The inductor arrangement of figure, with $L_1 = 40.0$ mH, $L_2 = 40.0$ mH, $L_3 = 10.0$ mH, and $L_4 = 12.0$ mH, is to be connected to a varying current source. What is the equivalent inductance of the arrangement?



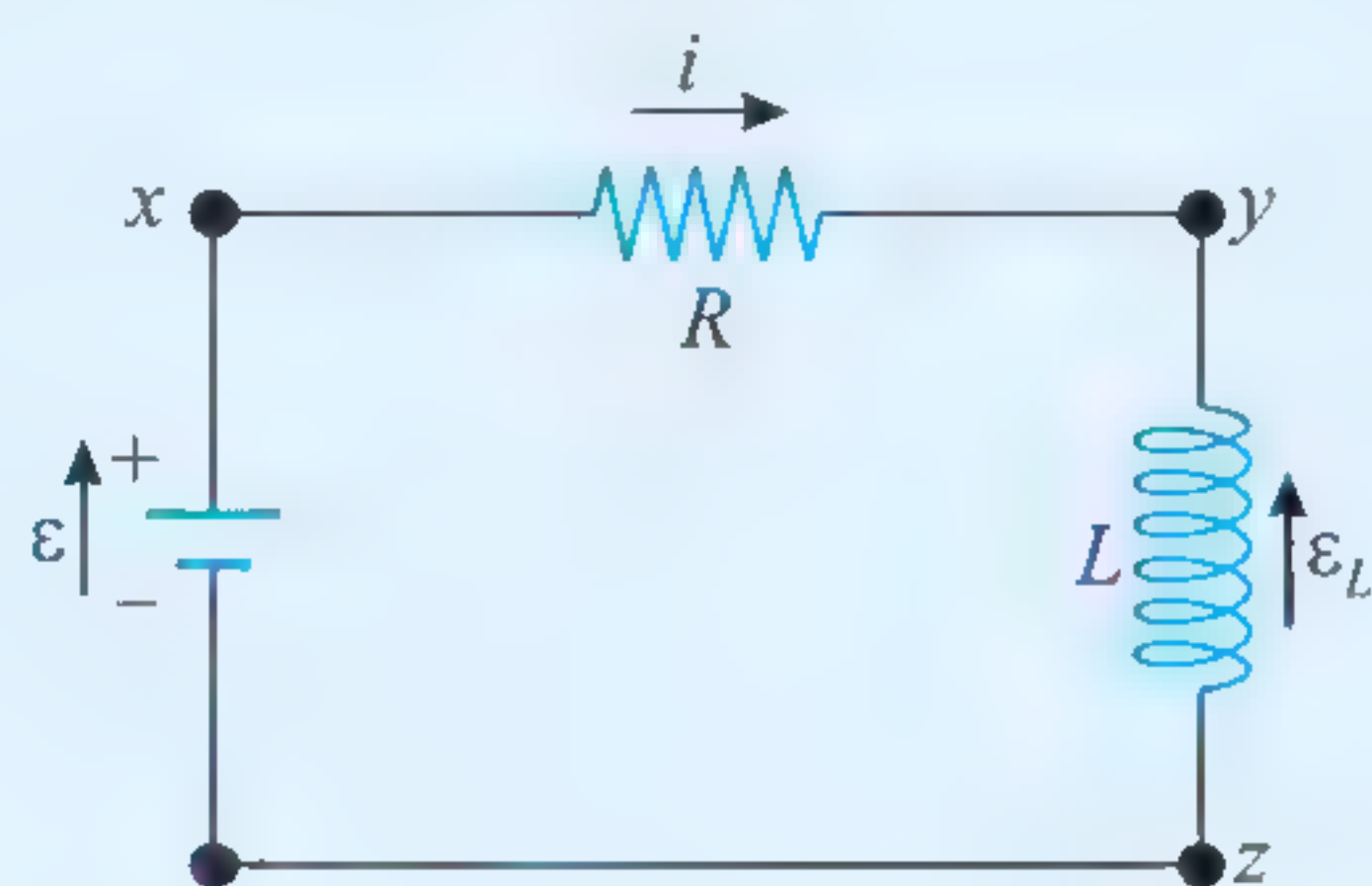
3. At a given instant the current and self-induced emf in an inductor are directed as indicated in figure. (a) Is the current increasing or decreasing? (b) The induced emf is 6.0 V, and the rate of change of the current is 1.5 kA/s; find the inductance.



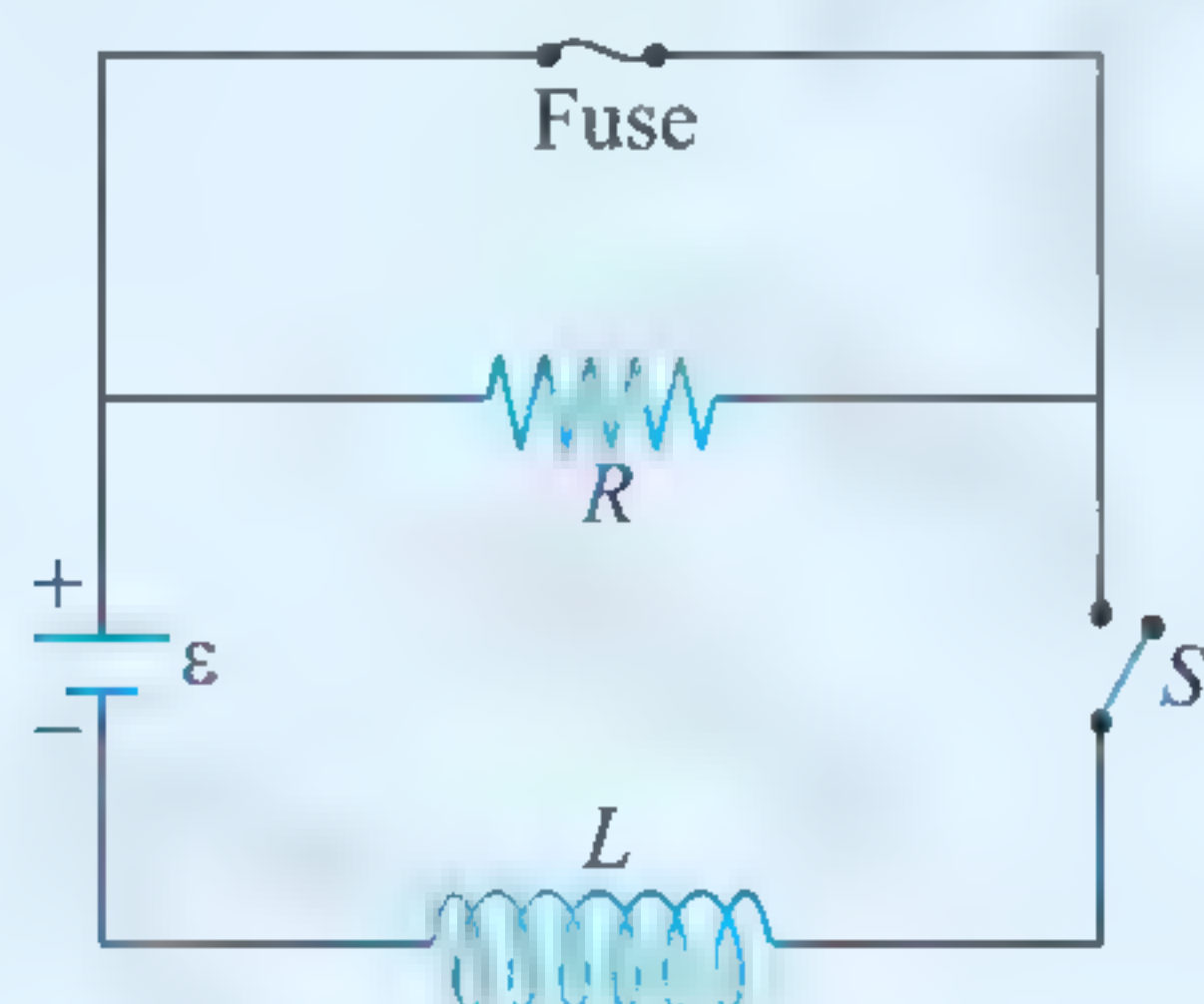
4. When the current in the portion of the circuit shown in figure is 2.00 A and increases at a rate of 0.500 A/s, the measured voltage is $\Delta V_{ab} = 9.00$ V. When the current is 2.00 A and decreases at the rate of 0.500 A/s, the measured voltage is $\Delta V_{ab} = 5.00$ V. Calculate the values of (a) L and (b) R .



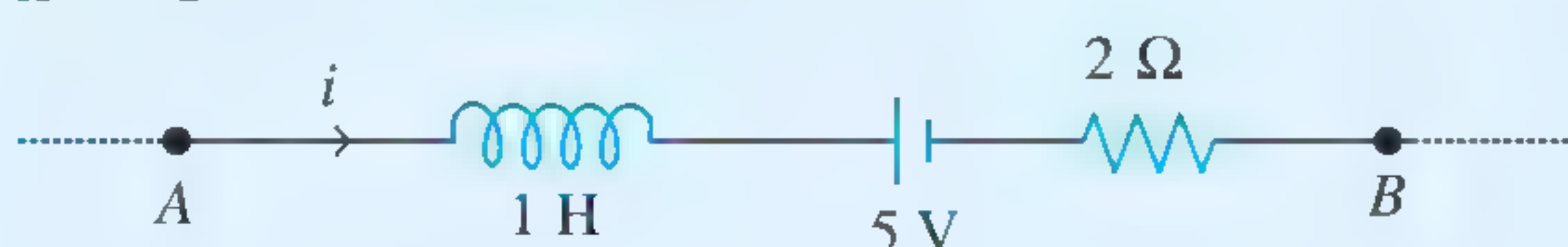
5. Suppose the emf of the battery in the circuit shown in figure varies with time t so that the current is given by $i(t) = 3.0 + 5.0t$, where i is in amperes and t is in seconds. Take $R = 4.0 \Omega$ and $L = 6.0$ H, and find an expression for the battery emf as a function of t .



6. In given figure, $R = 15 \Omega$, $L = 5.0$ H, the ideal battery has $\varepsilon = 10$ V, and the fuse in the upper branch is an ideal 3.0 A fuse. It has zero resistance as long as the current through it remains less than 3.0 A. If the current reaches 3.0 A, the fuse “blows” and thereafter has infinite resistance. Switch S is closed at time $t = 0$. When does the fuse blow?

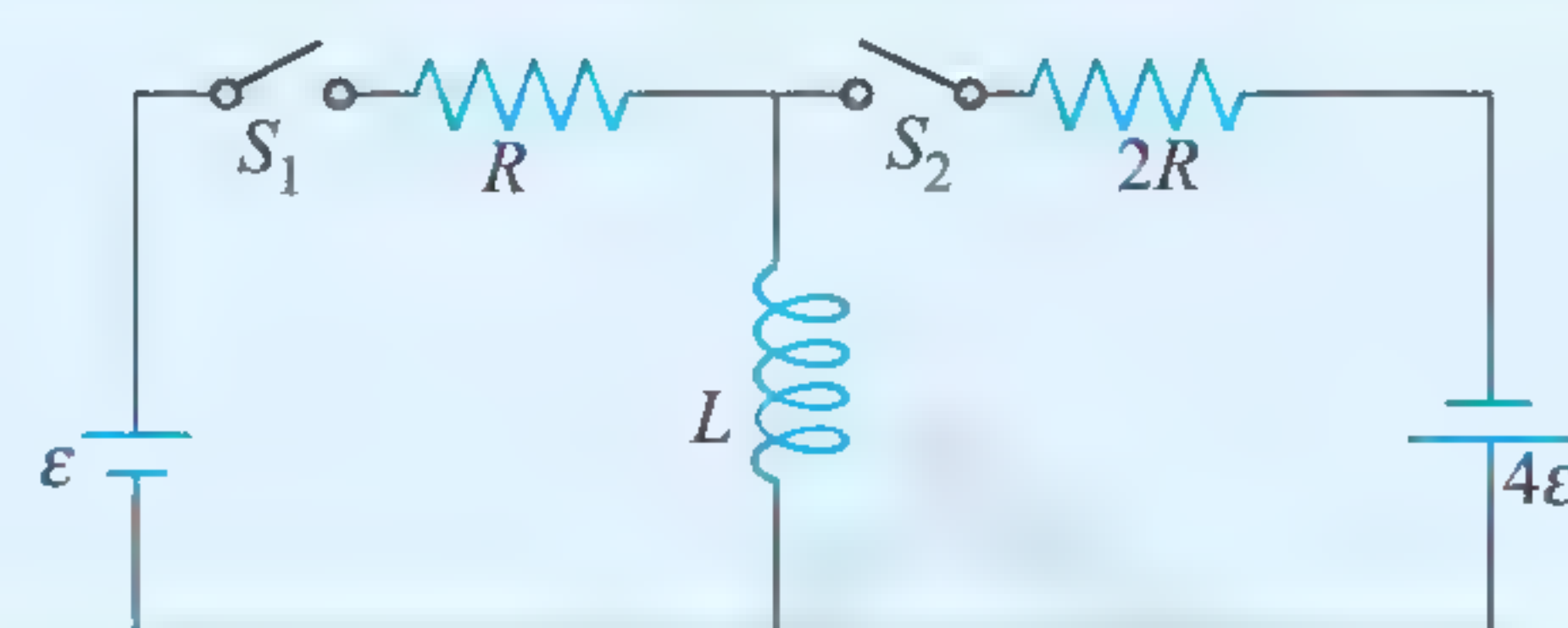


7. AB is a part of circuit. Find the potential difference $V_A - V_B$ if



- (a) current $i = 2$ A and is constant;
 (b) current $i = 2$ A and is increasing at the rate of 1 A s^{-1} ;
 (c) current $i = 2$ A and is decreasing at the rate 1 A s^{-1} .

8. In a circuit S_1 remains closed for a long time and S_2 remains open. Now S_2 is closed and S_1 is opened. Find out the di/dt in the right loop just after that moment.



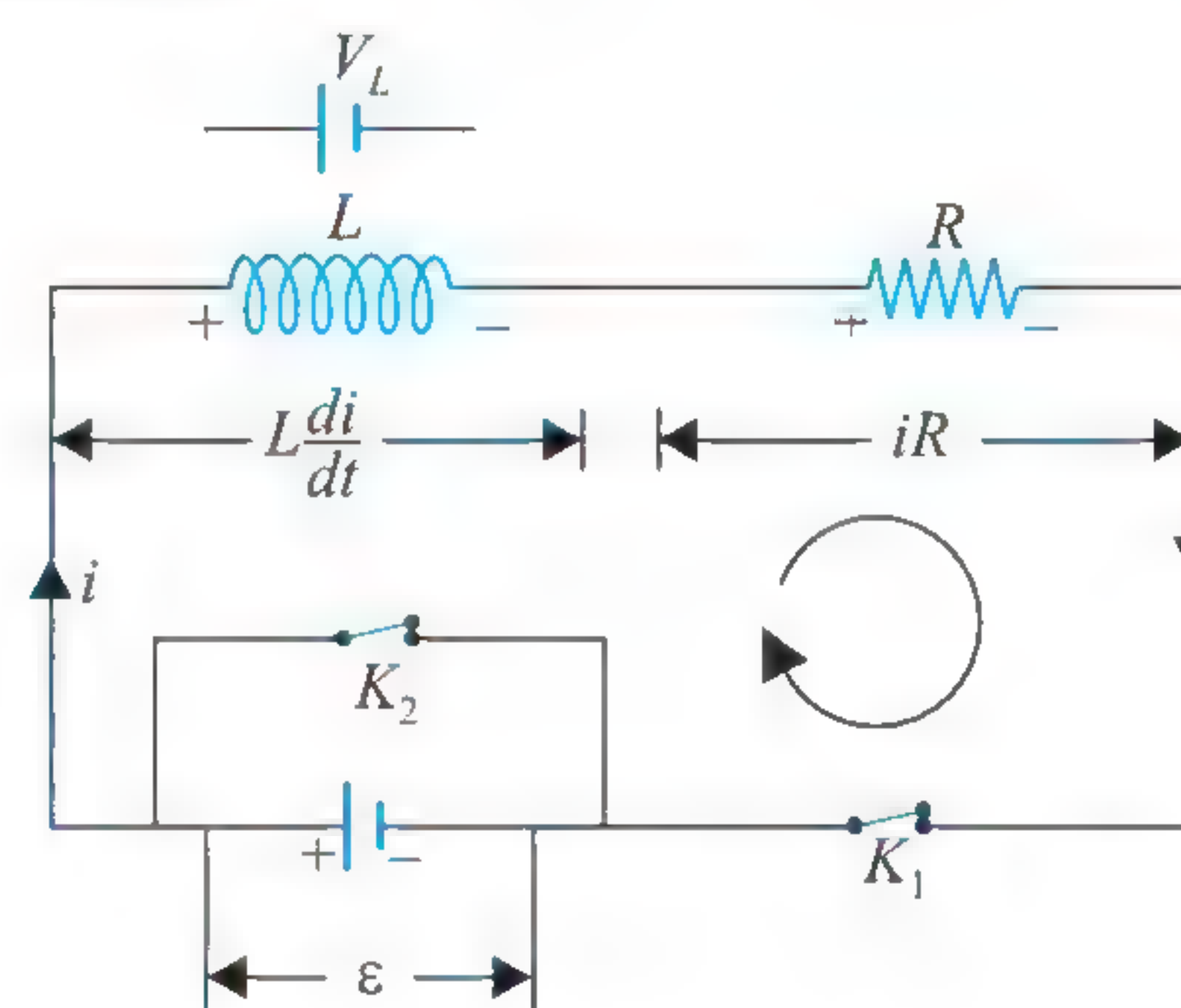
ANSWERS

2. 50 mH 3. (a) decreasing (b) 4.0 mH
 4. (a) 4.0 H (b) 3.5Ω 5. $(42 + 20t) \text{ V}$ 6. 1.5 s
 7. (a) 9 V (b) 10 V (c) 8 V 8. $-\frac{6\varepsilon}{L}$

GROWTH AND DECAY OF CURRENT

GROWTH OF CURRENT

In the figure given a circuit with an inductor of inductance L connected in series with a resistor of resistance R across a battery of emf ε . In this circuit when the switch is closed at $t = 0$, a current start flowing and if at time ' t ' current flowing in circuit is considered to be i .



Applying Kirchhoff's loop Law,

$$-L \frac{di}{dt} - iR + \varepsilon = 0 \quad \dots(i)$$

$$L \frac{di}{dt} + iR = \varepsilon \quad \text{or} \quad L \frac{di}{dt} = \varepsilon - iR$$

$$\text{or} \quad \frac{di}{\varepsilon - iR} = \frac{dt}{L}$$

Since, current increases from 0 to i during a time t , now integrating both sides

$$\text{or} \quad \int_0^i \frac{di}{\varepsilon - iR} = \frac{1}{L} \int_0^t dt$$

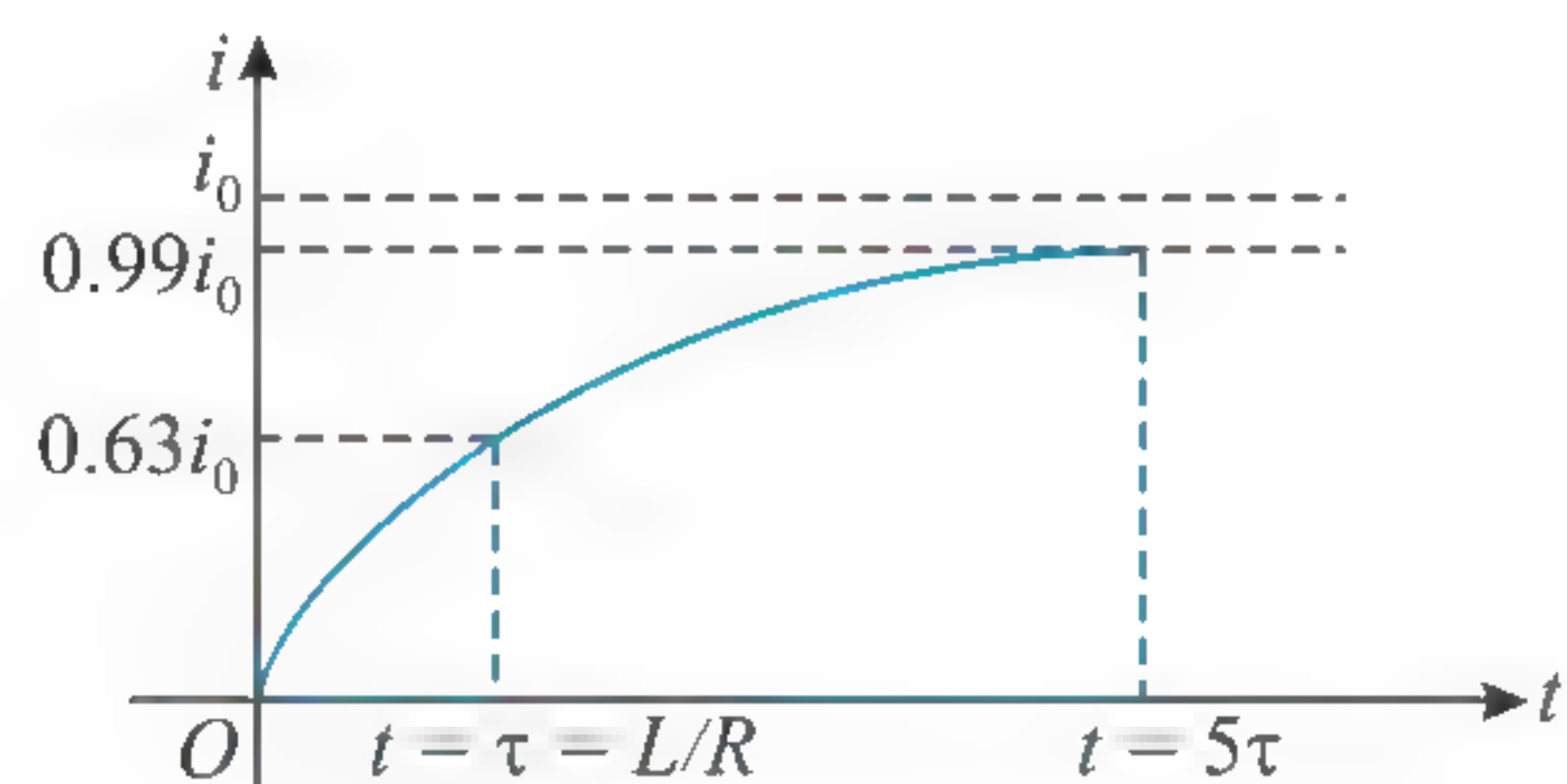
$$\text{or} \quad i = \frac{\varepsilon}{R} \left(1 - e^{-\frac{Rt}{L}} \right) = \frac{\varepsilon}{R} \left(1 - e^{-\frac{t}{\tau}} \right) \quad \dots(ii)$$

where $\tau = \frac{L}{R}$ is known as the time constant of L - R circuit.

Equation (ii) tells that, the current builds up exponentially to a steady value of magnitude $i_0 = \frac{\varepsilon}{R}$ after a long time ($t \rightarrow \infty$).

If we put $t = \tau$ in Eq. (ii) we have $i = i_0 \left(1 - \frac{1}{e}\right) = 0.63i_0$

During a time equal to one time constant from beginning, current builds upto 63% of the steady state current.



The current after five times the time constant then we have

$$i = i_0 (1 - e^{-5}) \Rightarrow i = 0.993i_0$$

From above equation, it can be seen that in duration equal to five times of time constant current attains 99.3% of steady state value and remaining 0.7% will be achieved theoretically in infinite time.

DECAY OF CURRENT

After reaching the current at steady state, the key K_2 is closed (say at $t = 0$), let the current fall at a rate of di/dt . Hence, the inductor acts as a battery to continue in sending the current in same direction. Substituting $\mathcal{E} = 0$ in Eq. (i), we get the differential equation

$$L \frac{di}{dt} + iR = 0$$

$$\text{or } \frac{di}{i} = -\frac{R}{L} dt$$

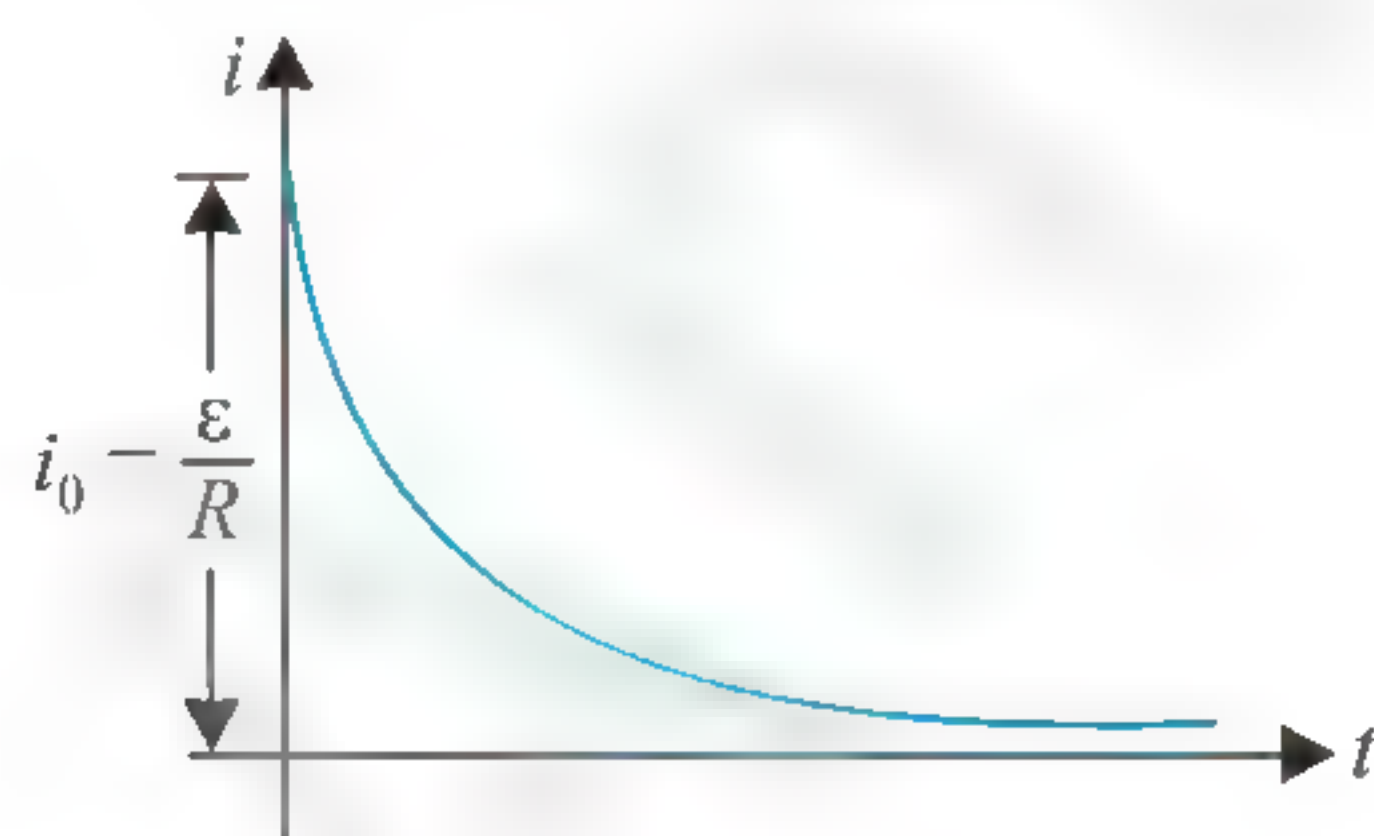
If the current falls down from i_0 to i during a time t after closing the key K_2 ,

$$\int_{i_0}^i \frac{di}{i} = -\frac{R}{L} \int_0^t dt \quad \dots(\text{iii})$$

After integrating Eq. (iii), we get

$$i = i_0 e^{-\frac{Rt}{L}} = i_0 e^{-\frac{t}{\tau}} \quad \dots(\text{iv})$$

It shows that the current decays exponentially, theoretically it takes infinite time ($t \rightarrow \infty$) for complete decay of the current.



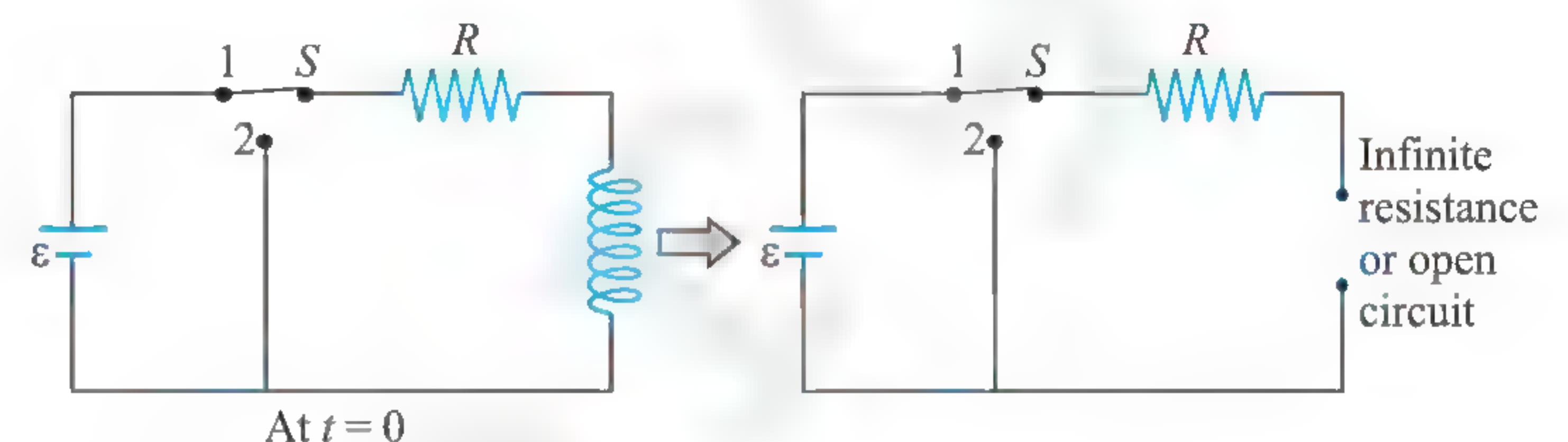
Instantaneous Behavior of an Inductor: An LR circuit is analyzed in three states:

- Initial state, i.e., just after closing the switch or just after opening the switch.
- Transient state or instantaneous state, i.e., any time after closing or opening the switch.
- Steady state, i.e., a long time after closing or opening the switch. In this state, current in the inductor does not varying with time, i.e., $\frac{di}{dt} = 0$

Rise of current in inductor circuit is given by, $i_t = \frac{\mathcal{E}}{R} \left(1 - e^{-\left(\frac{R}{L}\right)t}\right)$

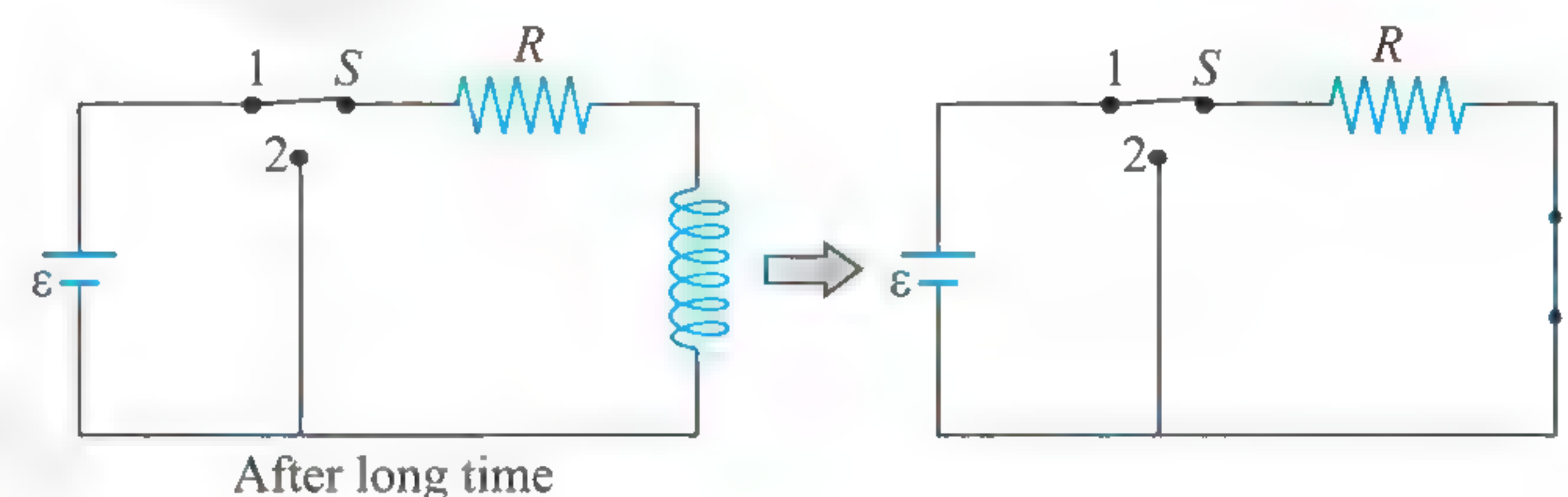
$$\text{At } t = 0, i_t = \frac{\mathcal{E}}{R} \left(1 - e^{-\left(\frac{R}{L}\right)0}\right) = 0$$

At $t = 0$, opposition produced by the inductor is infinite. Hence it acts as an infinite resistance or open circuit.



No current flow through the circuit at the instant of closing the switch.

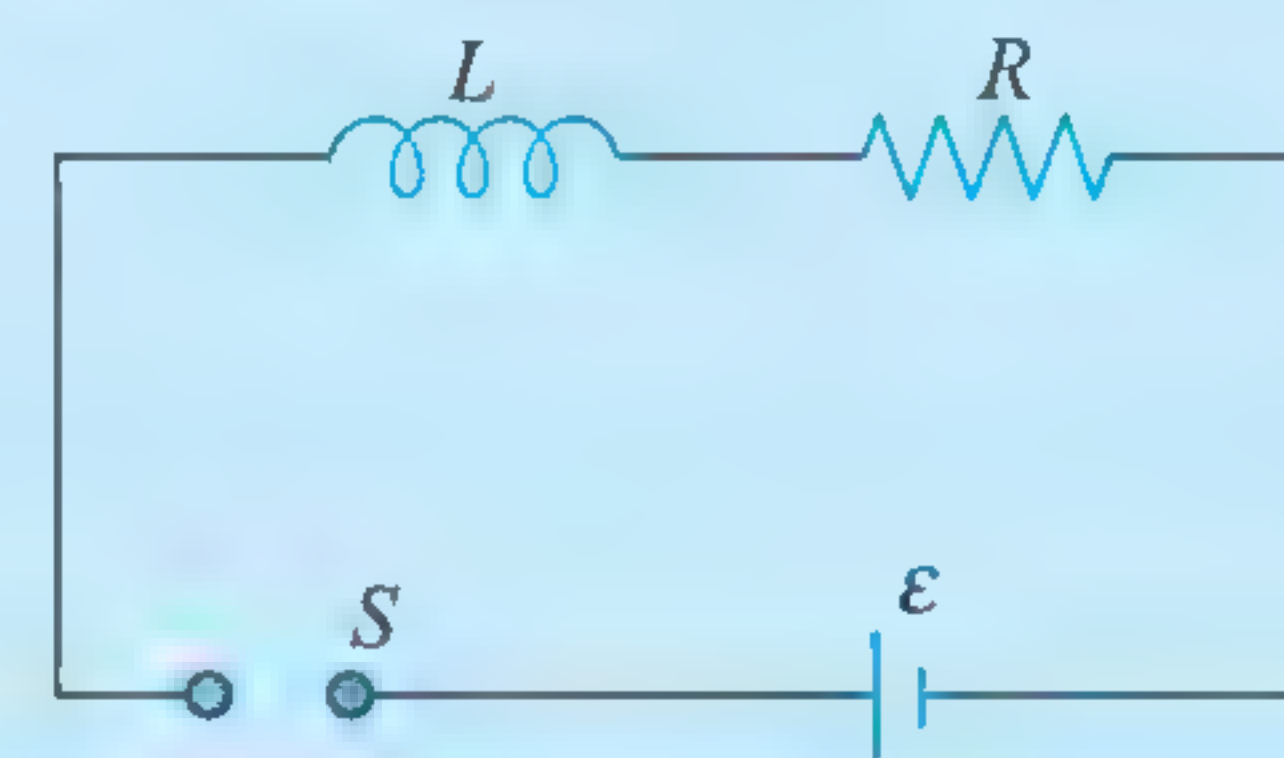
$$\text{After long time, at } t = \infty, i_t = \frac{\mathcal{E}}{R} \left(1 - e^{-\left(\frac{R}{L}\right)\infty}\right) = \frac{\mathcal{E}}{R}$$



At $t = \infty$, the current has risen to its maximum value, and the inductor does not produce any opposition, it acts as zero resistance or conducting wire.

ILLUSTRATION 5.16

In an LR circuit as shown in figure, when the switch is closed, how much time will it take for the current to grow to a value n times the maximum value of current (where $n < 1$)?



Sol. We know that $i = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau})$

$$i = n \frac{\mathcal{E}}{R} \quad (\text{given})$$

$$n \frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau})$$

$$\text{or } e^{-t/\tau} = 1 - n$$

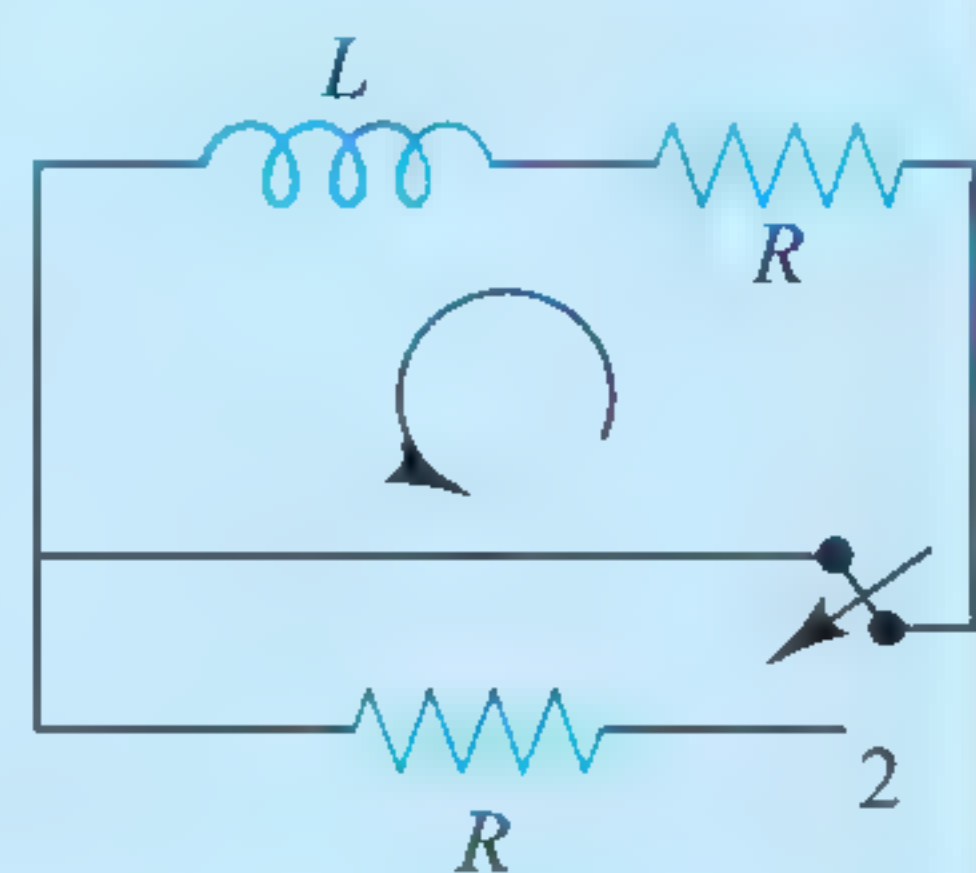
$$\text{or } t = \tau \ln \left(\frac{1}{1 - n} \right)$$

ILLUSTRATION 5.17

In the circuit shown in figure, the initial current through the inductor at $t = 0$ is I_0 . After a time $t = L/R$, the switch is quickly shifted to position 2.

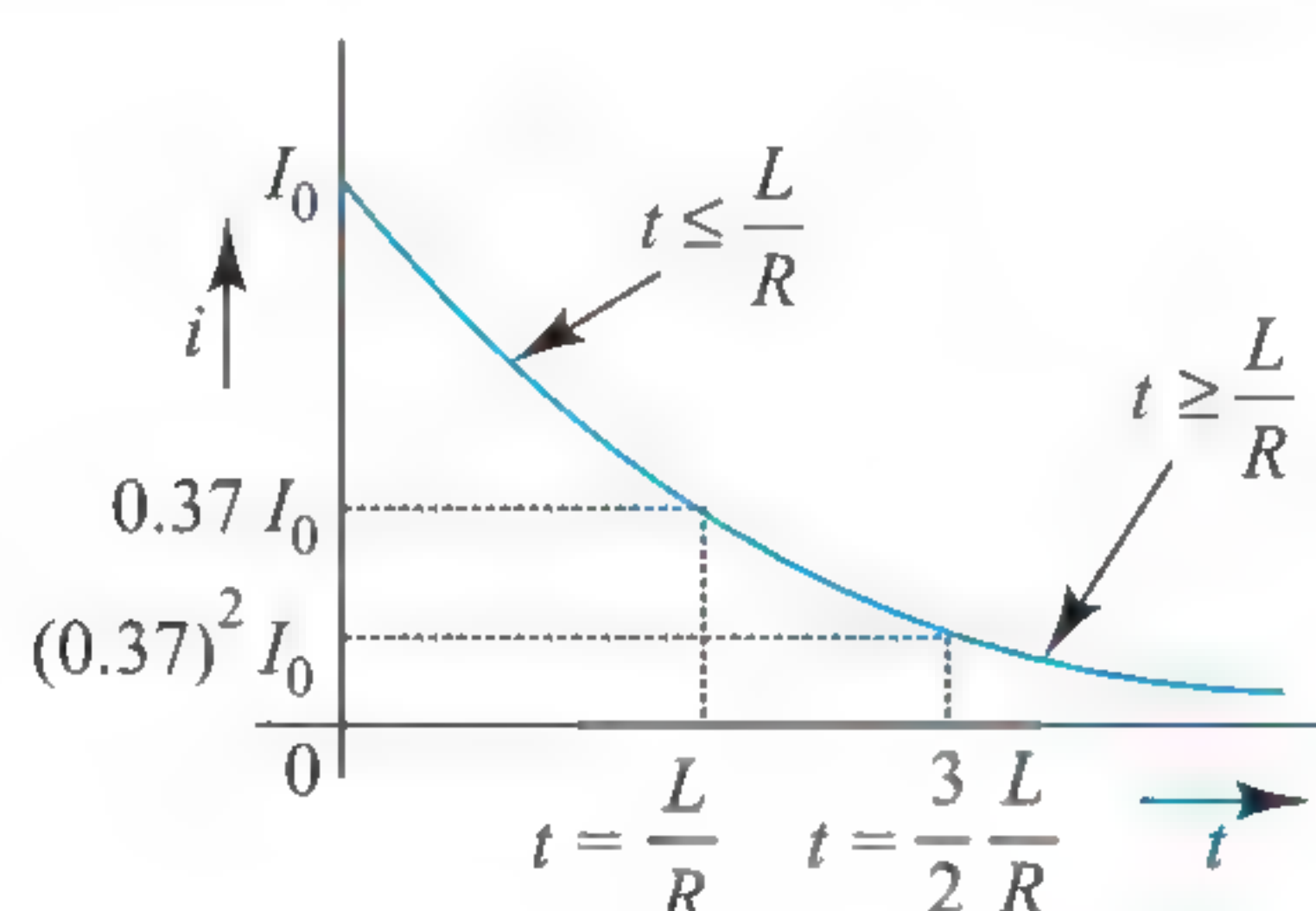
(a) Plot a graph showing the variation of current with time.

(b) Calculate the value of current in the inductor at $t = \frac{3L}{2R}$.

**Sol.**

(a) Shifting the position of switch increases the resistance of the circuit. This decreases the value of time constant. Consequently, the rate of decrease of current increases. The variation of current with time is shown in figure.

(b) The equations of the curves are $i = I_0 e^{-Rt/L}$ (for $0 \leq t \leq L/R$)



For $\frac{R}{L} \leq t \leq \infty$ and $i = (0.37) I_0 e^{-\frac{2R}{L}(t - \frac{L}{R})}$

At $t = \frac{3L}{2R}$; hence $i = (0.37) I_0 e^{-\frac{2R}{L}(\frac{3L}{2R} - \frac{L}{R})}$ or $i = (0.37)^2 I_0$

ILLUSTRATION 5.18

During the decay of current in an LR circuit, if the current falls to η times the initial value in time T , then determine the value of time constant.

Sol. We know that $I = I_{\max} e^{-t/T}$

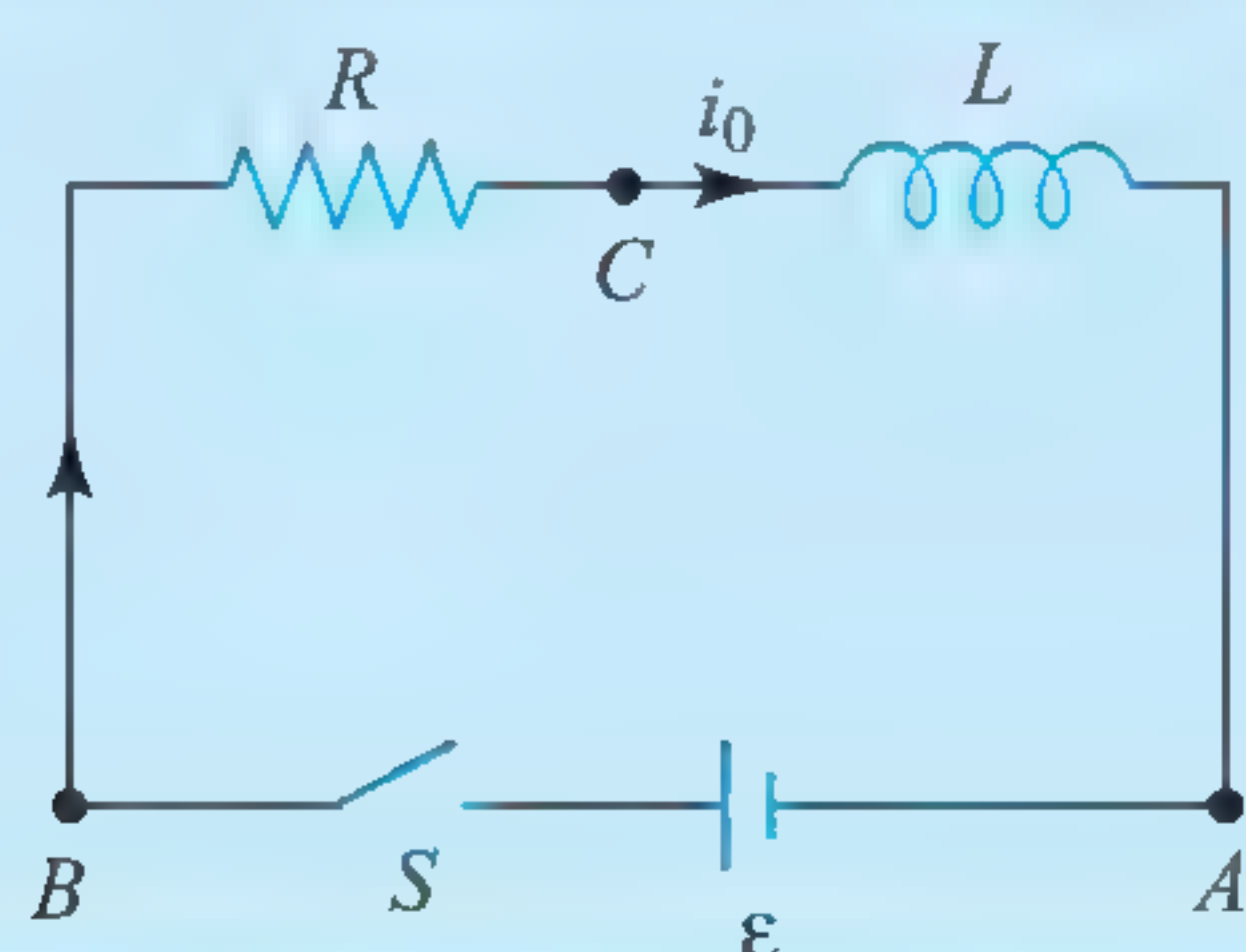
At $t = T$, $I = \eta I_{\max}$

$$\eta I_{\max} = I_{\max} e^{-t/T}$$

or $e^{-t/T} = \eta$; $T = \frac{t}{\ln |\frac{1}{\eta}|}$

ILLUSTRATION 5.19

Figure shows a circuit consisting of an ideal cell, an inductor L , and a resistor R , connected in series. Let switch S be closed at $t = 0$. Suppose at $t = 0$, the current in the inductor is i_0 , then find out the equation of current as a function of time.

**Sol.** Let at an instant t , the current in the circuit is i which is increasing at the rate di/dt .

Writing KVL along the circuit, we have

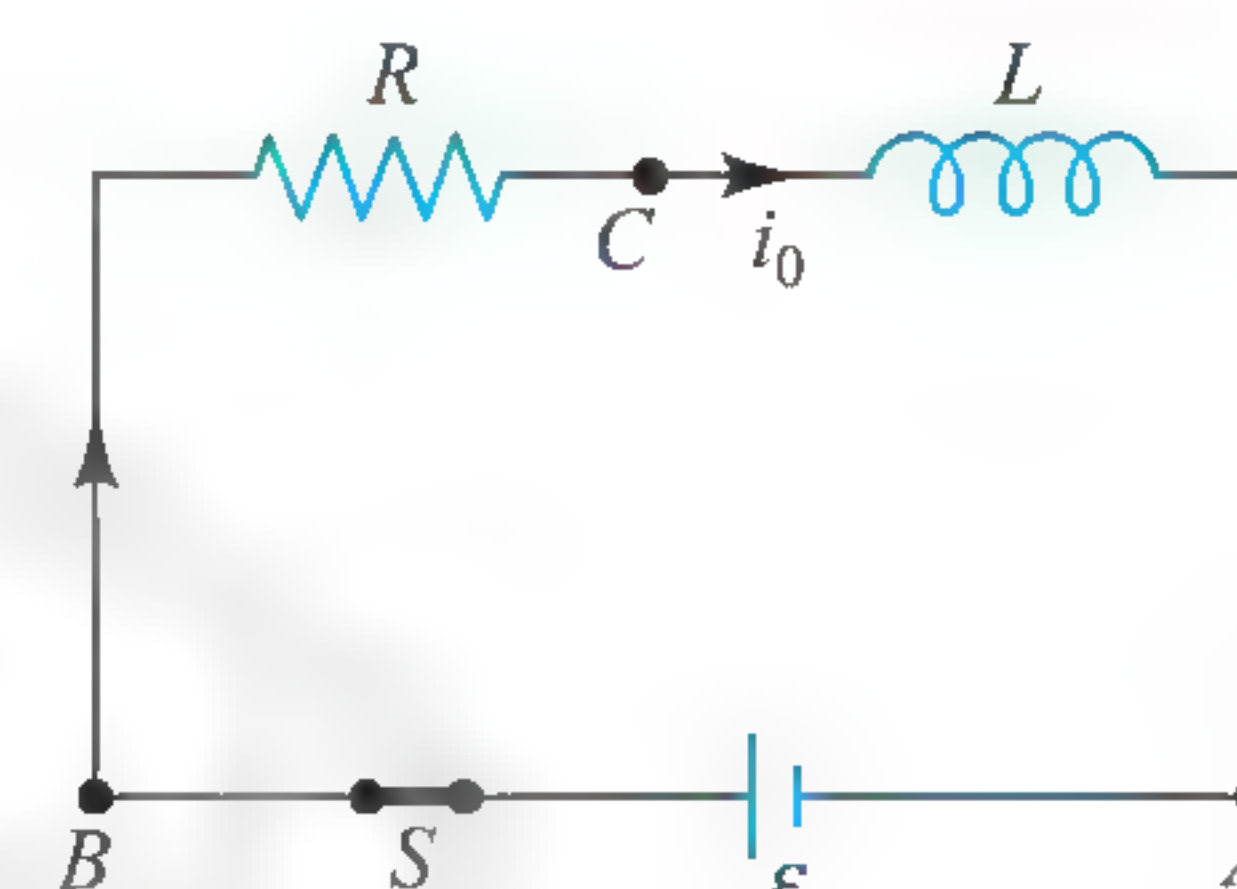
$$L \frac{di}{dt} = \varepsilon - iR$$

$$\Rightarrow \int_{i_0}^i \frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L}$$

$$\Rightarrow \ln \left(\frac{\varepsilon - iR}{\varepsilon - i_0 R} \right) = -\frac{Rt}{L}$$

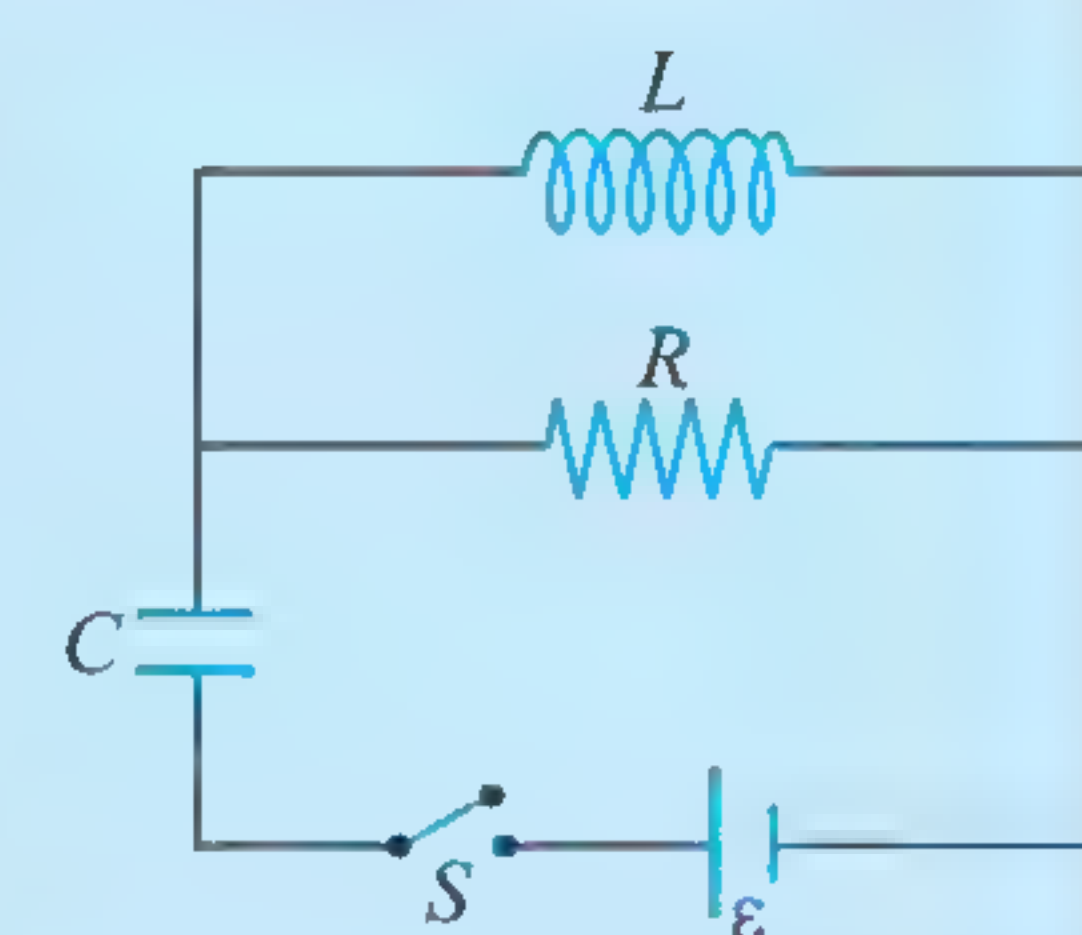
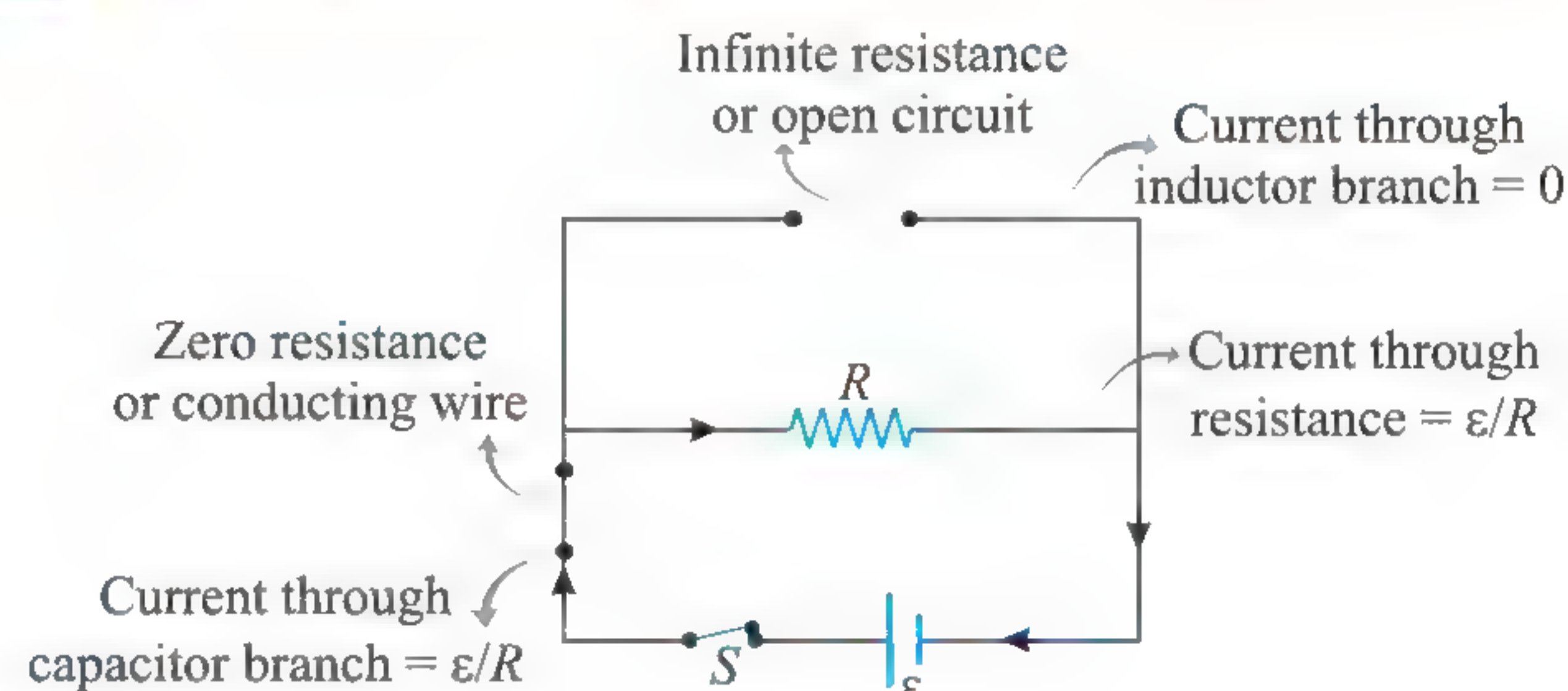
$$\Rightarrow \varepsilon - iR = (\varepsilon - i_0 R) e^{-Rt/L}$$

$$\Rightarrow i = \frac{\varepsilon - (\varepsilon - i_0 R) e^{-Rt/L}}{R}$$

**ILLUSTRATION 5.20**

The open switch in figure is closed at $t = 0$, before the switch is closed the capacitor is uncharged and all currents are zero.

Determine the currents in L , C , and R , the emf across L , and the potential differences across C and R at the instant after the switch is closed and long after it is closed.

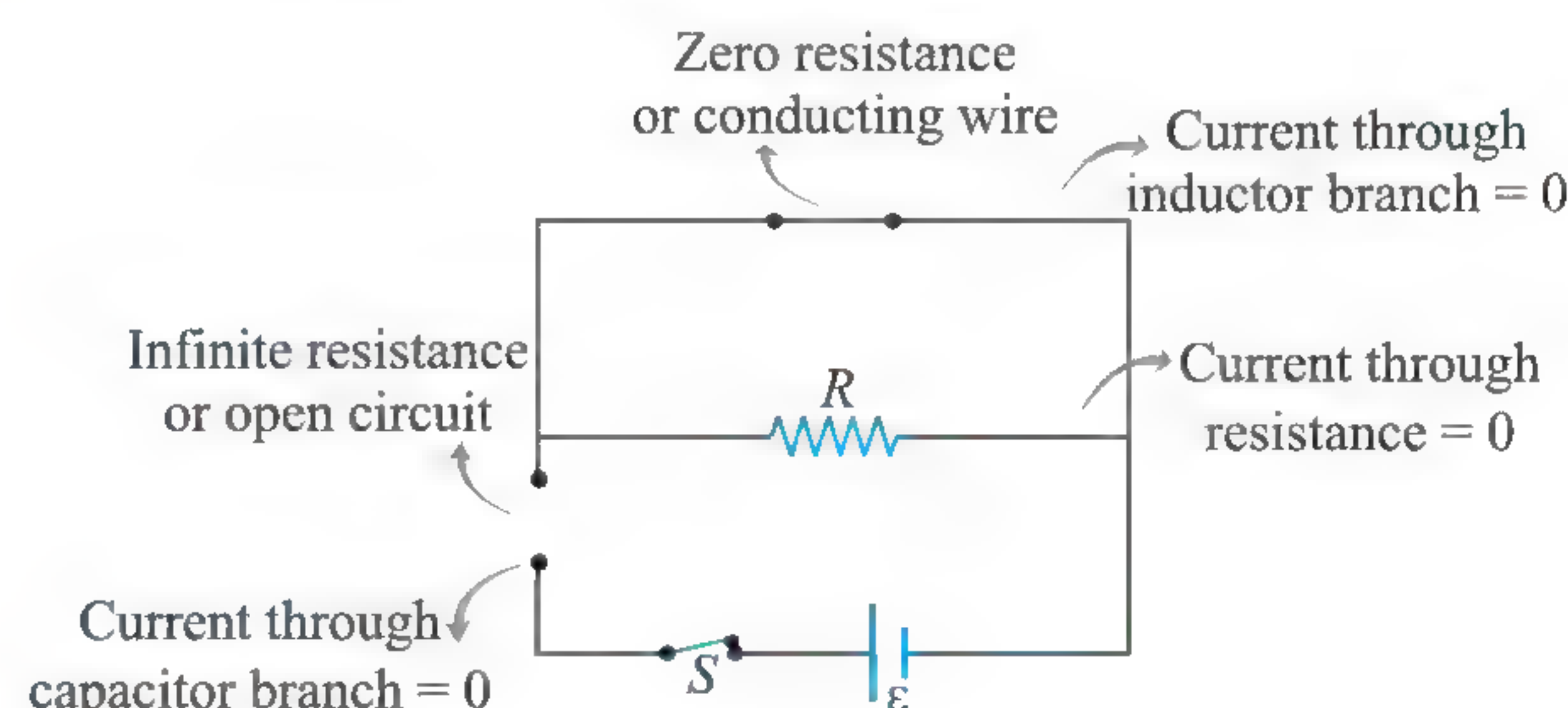
**Sol.** Just after closing the switch

Potential difference across capacitor = 0

Potential difference across resistance = ε

E.M.F. across inductor coil = ε

Long after closing the switch



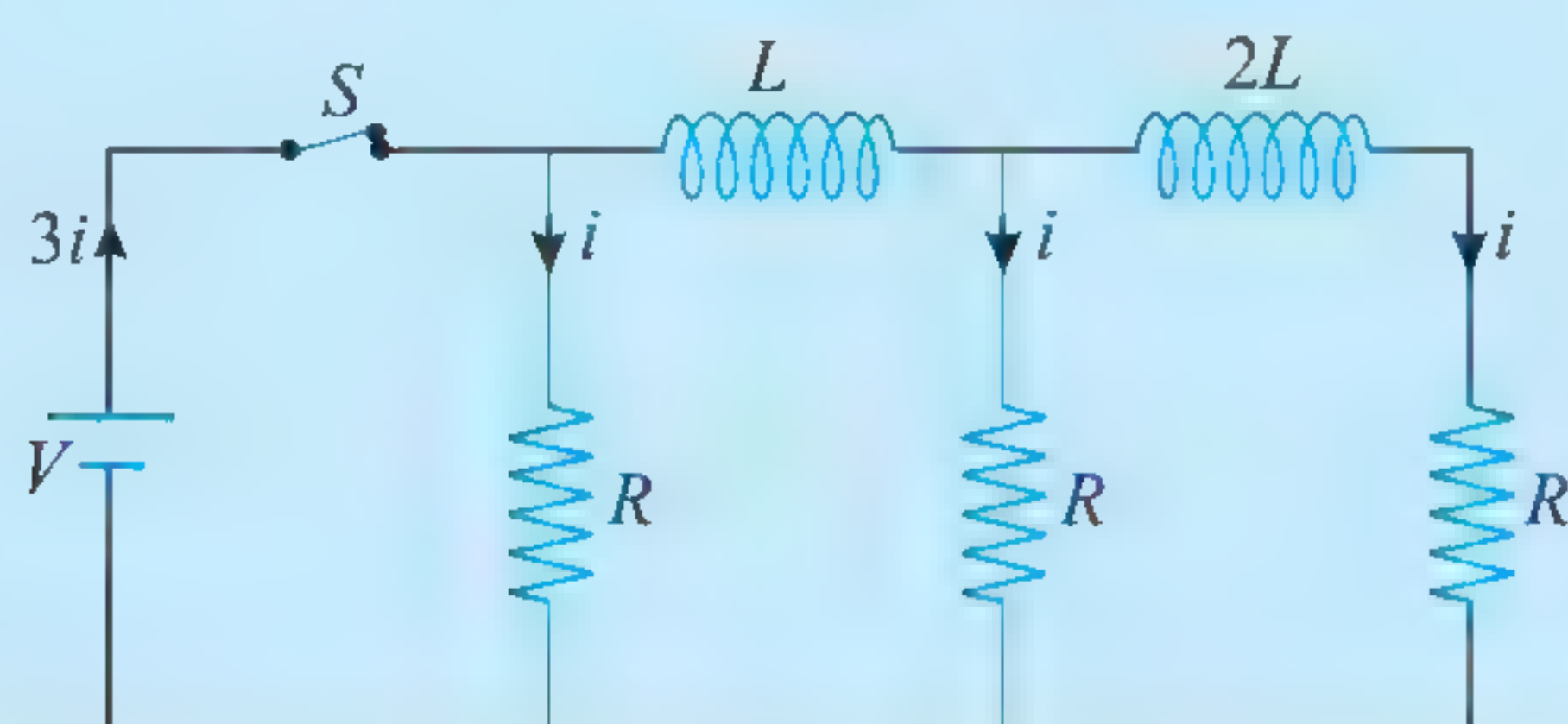
Potential difference across capacitor = ε

Potential difference across resistance = 0

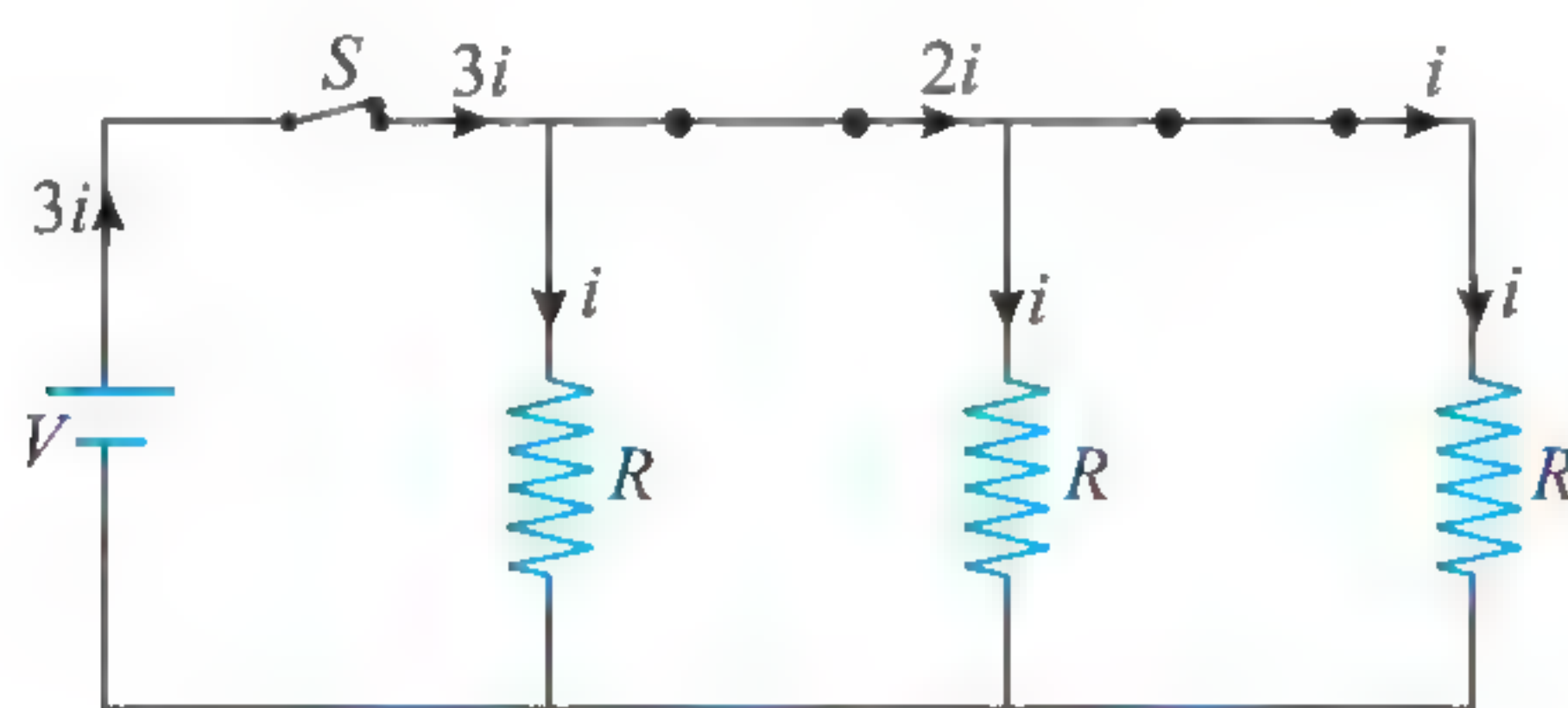
E.M.F. across inductor coil = 0

ILLUSTRATION 5.21

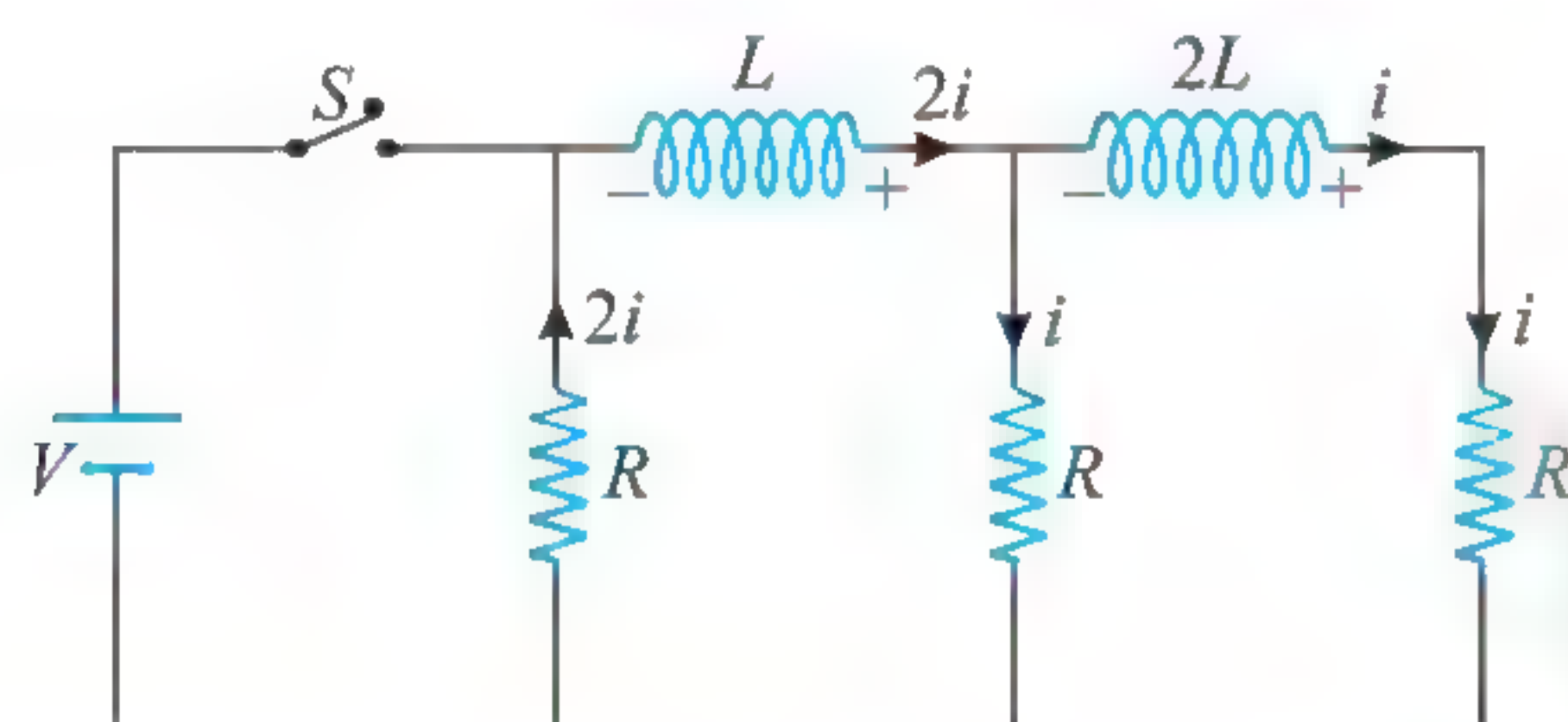
In the circuit shown in figure is at steady state, if $R_1 = R_2 = R_3 = R$ and the current through each resistor is i . Find the currents through the resistors immediately after the switch 'S' is opened.



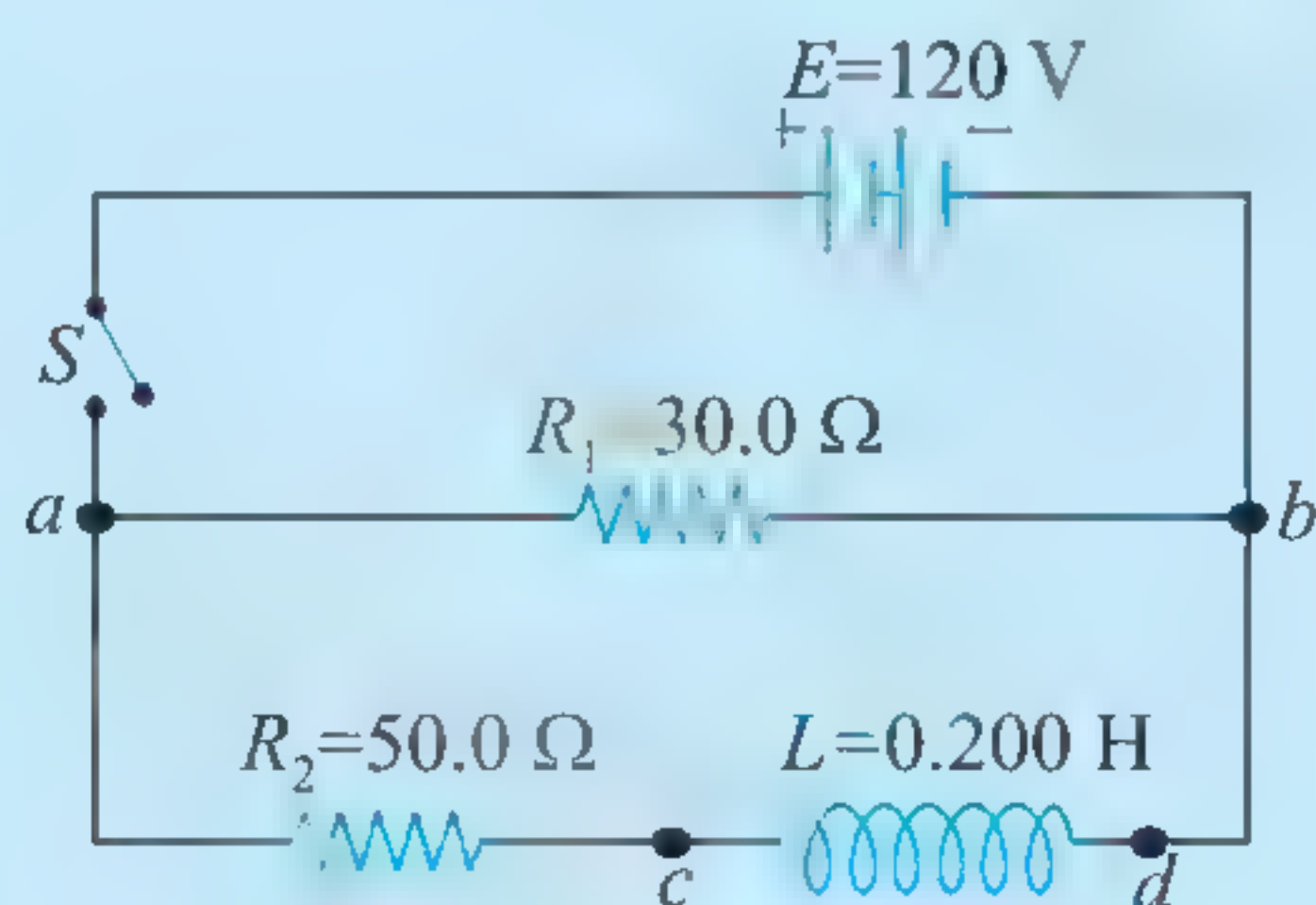
Sol. In steady state the inductor coils acts as conducting wires, the currents in different branches will be as shown in figure.



Immediately after opening the switch, the current through inductors will not change. Now at this stage the current will be supplied by the inductor coils only. The distribution of currents in different branches is shown in figure.

**ILLUSTRATION 5.22**

In the circuit shown in figure, $E = 120 \text{ V}$, $R_1 = 30.0 \Omega$, $R_2 = 50.0 \Omega$, and $L = 0.200 \text{ H}$. Switch S is closed at $t = 0$. Just after the switch is closed,



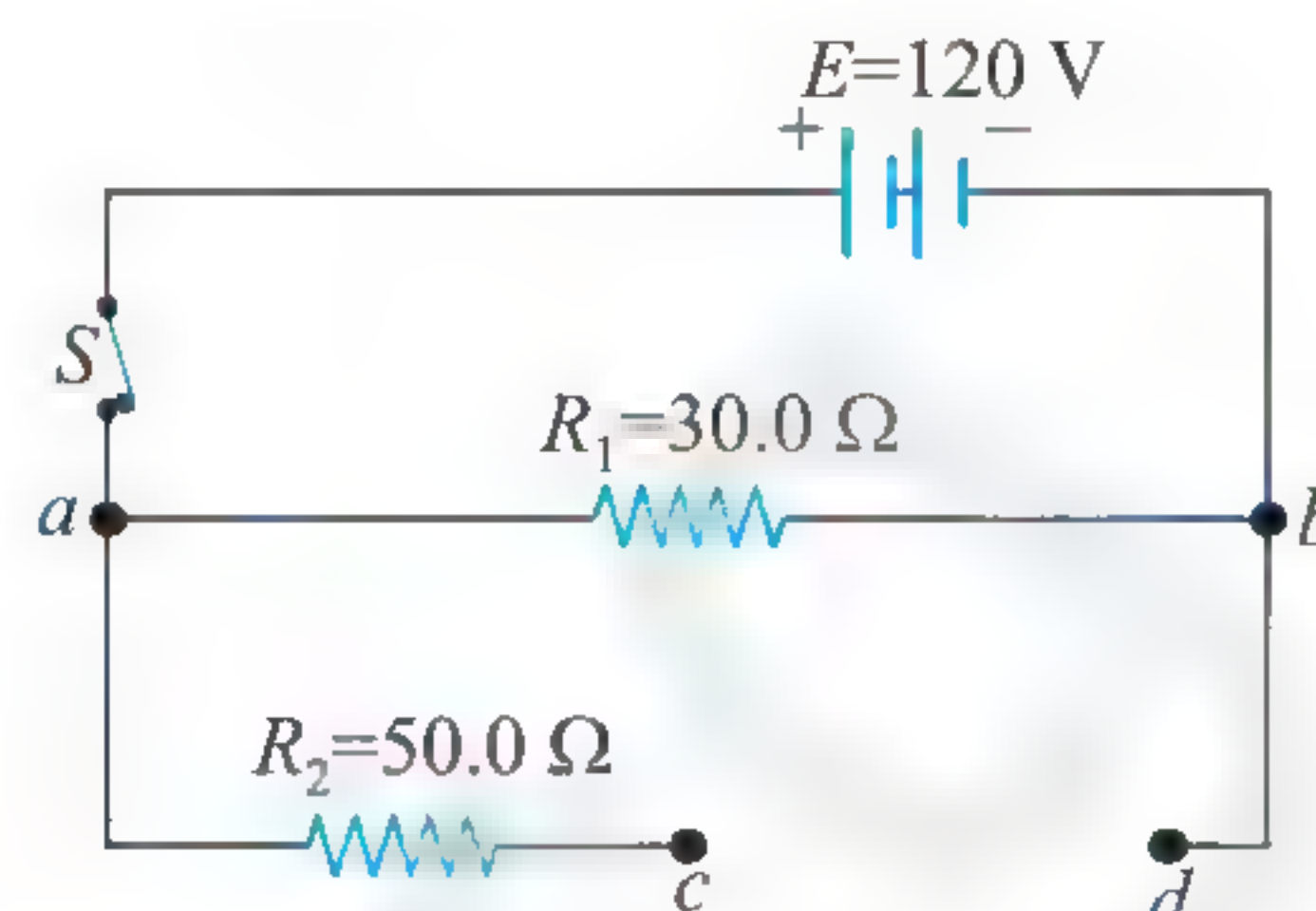
- What is the potential difference V_{ab} across the resistor R_1 ?
- Which point, c or d , is at higher potential?
- What is the potential difference V_{cd} across the inductor just after switch is opened?

The switch is left closed a long time and then it is opened.

- Which point, a or b , is at higher potential? Find $V_b - V_a = V_{ba} = ?$
- What is the potential difference V_{cd} across the inductor?

Sol.

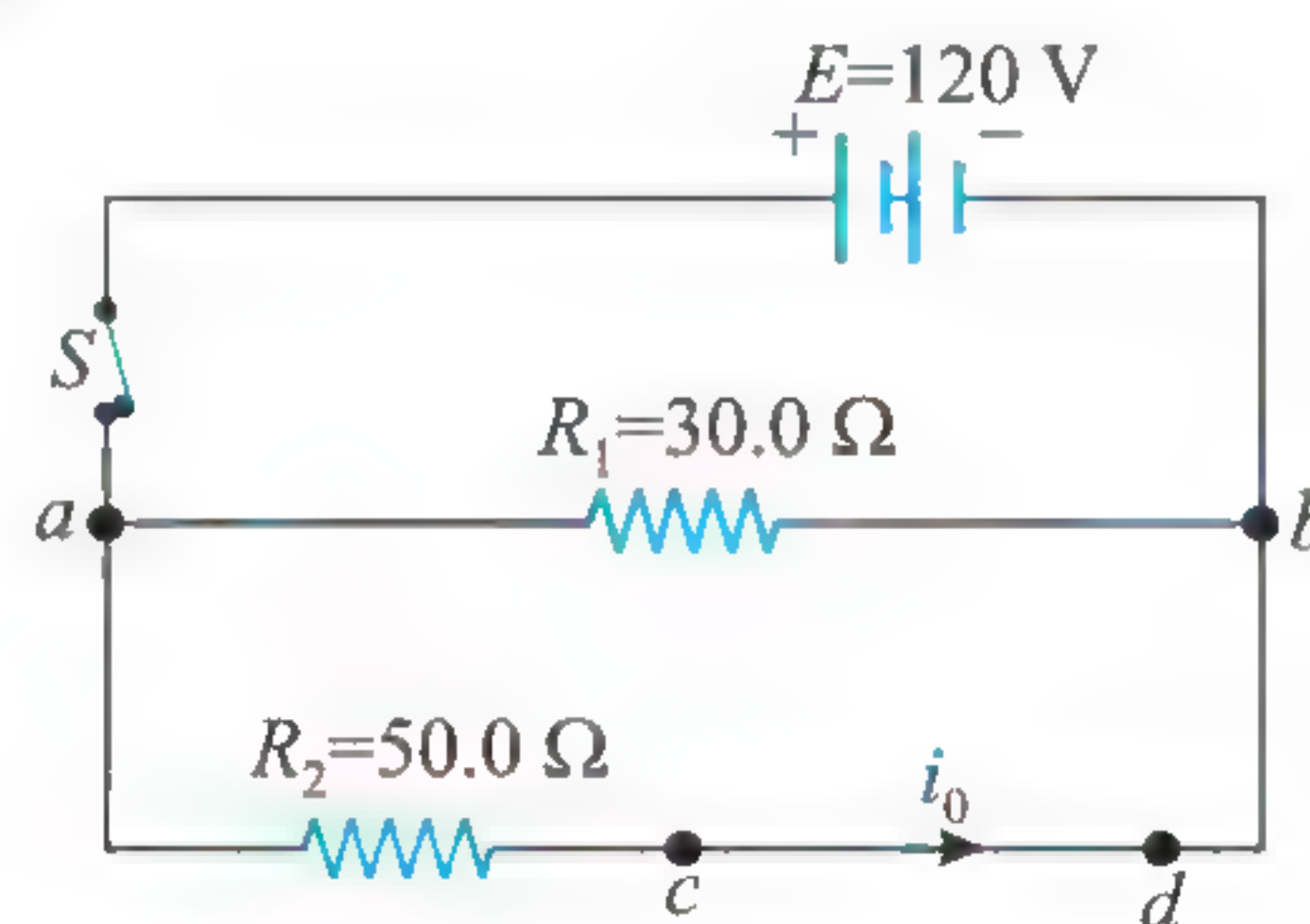
- Just after closing the switch, the inductor will act as infinite resistance, it will act as infinite resistance. Hence the current will flow through R_1 only



The potential difference across the resistor R_1 ,

$$V_{ab} = E = 120 \text{ V}$$

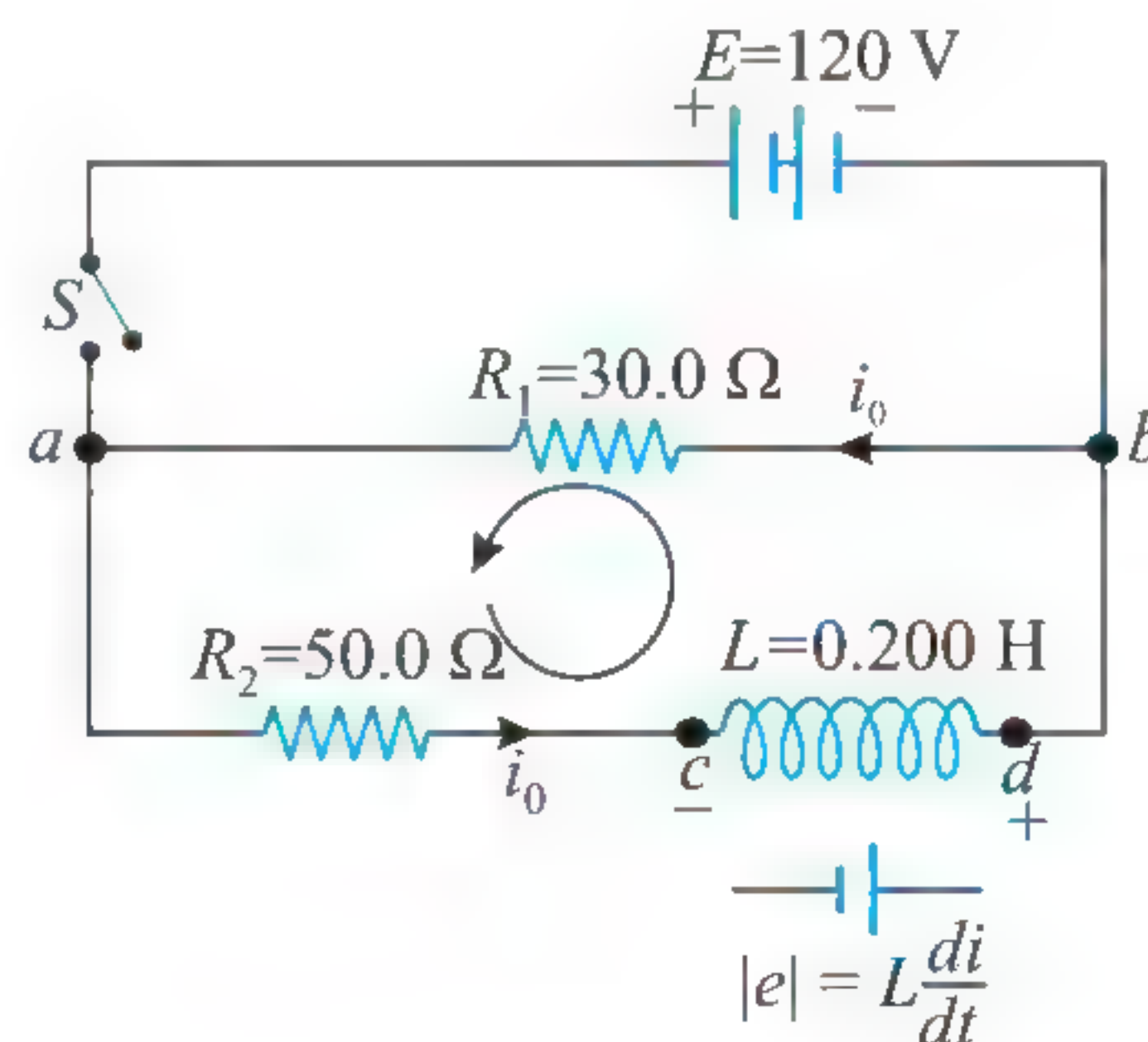
- The potential of point a = The potential of point c . Hence point c will be at higher potential.
- As no current in resistor R_2 , it means, $V_{cd} = V_{ab} = E = 120 \text{ V}$.
- The switch is left closed a long time, the inductor coil acts as conducting wire. Now the potential difference across resistor R_2 will be equal to 120 V. A constant current will flow through this branch.



Hence, just before opening the switch the current through inductor

$$I_0 = \frac{E}{R_2} = \frac{120}{50} = \frac{12}{5} \text{ A}$$

Now when the switch is opened, the inductor coil try to maintain the same current as it was flowing just before opening the switch hence end b of the coil will be at higher potential.



The potential difference across ba ,

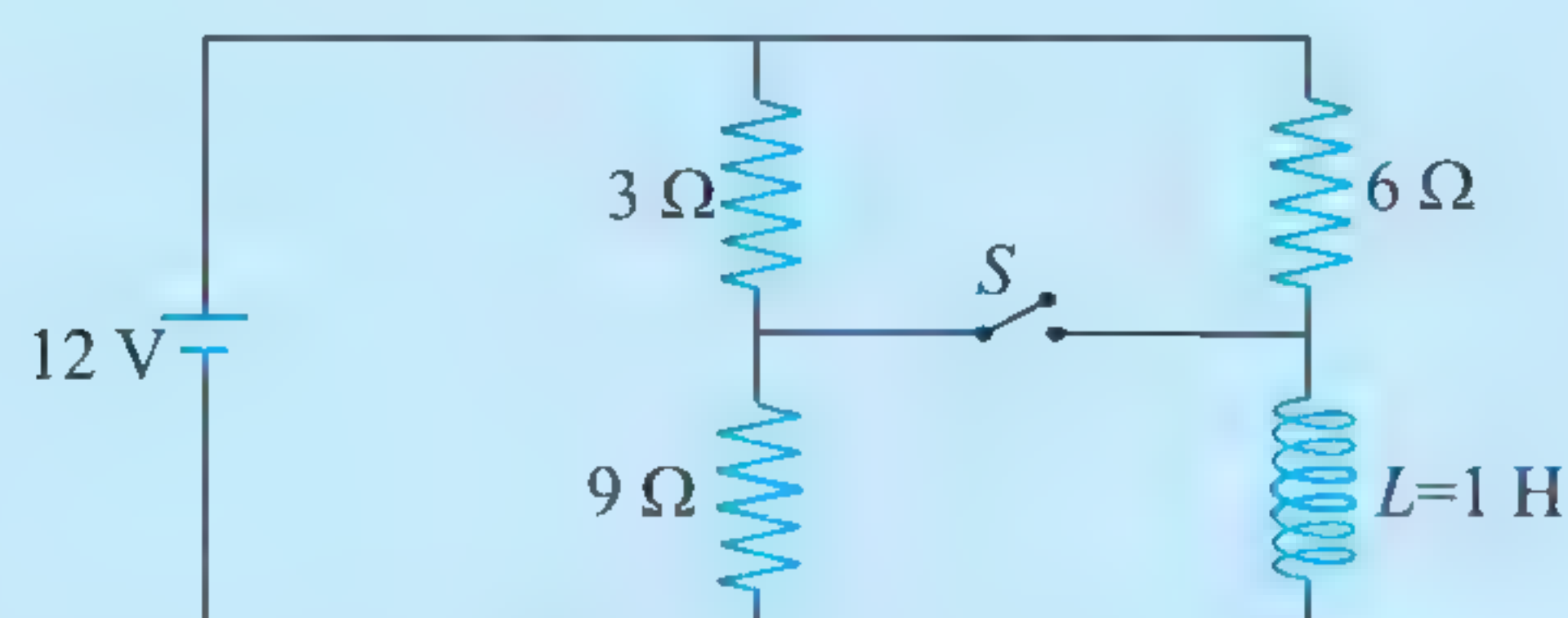
$$V_b - V_a = V_{ba} = R_1 i_0 = 30 \times \frac{12}{5} = 72 \text{ V}$$

- Just after opening the switch the current will flow in lower loop. The inductor coil will act as battery supplying equal current to both resistances R_1 and R_2 .

$$V_{d,c} = i_0 (R_1 + R_2) = \frac{12}{5} (30 + 50) = 192 \text{ V}$$

ILLUSTRATION 5.23

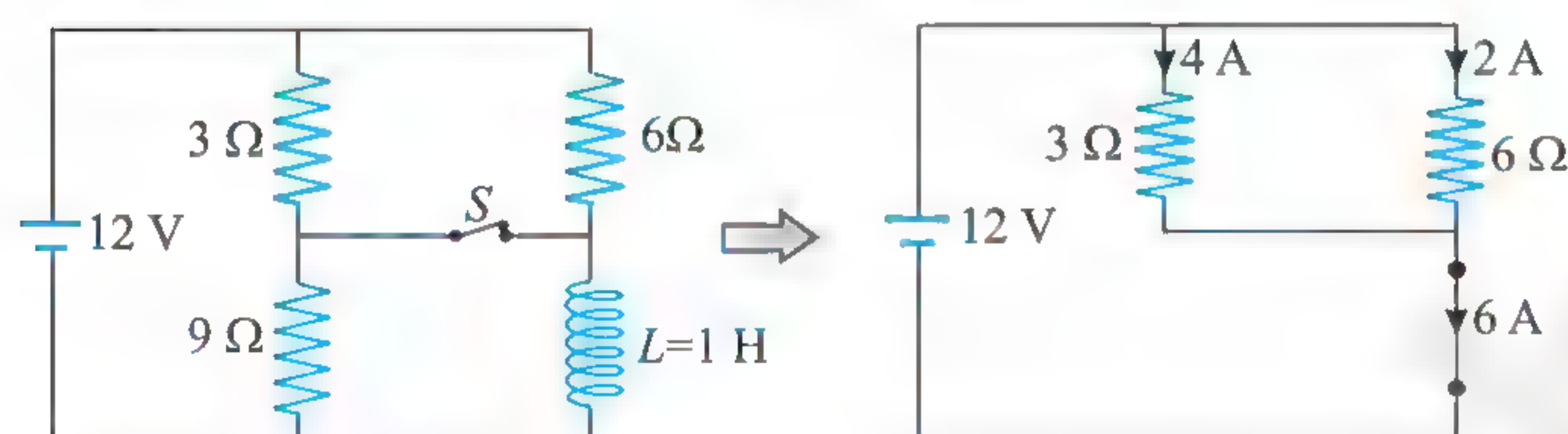
In the circuit shown, the switch 'S' has been closed for a long time and then opens at $t = 0$.



- Find the current through the inductor just before the switch is opened.
- Find the current through the inductor a long time after the switch is opened.
- Find current through the inductor as a function of time after the switch is opened. Also write the current through the cell as a function of time.

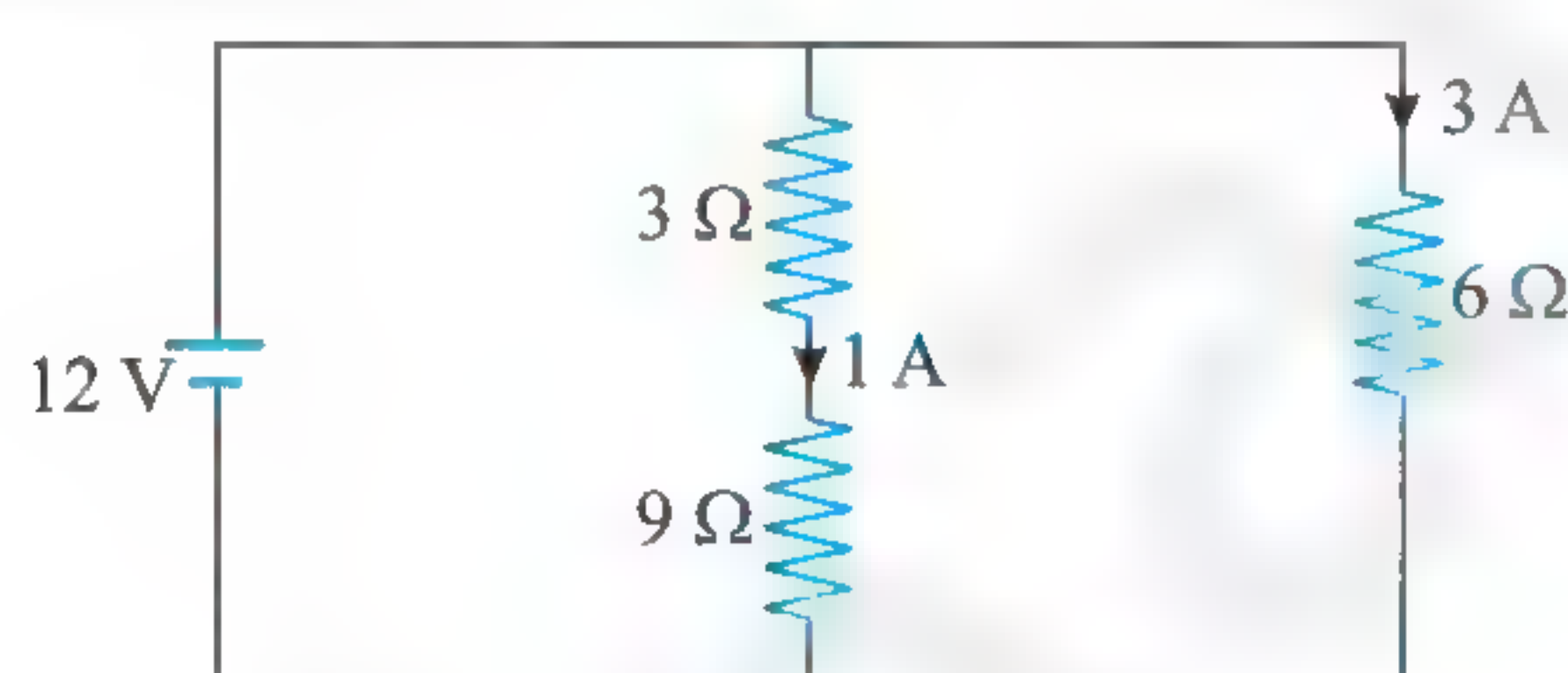
Sol.

- (a) When the switch 'S' is in closed position for a long time the circuit is in steady condition and the inductor acts as a short circuit or conducting wire.

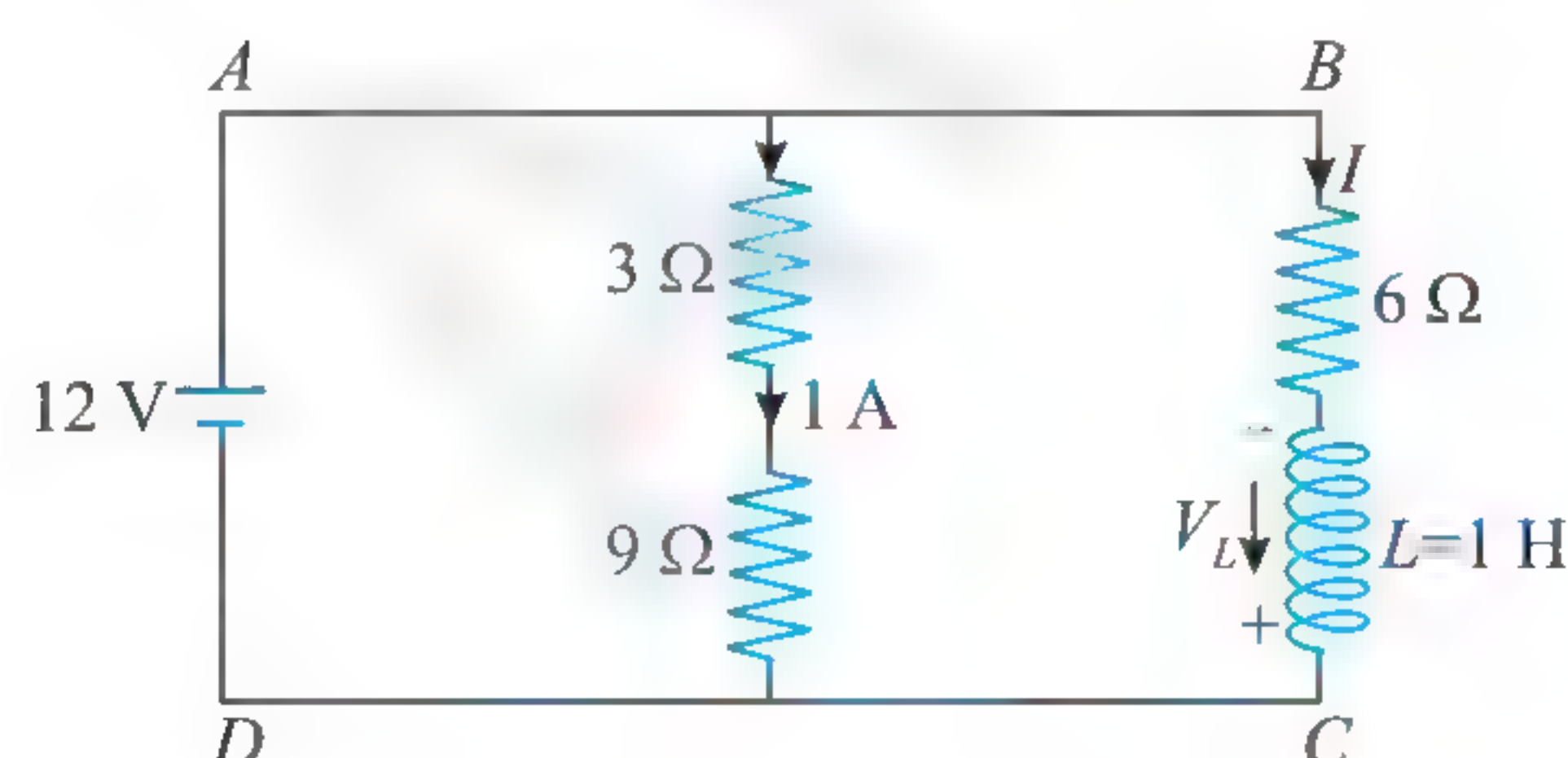


There is no potential difference across the inductor. It means no potential difference across 6Ω resistance is no current through 6Ω resistance. Effective circuit just before the switch is opened is as shown in figure. Required answer is 6 A.

- (b) After long time (at $t \rightarrow \infty$) once again the circuit will be in steady condition $I = 3$ A



- (c) At time 't' the circuit parameters are as shown in figure.



Applying Kirchhoff's Voltage law (KVL) in loop ABCDA:

$$6I + L \frac{dI}{dt} = 12$$

$$\int_0^I \frac{dI}{6I - 12} = - \int_0^t dt$$

$$= \frac{1}{2} [\ln(I - 2)]_6^I = -(t - 0)$$

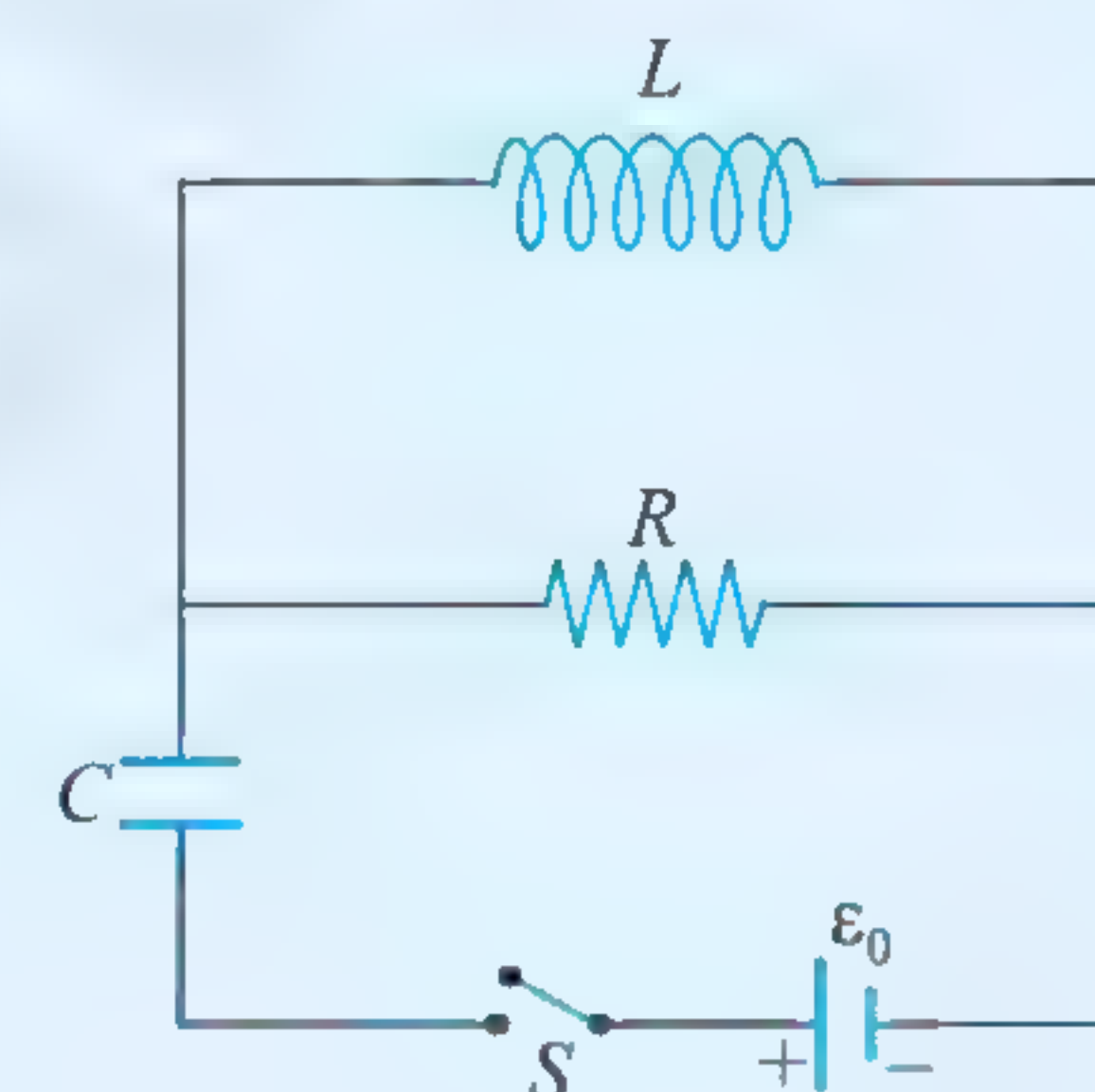
$$\ln[(I - 2) - \ln(6 - 2)] = -2t$$

$$\ln\left(\frac{I - 2}{4}\right) = -2t \Rightarrow \frac{I - 2}{4} = e^{-2t}$$

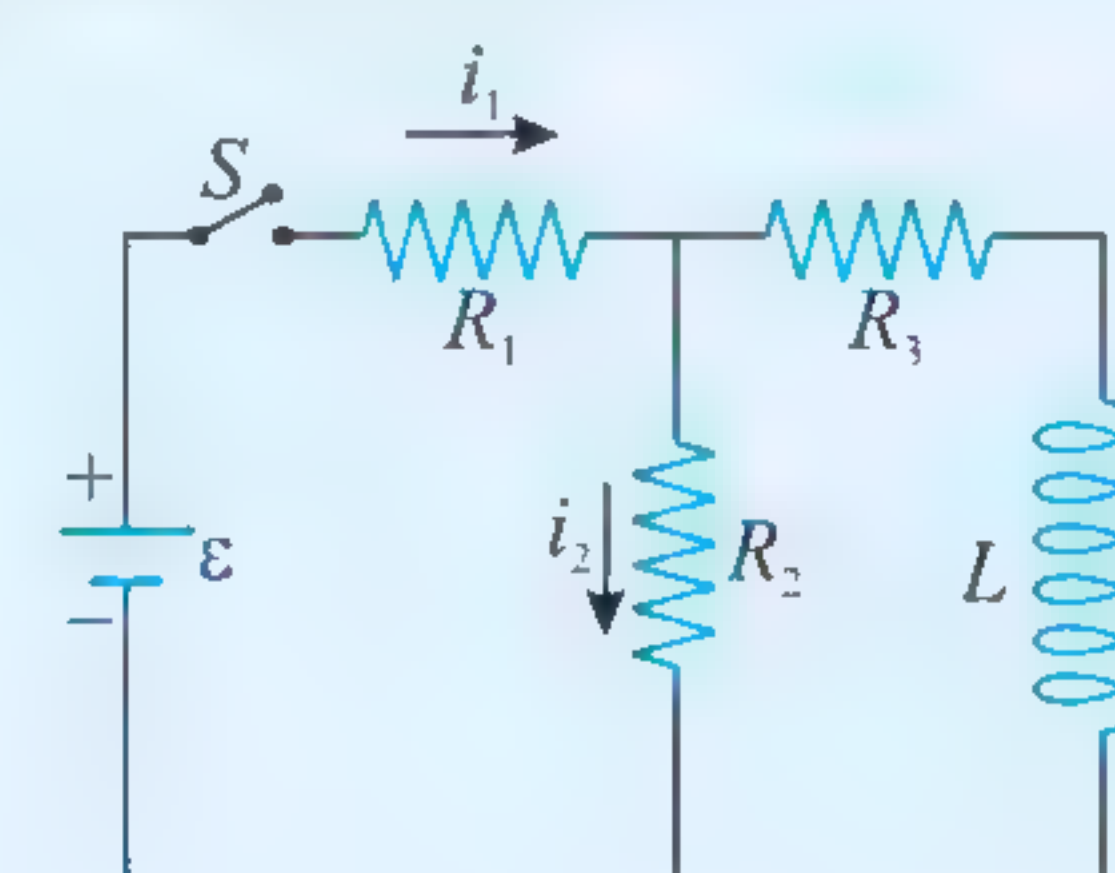
$$(I - 2) = 4e^{-2t} \Rightarrow I = 2 + 4e^{-2t}$$

CONCEPT APPLICATION EXERCISE 5.3

- The open switch in figure is thrown closed at $t = 0$. Before the switch is closed the capacitor is uncharged and all currents are zero. Determine the currents in L , C , and R , the emf across L , and the potential differences across C and R : (a) at the instant after the switch is closed and (b) long after it is closed.

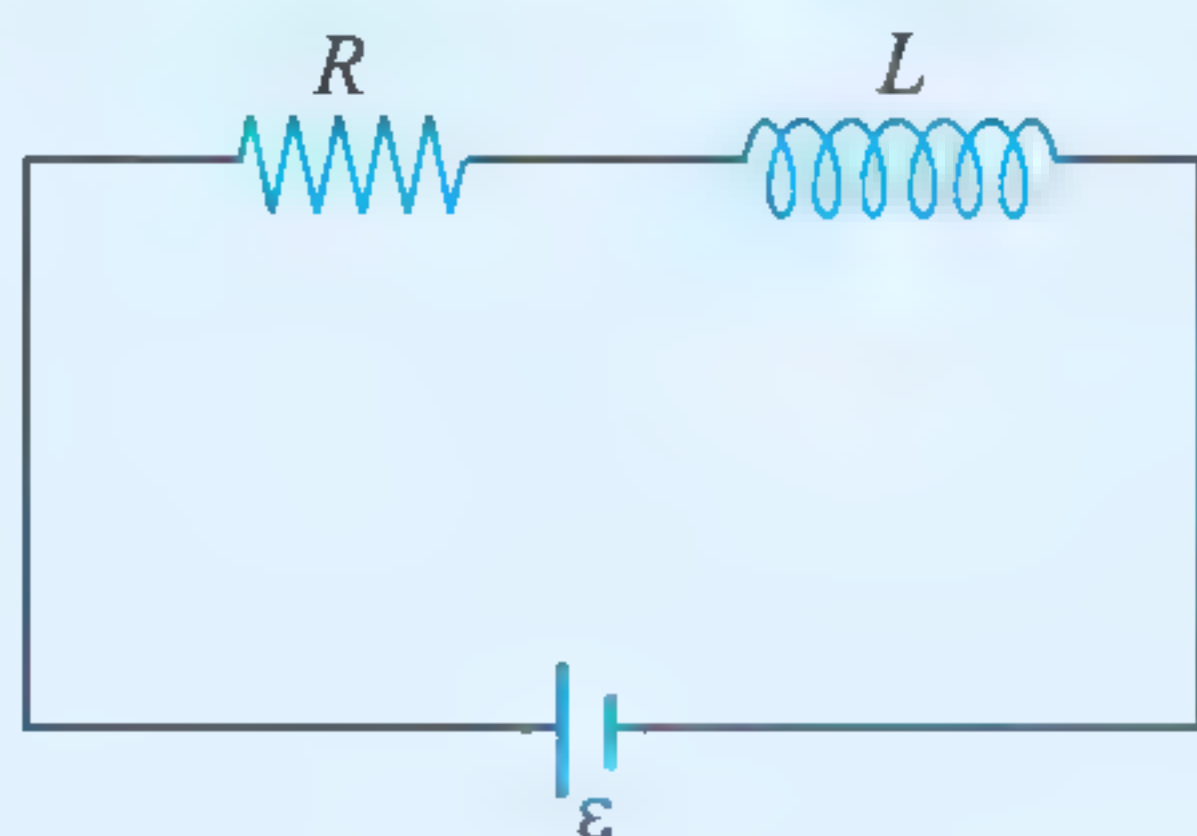


- In figure, $\varepsilon = 110$ V, $R_1 = 10.0\Omega$, $R_2 = 20.0\Omega$, $R_3 = 30.0\Omega$, and $L = 2.00$ H. Immediately after switch S is closed, what are (a) i_1 and (b) i_2 ? (Let currents in the indicated directions have positive values and currents in the opposite directions have negative values.) A long time later, what are (c) i_1 and (d) i_2 ? The switch is then reopened. Just then, what are (e) i_1 and (f) i_2 ? A long time later, what are (g) i_1 and (h) i_2 ?

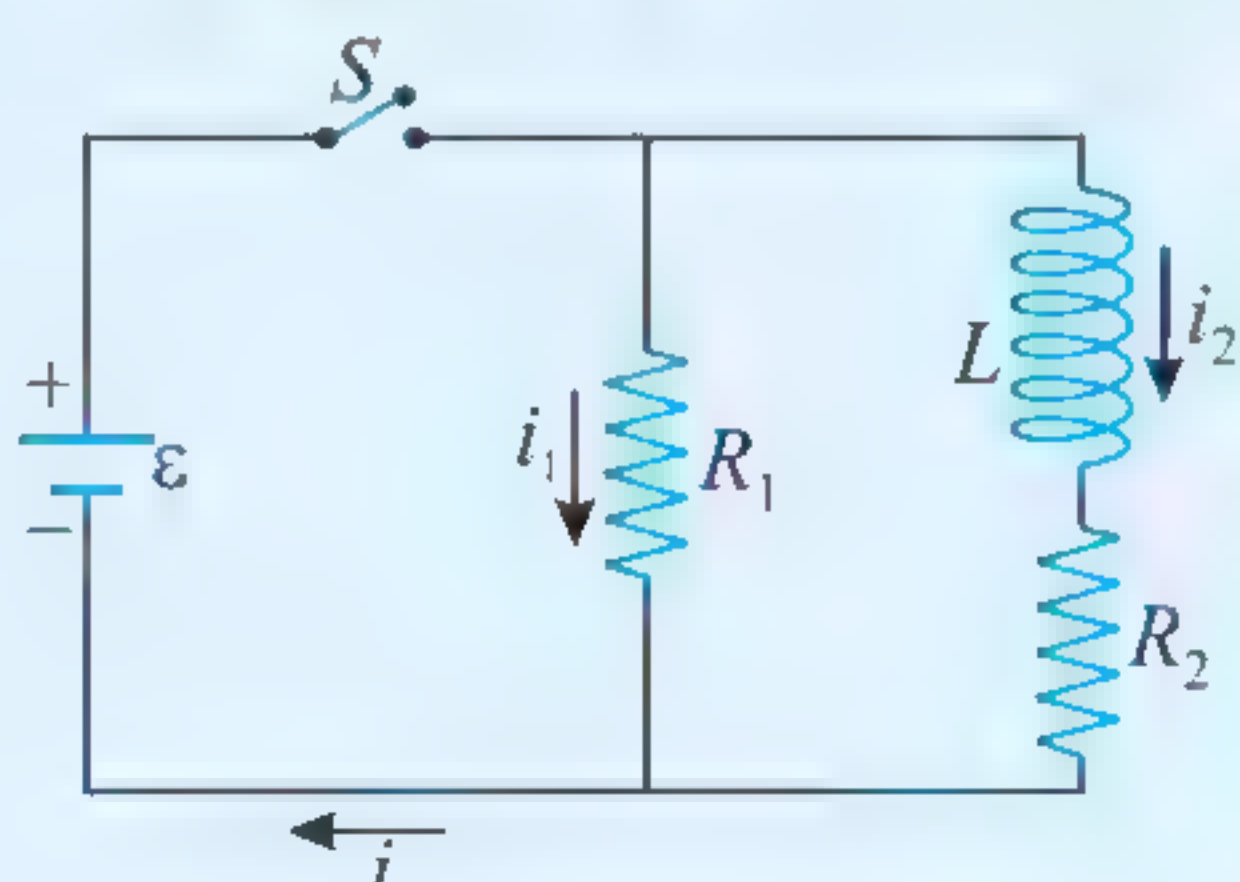


- A circuit consists of a coil, a switch, and a battery, all in series. The internal resistance of the battery is negligible compared with that of the coil. The switch is originally open. It is thrown closed, and after a time interval Δt , the current in the circuit reaches 80.0% of its final value. The switch then remains closed for a time interval much longer than Δt . The wires connected to the terminals of the battery are then short circuited with another wire and removed from the battery, so that the current is uninterrupted. (a) At an instant that is a time interval Δt after the short circuit, the current is what percentage of its maximum value? (b) At the moment $2\Delta t$ after the coil is short-circuited, the current in the coil is what percentage of its maximum value?

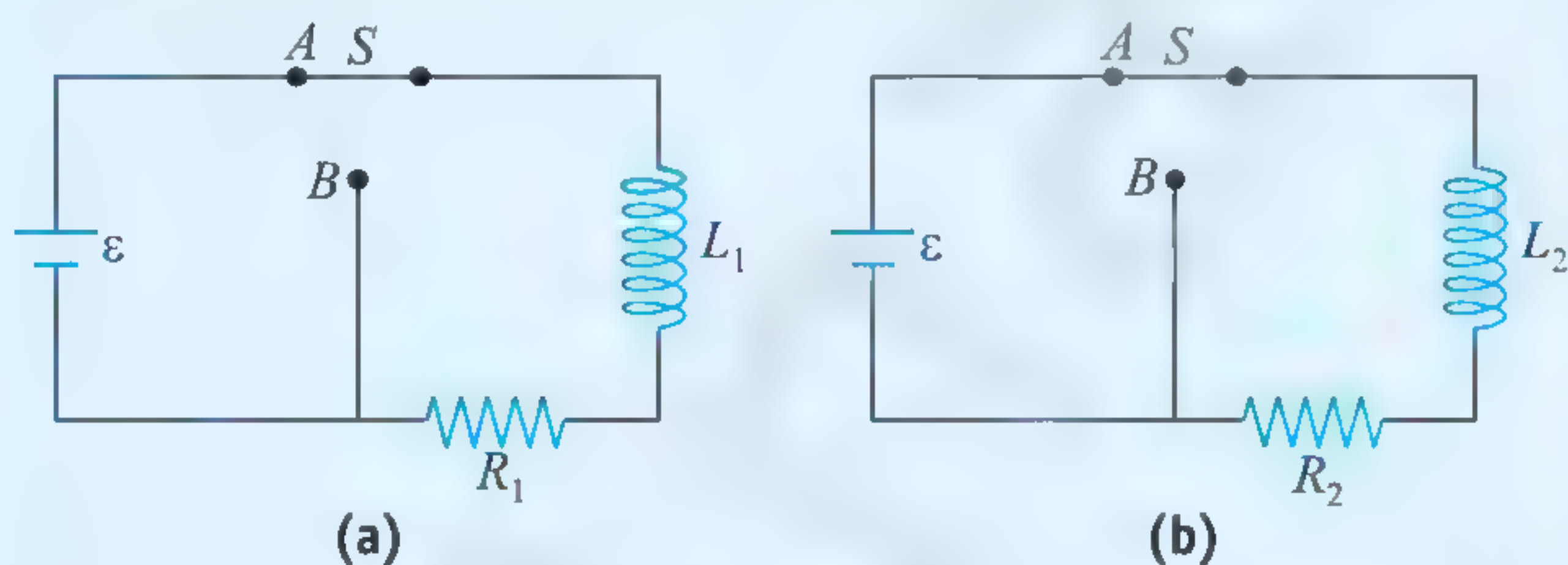
4. For the RL circuit shown in figure, let the inductance be 3.00 H , the resistance $8.00\ \Omega$, and the battery emf 36.0 V . (a) Calculate $\Delta V_R / \mathcal{E}_L$, that is, the ratio of the potential difference across the resistor to the emf across the inductor when the current is 2.00 A . (b) Calculate the emf across the inductor when the current is 4.50 A .



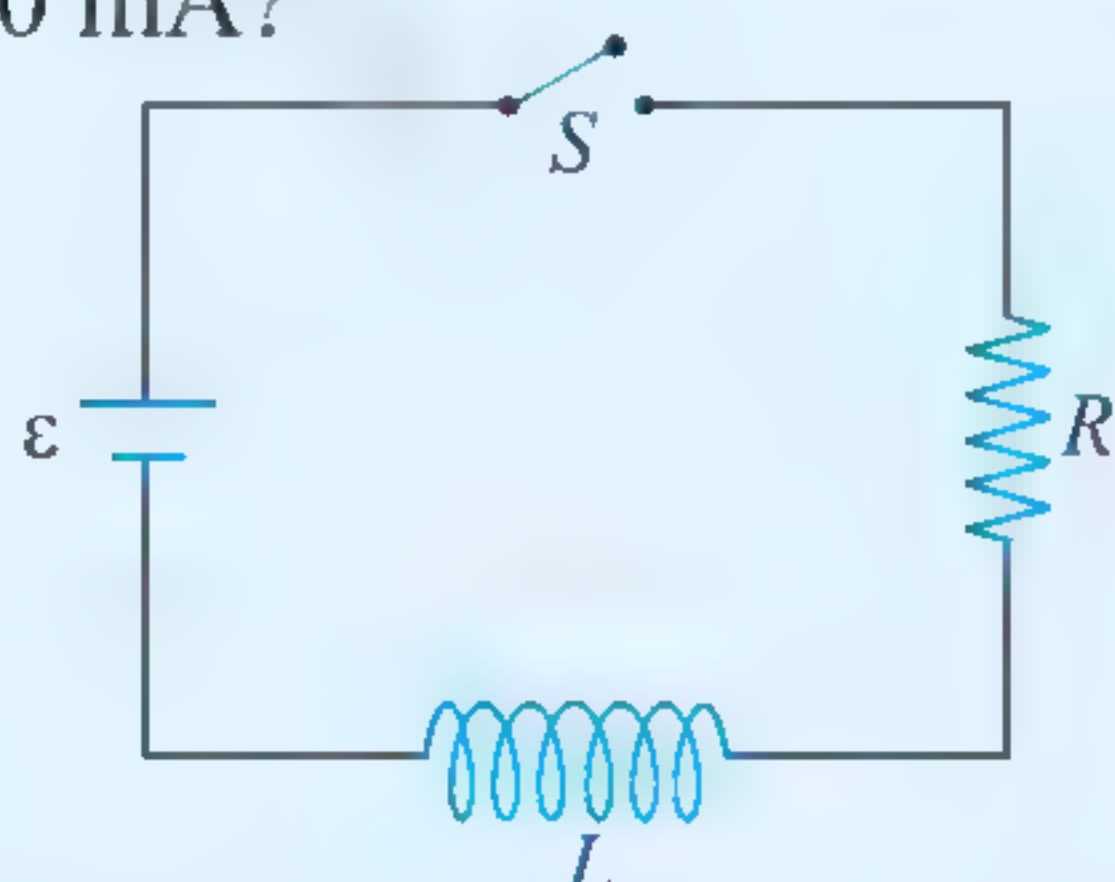
5. In figure, the battery is ideal and $\mathcal{E} = 10\text{ V}$, $R_1 = 5.0\ \Omega$, $R_2 = 10\ \Omega$, and $L = 5.0\text{ H}$. Switch S is closed at time $t = 0$. Just afterwards, what are (a) i_1 , (b) i_2 , (c) the current i_S through the switch, (d) the potential difference V_2 across resistor 2, (e) the potential difference V_L across the inductor, and (f) the rate of change di_2/dt ? A long time later, what are (g) i_1 , (h) i_2 , (i) i_S , (j) V_2 , (k) V_L , and (l) di_2/dt ?



6. In Fig. (a), switch S has been closed on A long enough to establish a steady current in the inductor of inductance $L_1 = 5.00\text{ mH}$ and the resistor of resistance $R_1 = 20.0\ \Omega$. Similarly, in Fig. (b), switch S has been closed on A long enough to establish a steady current in the inductor of inductance $L_2 = 6.00\text{ mH}$ and the resistor of resistance $R_2 = 30.0\ \Omega$. The ratio Φ_{02}/Φ_{01} of the magnetic flux through a turn in inductor 2 to that in inductor 1 is 2.0 . At time $t = 0$, the two switches are closed on B . At what time t is the flux through a turn in the two inductors equal?

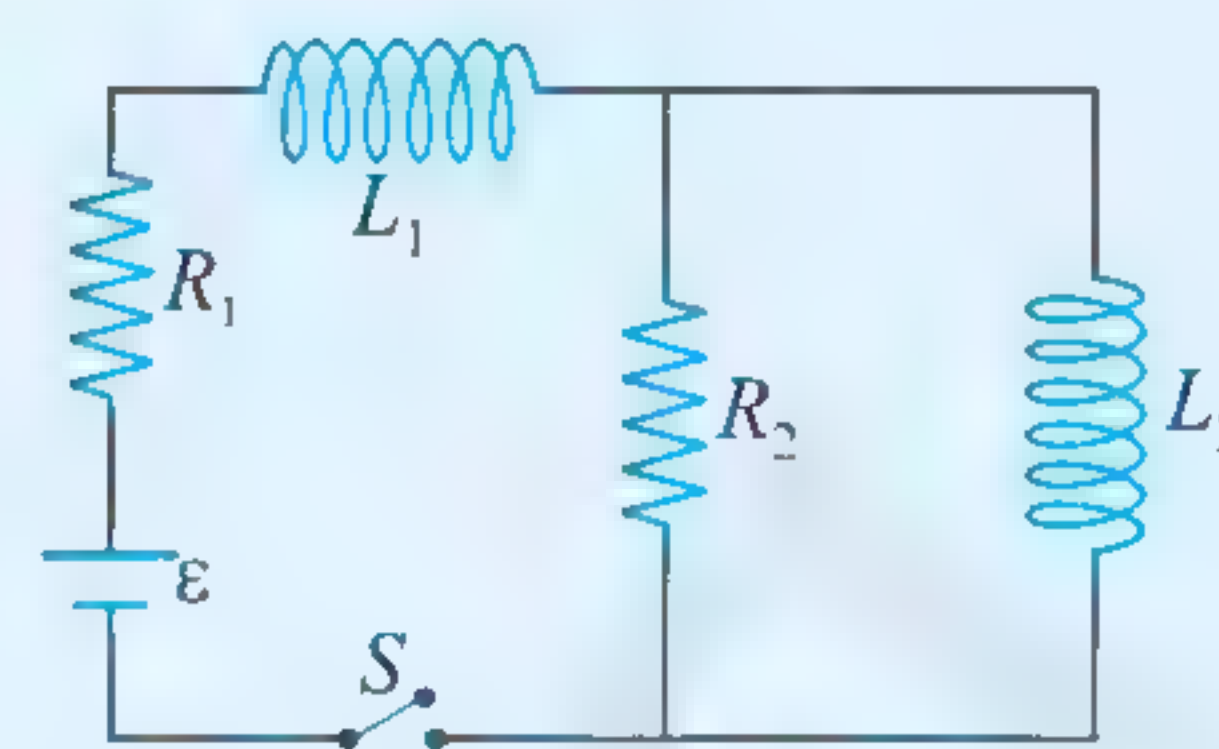


7. In given figure, $R = 4.0\text{ k}\Omega$, $L = 8.0\ \mu\text{H}$, and the ideal battery has $\mathcal{E} = 20\text{ V}$. How long after switch S is closed is the current 2.0 mA ?

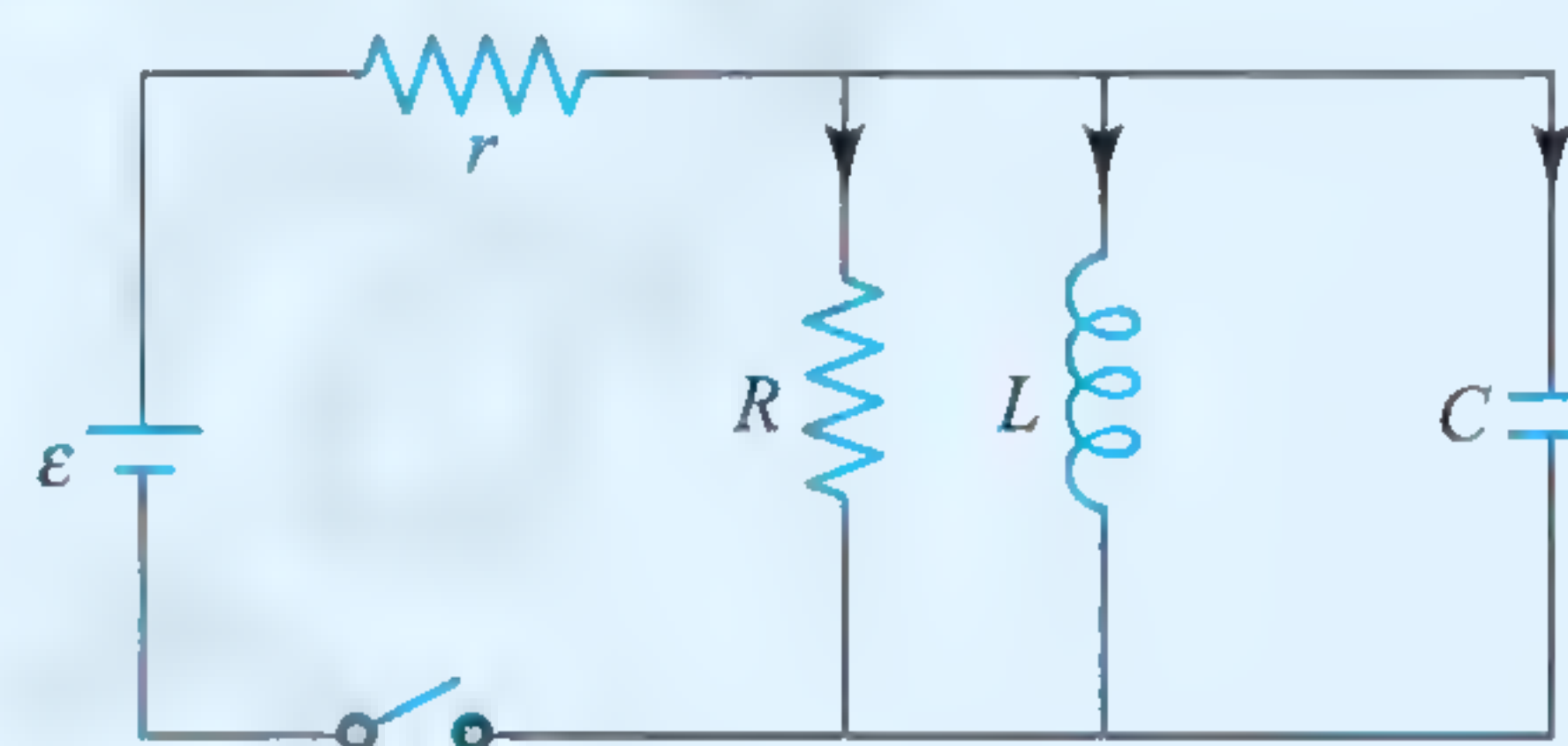


8. In given figure, $R_1 = 8.0\ \Omega$, $R_2 = 10\ \Omega$, $L_1 = 0.30\text{ H}$, $L_2 = 0.20\text{ H}$, and the ideal battery has $\mathcal{E} = 6.0\text{ V}$. (a) Just after

switch S is closed, at what rate is the current in inductor 1 changing? (b) When the circuit is in the steady state, what is the current in inductor 1?

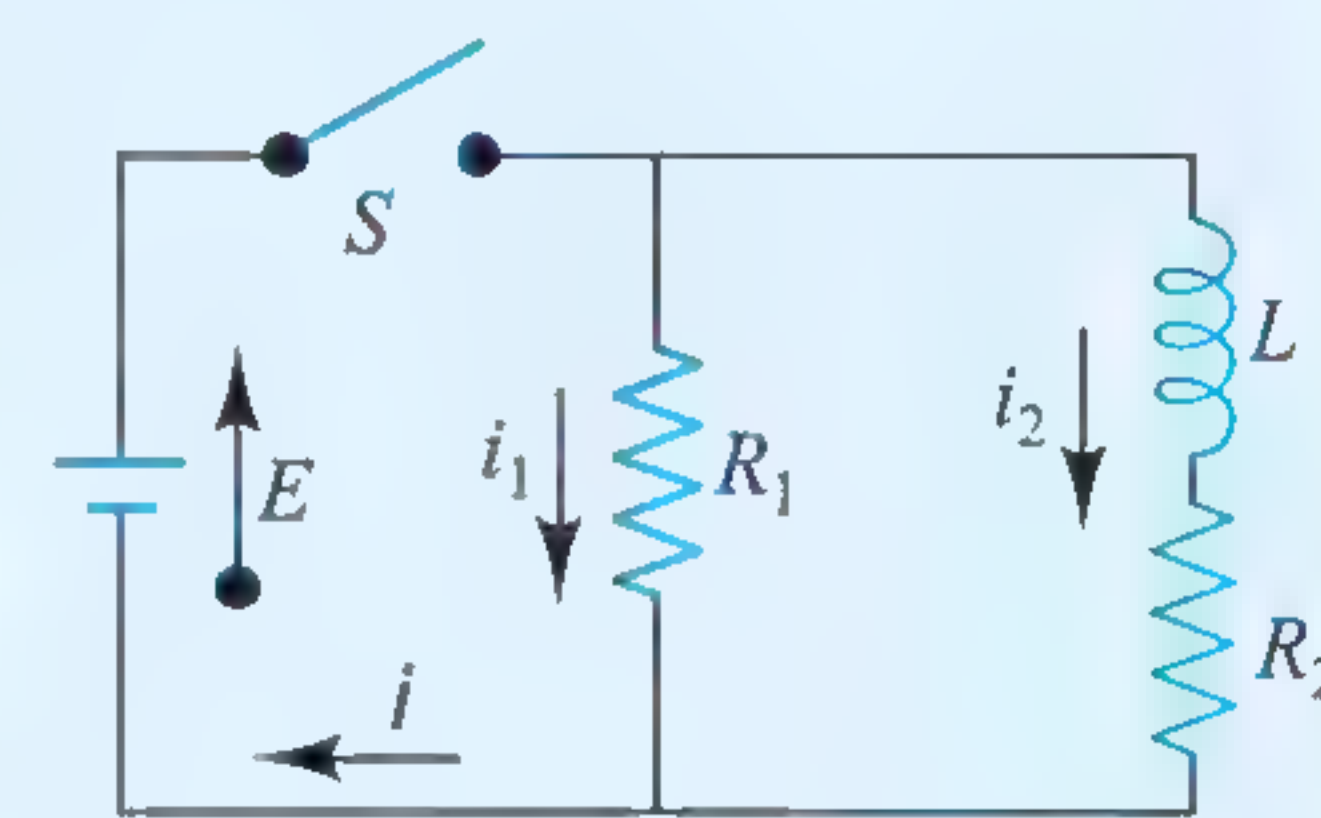


9. Figure shows an LCR circuit. When the switch is closed, the currents through resistor R , inductor L , and capacitor C are I_1 , I_2 , and I_3 , respectively. Determine the values of I_1 , I_2 , and I_3 .



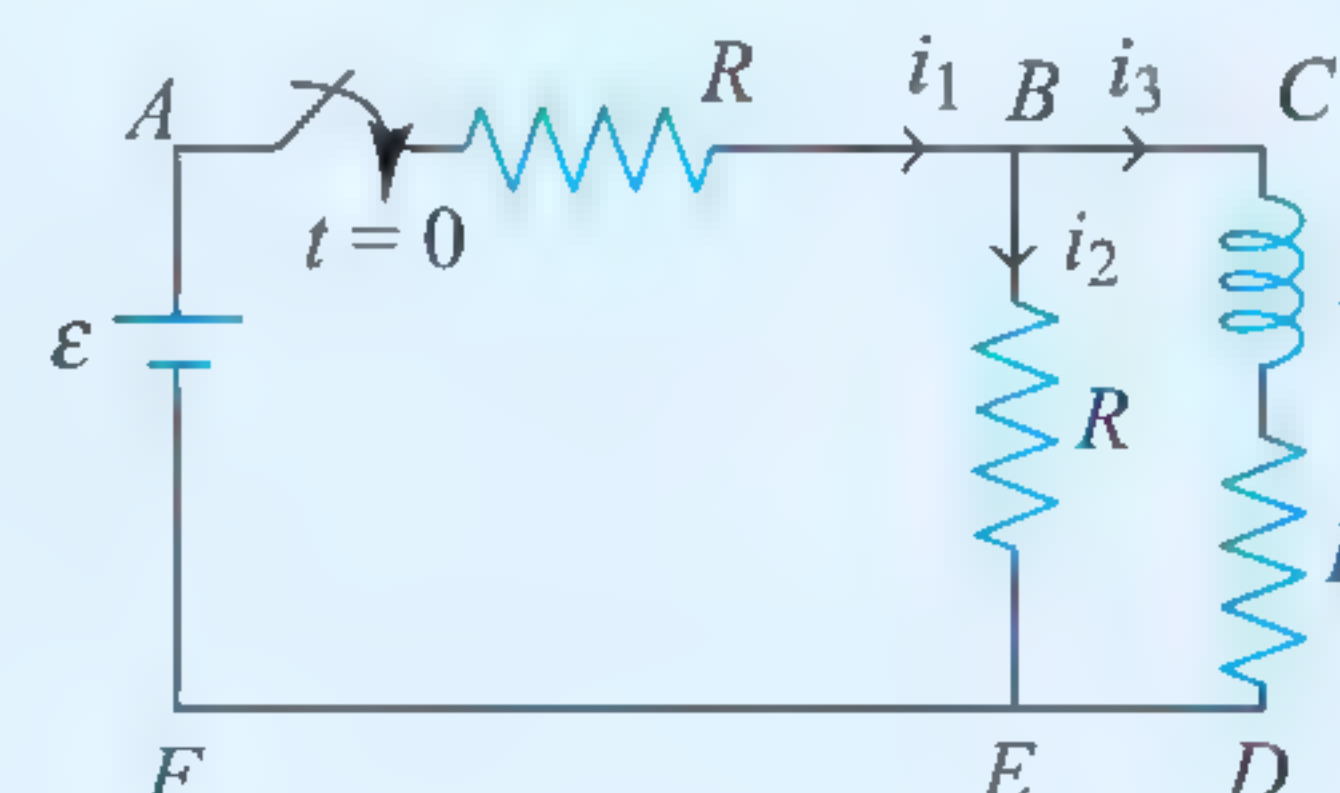
(a) at $t = 0$ (b) at $t = \infty$

10. In the circuit shown in figure, $E = 10\text{ V}$, $R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$, and $L = 5\text{ H}$. For the two separate conditions, (i) switch S is just closed and (ii) switch S is closed for a long time, calculate

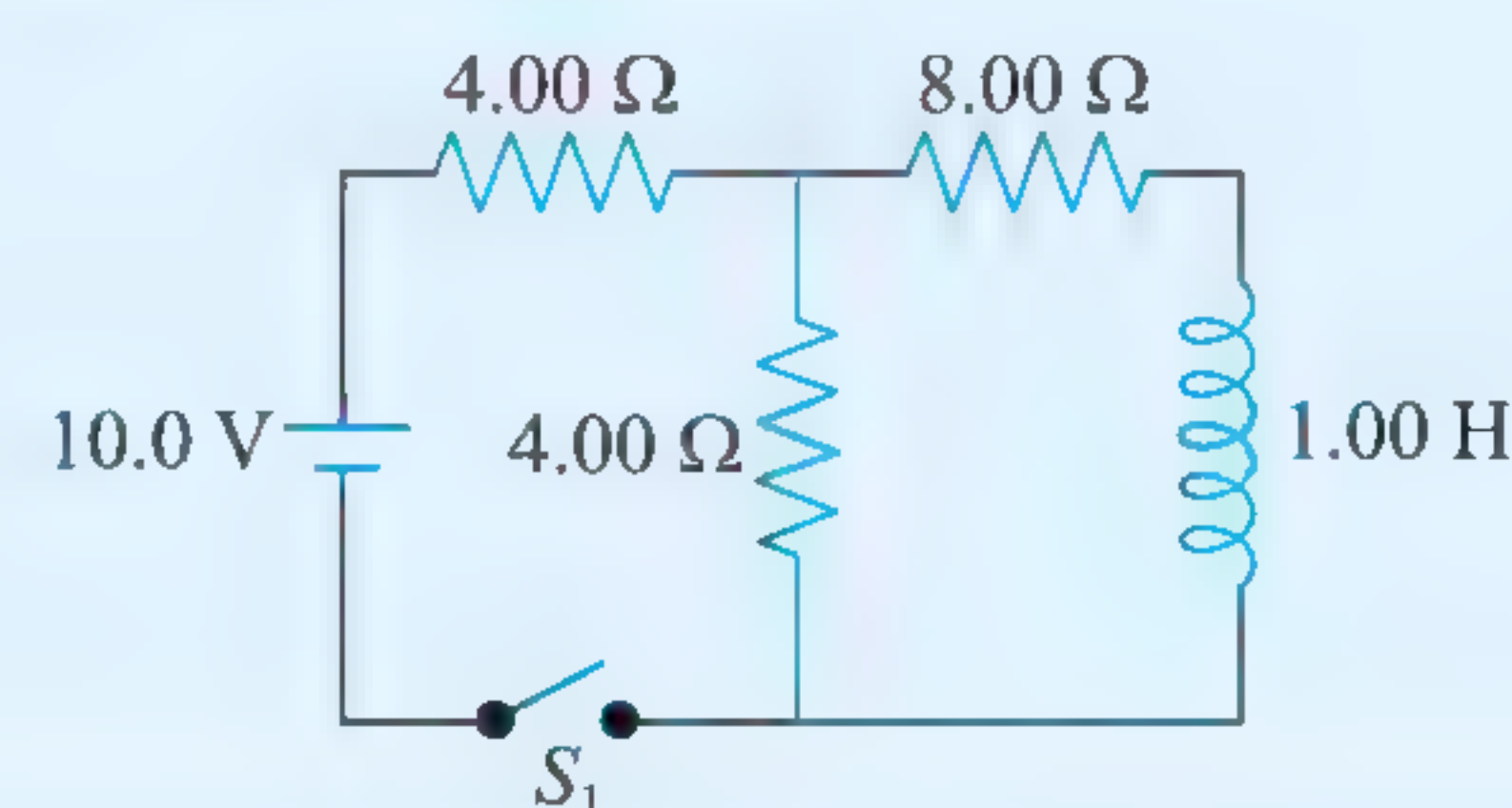


- (a) current i_1 through R_1 ,
(b) current i_2 through R_2 ,
(c) current i through the switches,
(d) the potential difference across R_2 ,
(e) the potential difference across L ,
(f) di_2/dt .

11. In the following circuit (figure), the switch is closed at $t = 0$. Find the currents i_1 , i_2 , i_3 and di_3/dt at $t = 0$ and at $t = \infty$. Initially, all currents are zero.



12. The switch in figure is closed at time $t = 0$. Find the current in the inductor and the current through the switch as functions of time thereafter.

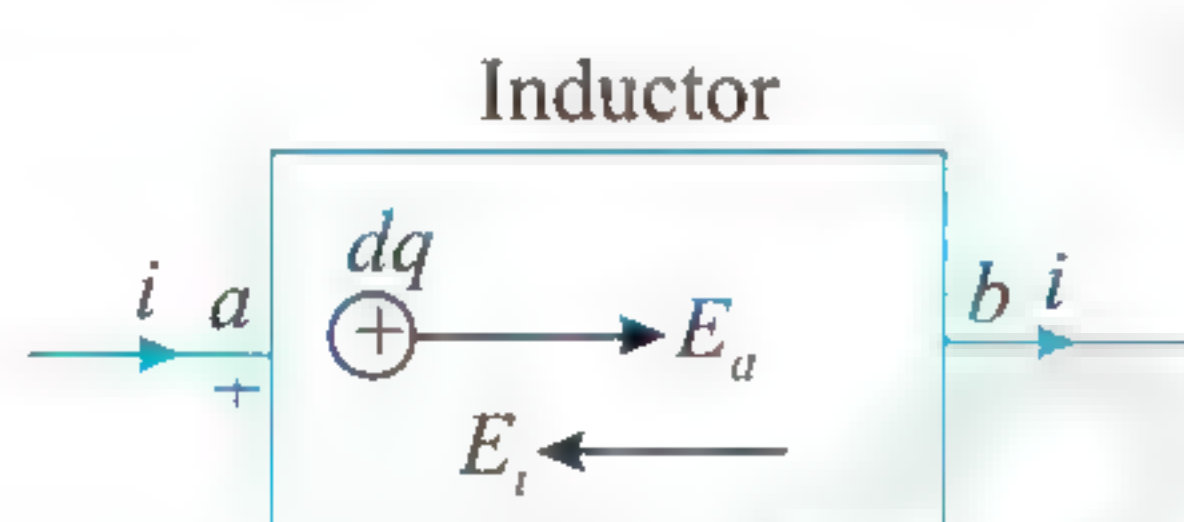


ANSWERS

1. (a) $I_L = 0, I_c = \frac{\varepsilon_0}{R}, I_R = \frac{\varepsilon_0}{R}, \Delta V_L = \varepsilon_0, \Delta V_c = 0, \Delta V_R = \varepsilon_0$
 (b) $I_L = 0, I_c = 0, I_R = 0, \Delta V_L = 0, \Delta V_c = \varepsilon_0, \Delta V_R = 0$
2. (a) $\frac{11}{3}$ A (b) $\frac{11}{3}$ A (c) 5.0 A (d) 3.0 A (e) 0
 (f) -2.0 A (reversed) (g) 0 (h) 0
3. (a) 20% (b) 4% 4. (a) 0.80 (b) zero
5. (a) 2.0 A (b) 0 (c) 2.0 A (d) 0 (e) 10 V (f) 2.0 A/s
 (g) 2.0 A (h) 1.0 A (i) 3.0 A (j) 10 V (k) 0 (l) 0
6. $\frac{\ln(2.0)}{1000}$ s 7. 1.0 ns 8. (a) 20 A/s; (b) 0.75 A
9. (a) $I_1 = 0, I_2 = 0, I_3 = \frac{\varepsilon}{r}$ (b) $I_1 = 0, I_2 = \frac{\varepsilon}{r}, I_3 = 0$
10. (i) (a) 2 A (b) 0 (c) 2 A (d) 0 (e) 10 V (f) 2 A/s.
 (ii) (a) 2 A (b) 1 A (c) 3 A (d) 10 V (e) 0 V (f) 0
11. At $t = 0, i_1 = i_2 = \frac{\varepsilon}{2R}, i_3 = 0, \frac{di_3}{dt} = \frac{\varepsilon}{2L}$
 At $t = \infty, i_2 = i_3 = \frac{\varepsilon}{3R}, i_1 = \frac{2\varepsilon}{3R}, \frac{di_3}{dt} = 0$
12. $\frac{1}{2}(1 - e^{-10t}), \frac{5}{4} + \frac{1}{4}(1 - e^{-10t})$

ENERGY STORED IN THE MAGNETIC FIELD OF AN INDUCTOR

When a current builds up at certain rate in an inductor, a self-induced voltage is developed in the inductor coil which oppose the applied emf. In this process, the source performs a positive work in pushing the charges against the induced voltage.



Let an elementary charge dq is pushed by the source from a to b against the induced voltage V_L during a time dt , constituting an electric current i . The work dW performed by the applied source during the transportation of the elementary charge dq between the terminals of the inductor is given as

$$dW = V_L dq; \text{ then substituting } V_L = L \frac{di}{dt} \text{ and } dq = i dt,$$

$$\text{we have } dW = L \frac{di}{dt} i dt = L i di$$

Integrating both sides, the total work done by the source in establishing a current i is given by,

$$W = \int dW = L \int_0^i i di = \frac{1}{2} L i^2$$

This work done by applied source will be stored as magnetic energy in inductor coil.

$$\text{Hence magnetic energy stored } U = \frac{1}{2} L i^2 \quad \dots(i)$$

As the flux associated with coil is given by,

$$\phi = L i \Rightarrow i = \frac{\phi}{L} \text{ or } L = \frac{\phi}{i} \quad \dots(ii)$$

$$\text{From (i) and (ii) we can write, } U = \frac{1}{2} L i^2 = \frac{1}{2} \frac{\phi^2}{L} = \frac{1}{2} \phi i$$

The energy in an inductor is actually stored in the magnetic field within the coil. We can develop magnetic energy density u (energy stored per unit volume) analogous to those we obtained in electrostatics. We will concentrate on one simple case of an ideal long cylindrical solenoid. For a long solenoid, its magnetic field can be assumed completely within the solenoid. Energy U stored in the solenoid with current i is

$$U = \frac{1}{2} L i^2 = \frac{1}{2} (\mu_0 n^2 V) i^2 \text{ as } L = \mu_0 n^2 V$$

The energy per unit volume is $u = U/V$.

$$u = \frac{U}{V} = \frac{1}{2} \mu_0 n^2 i^2 = \frac{1}{2} \frac{(\mu_0 n i)^2}{\mu_0} = \frac{1}{2} \frac{B^2}{\mu_0} \text{ as } B = \mu_0 n i$$

$$\text{Thus, } u = \frac{1}{2} \frac{B^2}{\mu_0}$$

This expression is similar to $u = \frac{1}{2} \varepsilon_0 E^2$ used in electrostatics.

Although we have derived it for one special situation, it turns out to be correct for any magnetic field configuration.

ILLUSTRATION 5.24

Consider the RL circuit in figure. When the switch is closed in position 1 and opens in position 2, electrical work must be performed on the inductor and on the resistor. The energy stored in the inductor is for the magnetic field inside it which increases as I increases. In the resistor energy appears as heat.

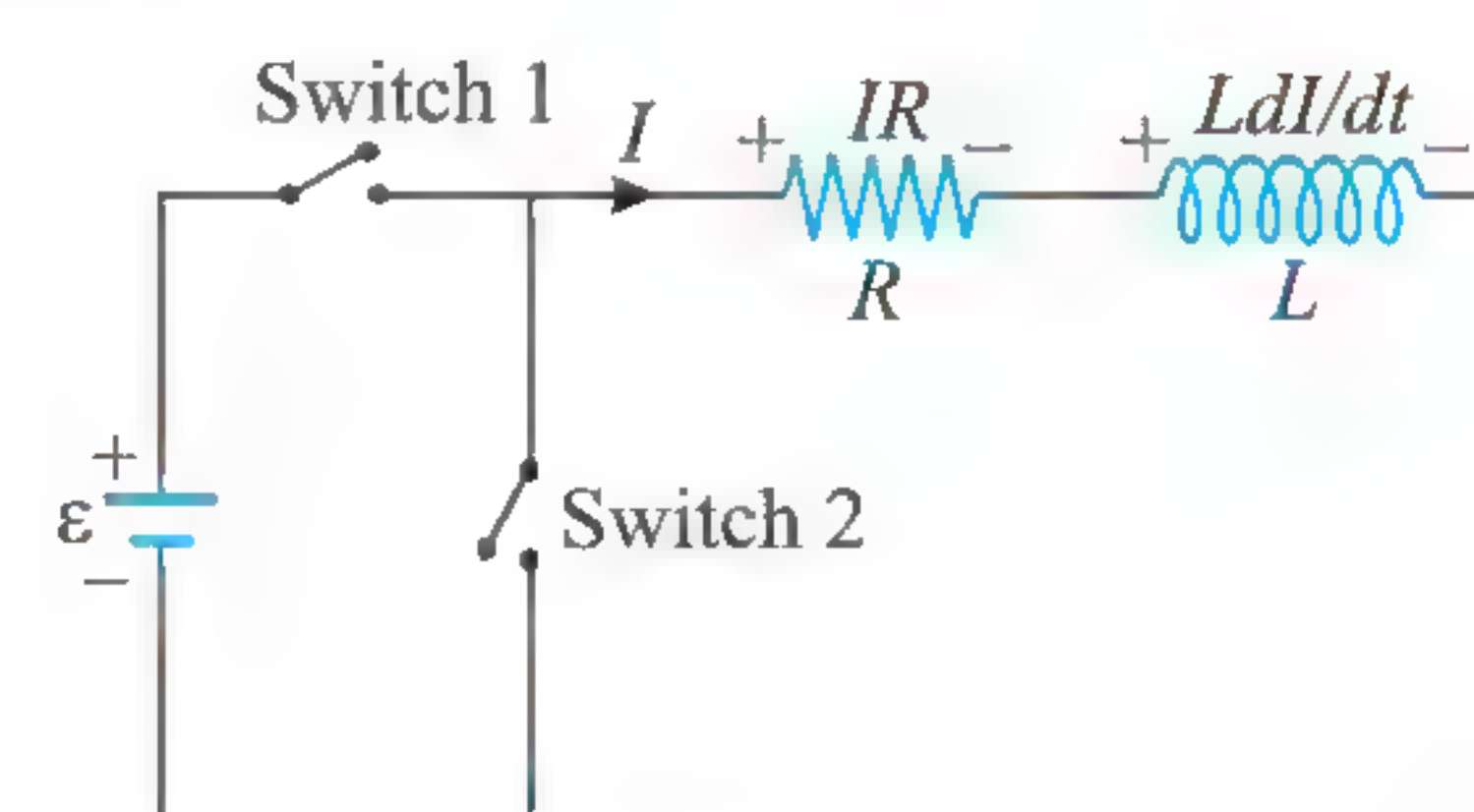
- (a) What is the ratio of P_L/P_R of the rate at which energy is stored in the inductor to the rate at which energy is dissipated in the resistor?
 (b) Express the ratio P_L/P_R as a function of time.
 (c) If the time constant of circuit is t , what is the time at which $P_L = P_R$?

Sol.

- (a) The energy stored in the inductor is $U_L = \frac{1}{2} L I^2$

$$\text{Power } P_L = \frac{dU}{dt} = \frac{d}{dt} \left(\frac{1}{2} L I^2 \right) = L I \frac{dI}{dt} \quad \dots(i)$$

$$\text{The power of a resistor } P_R = I^2 R \quad \dots(ii)$$



$$\text{So the ratio is } \frac{P_L}{P_R} = \frac{L I dI/dt}{I^2 R} = \frac{\tau (dI/dt)}{I} \quad \dots(iii)$$

$$(b) \text{ As } I = \frac{E}{R} [1 - e^{-(t/\tau)}] \quad \dots(iv)$$

$$\frac{dI}{dt} = \frac{d}{dt} \left[\frac{E}{R} (1 - e^{-(t/\tau)}) \right] = \frac{E}{R\tau} e^{-(t/\tau)} \quad \dots(v)$$

Inserting the value of dI/dt and I in Eq. (iii), we have

$$\frac{P_L}{P_R} = \tau \frac{\frac{E}{R\tau} e^{-t/\tau}}{\frac{E}{R} (1 - e^{-t/\tau})} = \frac{e^{-(t/\tau)}}{1 - e^{-(t/\tau)}} = \frac{1}{e^{(t/\tau)} - 1} \quad \dots(vi)$$

When $t = 0$, this ratio tends to infinity, due to large initial value of dI/dt and small initial value of I . When t tends to infinity, after many time constants, the current tends to zero. As a result, P_L vanishes and the ratio goes to zero.

(c) From Eq. (vi), $P_L = P_R$ when $\frac{1}{e^{(t/\tau)} - 1} = 1$ which on simplification yields $e^{(t/\tau)} = 2$, $\frac{t}{\tau} = \ln 2 \Rightarrow t = 0.693 \tau$

ILLUSTRATION 5.25

A long cylindrical wire of radius R and carrying a current I . Find the total field energy of magnetic field stored per unit length inside the wire.

Sol. Let us consider an elemental shell of radius x and wall width dx inside the cylinder as shown in figure.

The magnetic field inside the wire at a distance x from axis,

$$B = \frac{\mu_0 I x}{2\pi R^2}$$

The volume of the elemental shell of length 1 m is given as

$$dV = 2\pi x dx \times 1$$

Field energy stored in elemental cylindrical sheet is given as

$$dU = \frac{B^2}{2\mu_0} \times dV = \frac{1}{2\mu_0} \left(\frac{\mu_0 I x}{2\pi R^2} \right)^2 \times 2\pi x dx$$

$$dU = \frac{\mu_0 I^2 x^3 dx}{4\pi R^4}$$

Total field energy inside the cylinder per unit length is given by integrating above expression within limits from 0 to R which is given as

$$U = \int dU = \int_0^R \frac{\mu_0 I^2}{4\pi R^4} \cdot x^3 dx$$

$$U = \frac{\mu_0 I^2}{4\pi R^4} \cdot \frac{R^4}{4} = \frac{\mu_0 I^2}{16\pi}$$

the circuit and the charge on the capacitor oscillate between maximum positive and negative values. If the resistance of the circuit is zero, no energy is transformed to internal energy. In the following analysis, the resistance in the circuit is neglected. We also assume an idealized situation in which energy is not radiated away from the circuit.

Assume the capacitor has an initial charge $Q_{\max}$ (the maximum charge) and the switch is open for $t < 0$ and then closed at $t = 0$.

Consider some arbitrary time t after the switch is closed so that the capacitor has a charge $q < Q_{\max}$ and the current is $i < I_{\max}$. At this time, both circuit elements store energy, but the sum of the two energies must equal the total initial energy U stored in the fully charged capacitor at $t = 0$:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2} Li^2 = \frac{Q_{\max}^2}{2C} \quad \dots(i)$$

The system of the circuit is isolated: the total energy of the system must remain constant in time. We describe the constant energy of the system mathematically by setting $dU/dt = 0$. Therefore, by differentiating Eq. (i) with respect to time while noting that q and i vary with time gives

$$\frac{dU}{dt} = \frac{d}{dt} \left(\frac{q^2}{2C} + \frac{1}{2} Li^2 \right) = \frac{q}{C} \frac{dq}{dt} + Li \frac{di}{dt} = 0 \quad \dots(ii)$$

We can reduce this result to a differential equation in one variable by remembering that the current in the circuit is equal to the rate at which the charge on the capacitor changes.

$i = -\frac{dq}{dt}$ (negative sign because the charge in capacitor is decreasing with time)

It then follows that $\frac{di}{dt} = -\frac{d^2q}{dt^2}$. Substitution of these relationships into Eq. (ii) gives

$$\frac{q}{C} (-i) + Li \left(-\frac{d^2q}{dt^2} \right) = 0 \Rightarrow \frac{q}{C} + L \frac{d^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} = -\frac{1}{LC} q \quad \dots(iii)$$

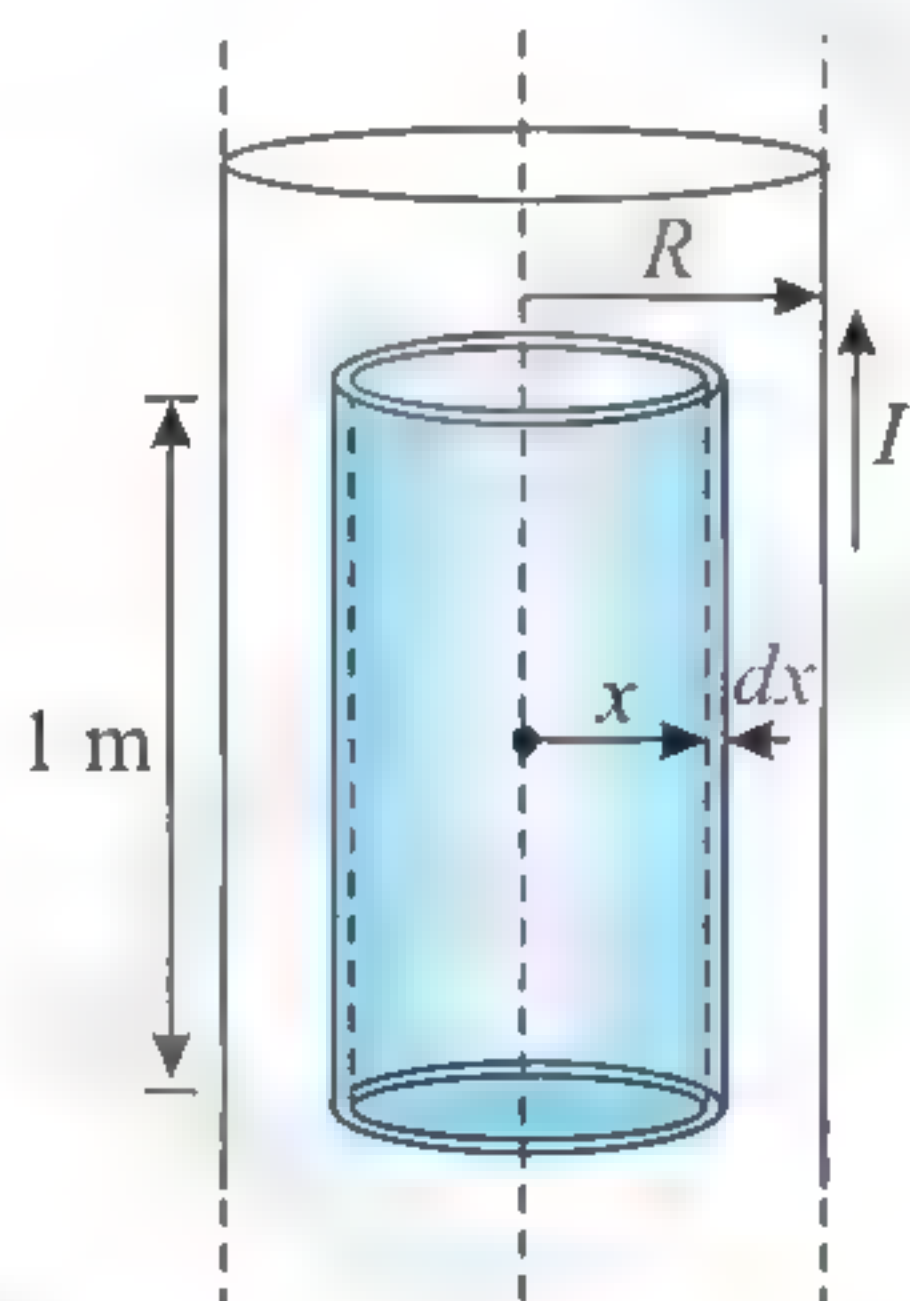
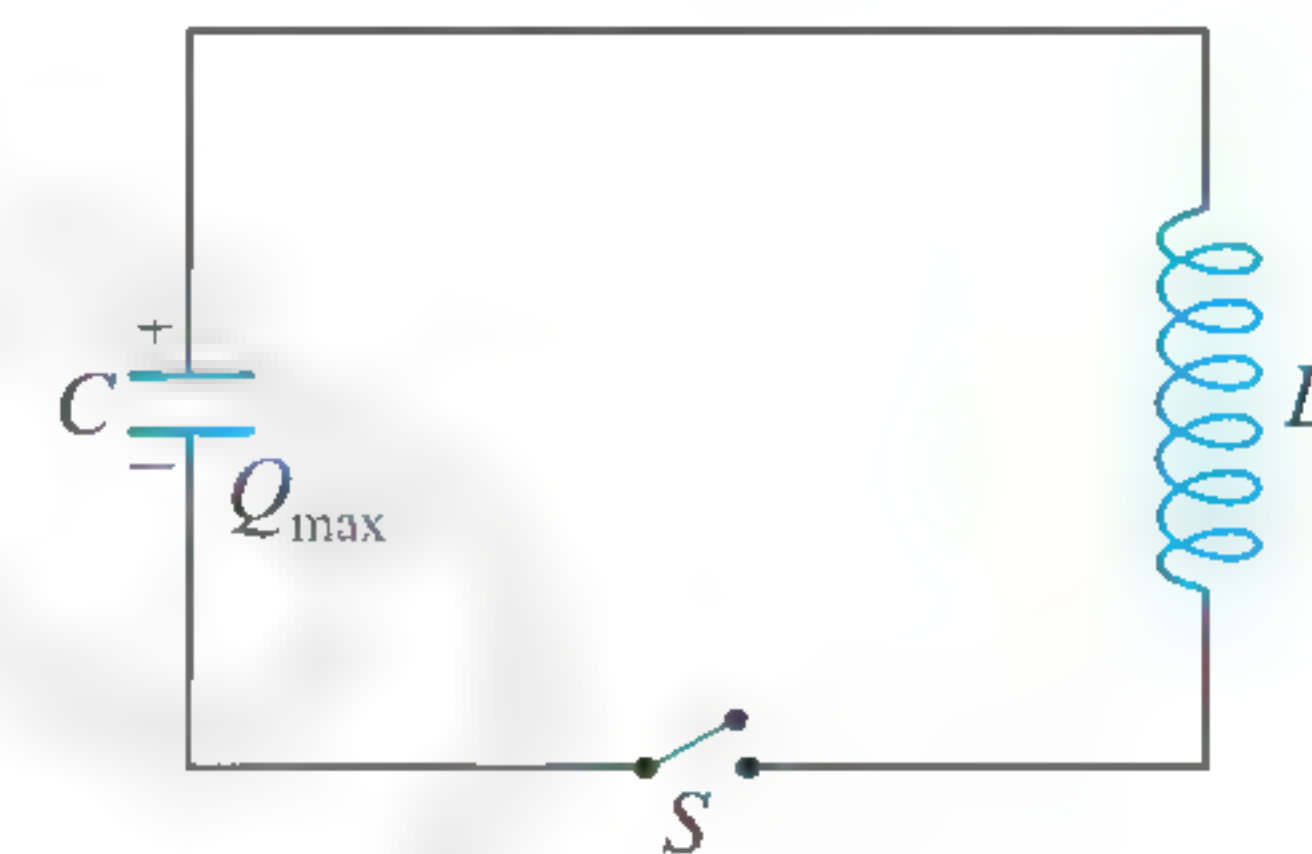
Let's solve for q by noting that this expression is of the same form as the analogous equations for a particle in simple harmonic motion:

$$\frac{d^2x}{dt^2} = -\frac{k}{m} x = -\omega^2 x$$

where k is the spring constant, m is the mass of the block, and $\omega = \sqrt{k/m}$. The solution of this mechanical equation has the general form

$$x = A \cos(\omega t + \phi)$$

where A is the amplitude of the simple harmonic motion (the maximum value of x), ω is the angular frequency of this motion, and ϕ is the phase constant; the values of A and ϕ depend on the initial conditions. Because Eq. (iii) is of the same mathematical form as the differential equation of the simple harmonic oscillator, it has the solution



OSCILLATIONS IN AN LC CIRCUIT

When a capacitor is connected to an inductor as illustrated in figure, the combination is an LC circuit. If the capacitor is initially charged and the switch is then closed, both the current in

$$q = Q_{\max} \cos(\omega t + \phi) \quad \dots(\text{iv})$$

$$\text{The angular frequency } \omega = \frac{1}{\sqrt{LC}} \quad \dots(\text{v})$$

Note that the angular frequency of the oscillations depends solely on the inductance and capacitance of the circuit. Equation (v) gives the natural frequency of oscillation of the LC circuit.

Because q varies sinusoidally with time, the current in the circuit also varies sinusoidally. We can show that by differentiating Eq. (iv) with respect to time:

$$i = -\frac{dq}{dt} = \omega Q_{\max} \sin(\omega t + \phi) \quad \dots(\text{vi})$$

Current as a Function of Time for an Ideal LC Current

To determine the value of the phase angle ϕ , let's examine the initial conditions, which in our situation require that at $t = 0$, $i = 0$, and $q = Q_{\max}$. Setting $i = 0$ at $t = 0$ in Eq. (vi) gives

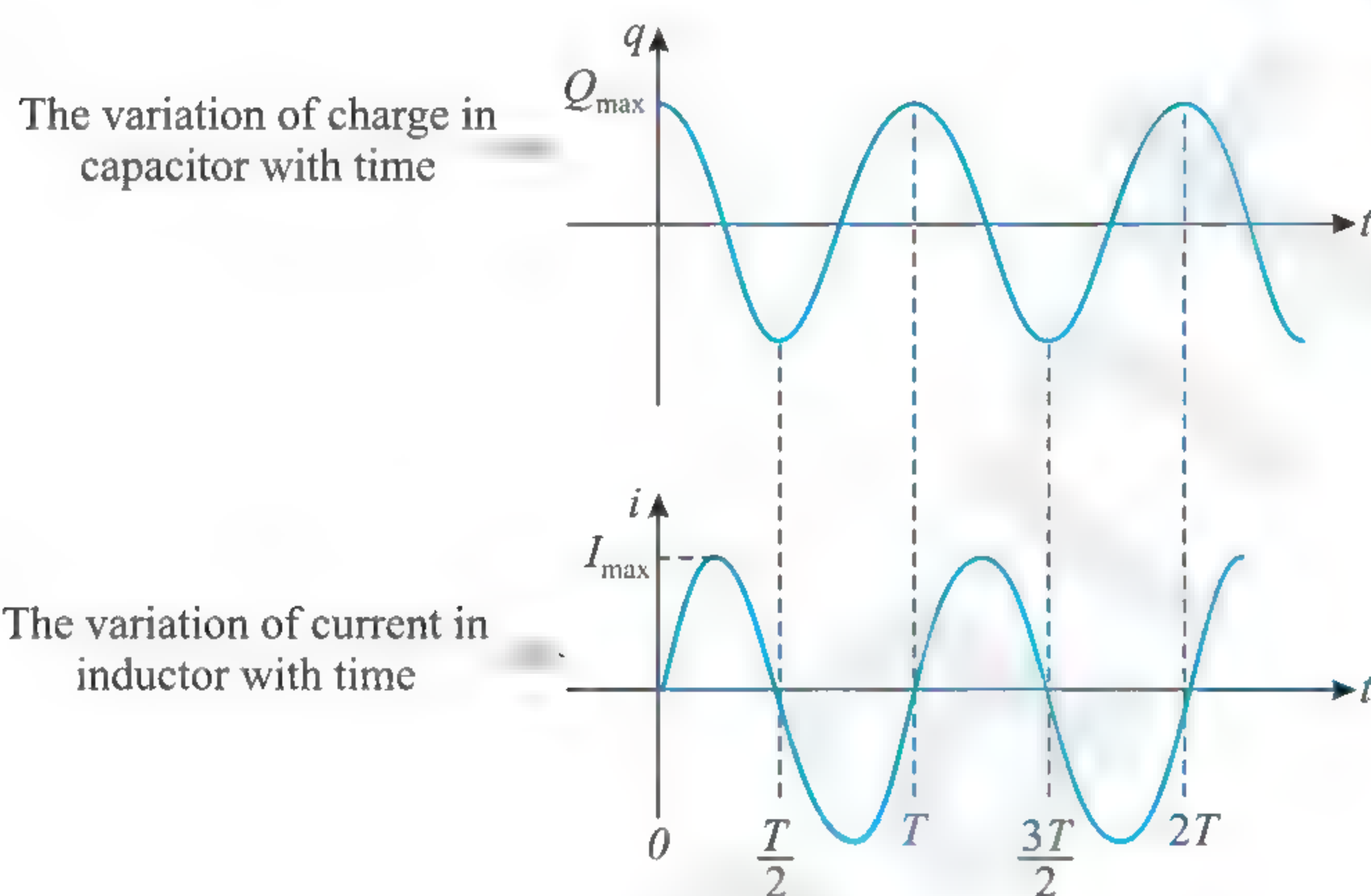
$$0 = \omega Q_{\max} \sin \phi$$

which shows that $\phi = 0$. This value for ϕ also is consistent with Eq. (iv) and the condition that $q = Q_{\max}$ at $t = 0$. Therefore, in our case, the expressions for q and i are

$$\text{Charge as a function of time } q = Q_{\max} \cos \omega t \quad \dots(\text{vii})$$

Current as a function of time,

$$i = \omega Q_{\max} \sin \omega t = I_{\max} \sin \omega t \quad \dots(\text{viii})$$

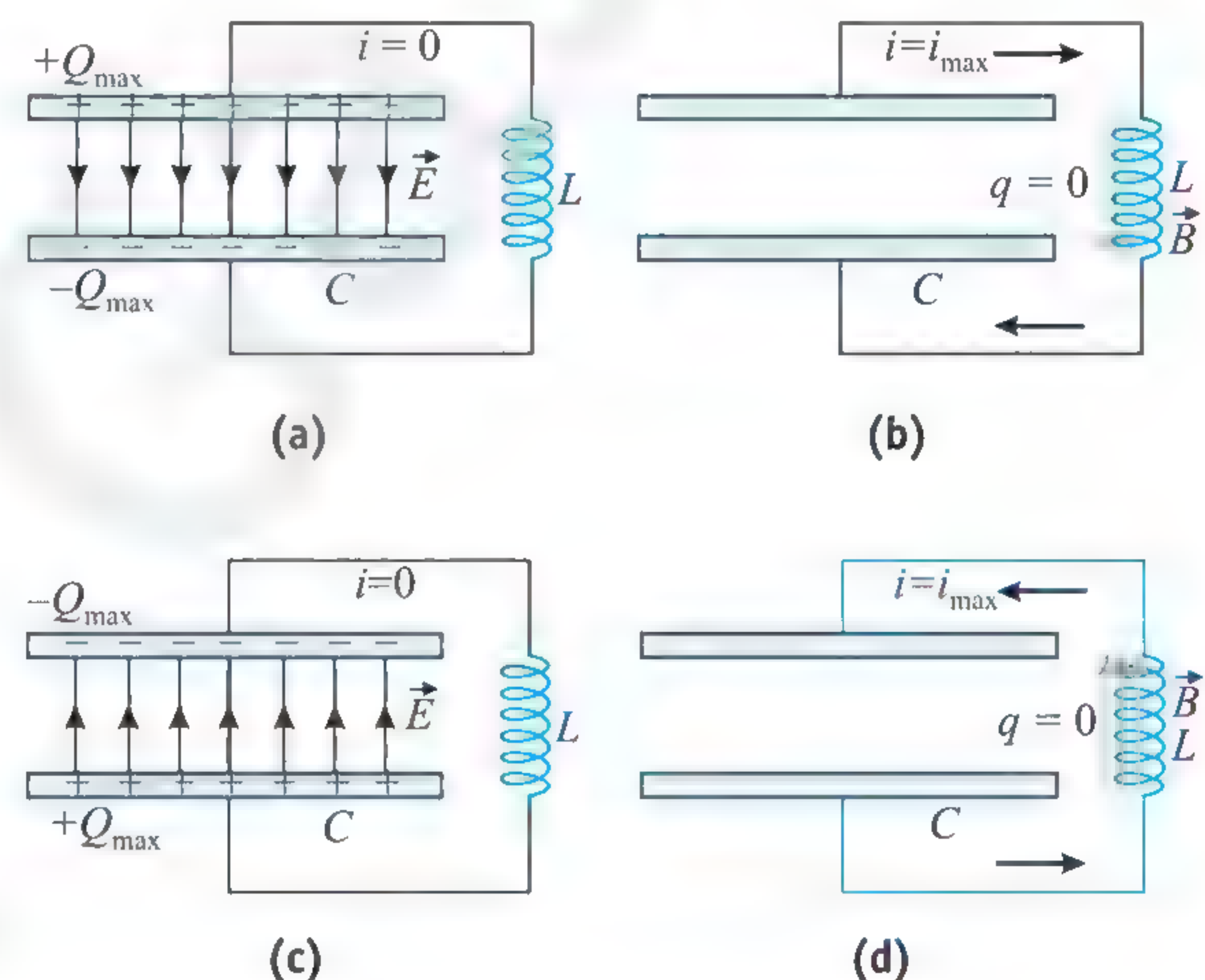


OSCILLATION OF ELECTRICAL AND MAGNETIC ENERGY

When the capacitor is fully charged, the energy U in the circuit is stored in the capacitor's electric field and is equal to $Q^2/2C$. At this time, the current in the circuit is zero; therefore, no energy is stored in the inductor [Fig. (a)]. After the switch is closed, the rate at which charges leave or enter the capacitor plates (which is also the rate at which the charge on the capacitor changes) is equal to the current in the circuit. After the switch is closed and the capacitor begins to discharge, the energy stored in its electric field decreases. The capacitor's discharge represents a current in the circuit, and some energy is now stored in the magnetic field of the inductor. Therefore, energy is transferred from the electric

field of the capacitor to the magnetic field of the inductor. When the capacitor is fully discharged, it stores no energy. At this time, the current reaches its maximum value and all the energy in the circuit is stored in the inductor [Fig. (b)]. The current continues in the same direction, decreasing in magnitude, with the capacitor eventually becoming fully charged again but with the polarity of its plates now opposite the initial polarity [Fig. (c)]. This process is followed by another discharge until the circuit returns to its original state of maximum charge Q_0 and the plate polarity [Fig. (a)]. The energy continues to oscillate between inductor and capacitor.

The oscillations of the LC circuit are an electromagnetic analog to the mechanical oscillations of the particle in simple harmonic motion.



$$\text{Electrical energy in relation with time } U_E = \frac{Q_{\max}^2}{2C} \cos^2 \omega t$$

$$\text{Magnetic energy in relation with time } U_B = \frac{1}{2} L I_{\max}^2 \sin^2 \omega t$$

$$\text{The total energy, } U = U_E + U_B = \frac{Q_{\max}^2}{2C} \cos^2 \omega t + \frac{1}{2} L I_{\max}^2 \sin^2 \omega t$$

Plots of the time variations of U_E and U_B are shown in figure. The sum $U_E + U_B$ is a constant and is equal to the total energy $\frac{Q_{\max}^2}{2C}$, or $\frac{1}{2} L I_{\max}^2$.

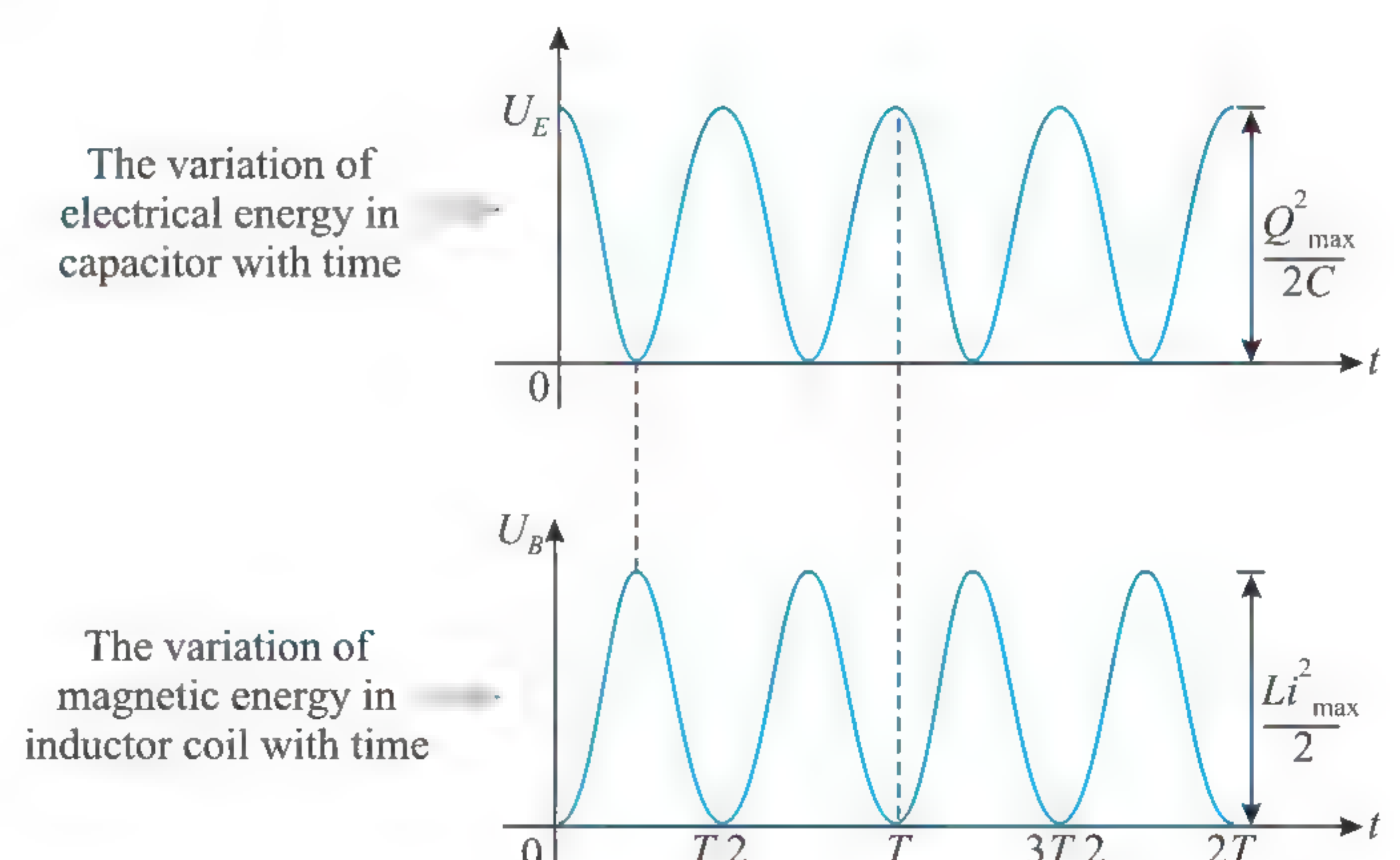


ILLUSTRATION 5.26

In an LC circuit as shown in figure, the switch is closed at $t = 0$. $Q_{\max} = 100 \mu\text{C}$; $L = 40 \text{ mH}$; $C = 100 \mu\text{F}$.



- Determine the equation for instantaneous charge on the capacitor.
- Determine the equation for instantaneous current in the circuit.

Sol.

- (a) $q = Q_{\max} \cos \omega t$, Here $Q_{\max} = 100 \mu\text{C}$;

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(40 \times 10^{-3})(100 \times 10^{-6})}} = 500 \text{ rad s}^{-1}$$

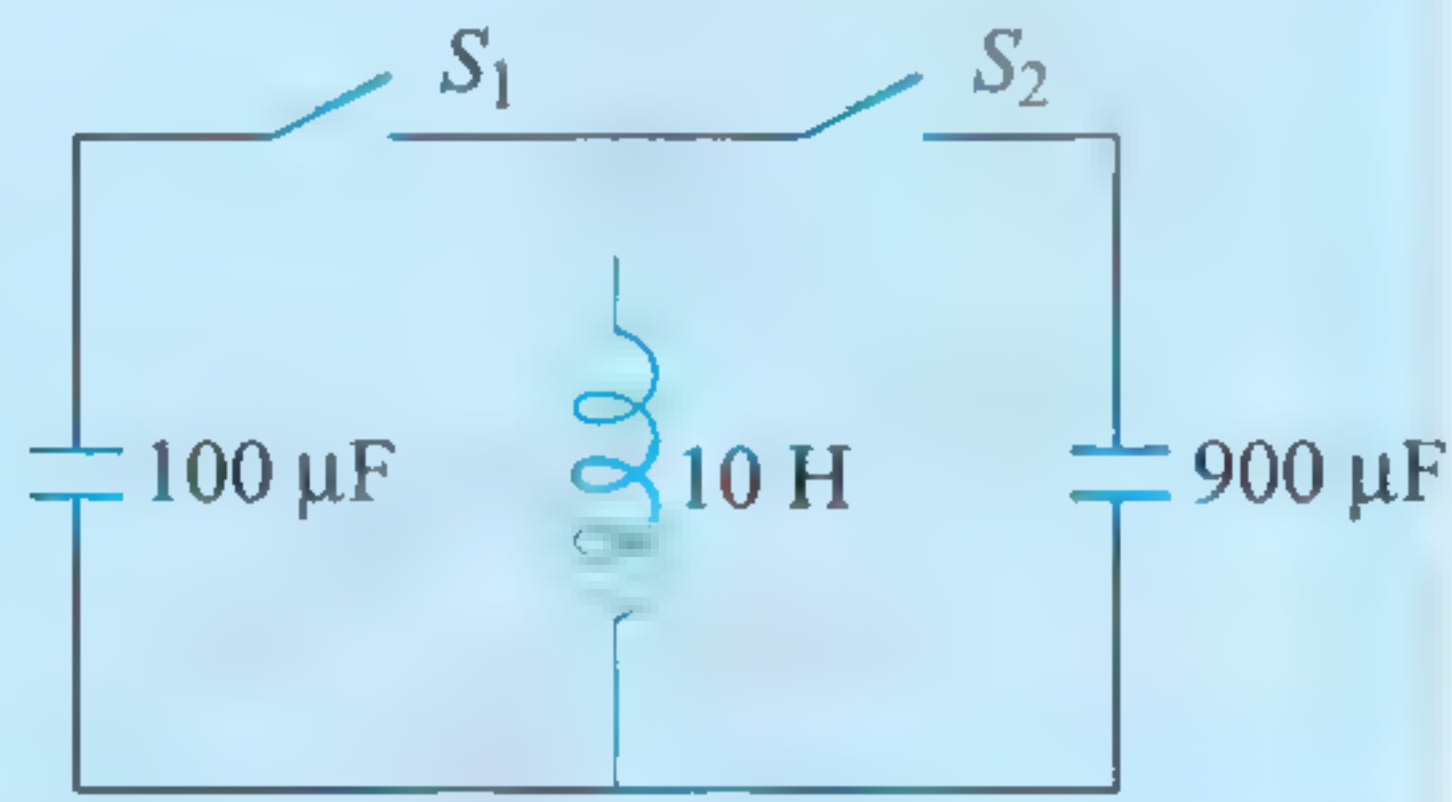
$$q = 100 \cos(500t) \mu\text{C}$$

- (b) $i = \frac{-dq}{dt} = (100)(500) \sin(500t) \mu\text{A}$

$$\text{or } i = 50000 \sin(500t) \mu\text{A}$$

ILLUSTRATION 5.27

Initially the $900 \mu\text{F}$ capacitor is charged to 100 V and the $100 \mu\text{F}$ capacitor is uncharged in figure. Then switch S_2 is closed for time t_1 , after which it is opened and at the same instant switch S_1 is closed for time t_2 and then opened. It is now found that $100 \mu\text{F}$ capacitor is charged to 300 V . Find the minimum possible values of the time interval t_1 and t_2 .



Sol. Initial energy stored in the $900 \mu\text{F}$ capacitor is

$$U_1 = (1/2) \times 900 \times 10^{-6} \times (100)^2 = 4.5 \text{ J}$$

Finally, energy stored in the $100 \mu\text{F}$ capacitor is

$$U_2 = (1/2) \times 100 \times 10^{-6} \times (300)^2 = 4.5 \text{ J}$$

The entire energy of the $900 \mu\text{F}$ capacitor has been transferred to the $100 \mu\text{F}$ capacitor. First, electrical energy of the $900 \mu\text{F}$ capacitor is converted into magnetic energy in the inductor and then this energy is converted into electrical energy once again using S_2 and S_1 appropriately.

In an LC circuit, the transfer of electrical energy into magnetic energy and vice versa takes place in time $T/4$ where $T = 2\pi\sqrt{LC}$ is the time period of the electrical oscillations. Thus,

$$T_1 = 2\pi\sqrt{10 \times 900 \times 10^{-6}} = 0.6 \text{ s}$$

$$T_2 = 2\pi\sqrt{10 \times 100 \times 10^{-6}} = 0.2 \text{ s}$$

Therefore, switch S_2 is first closed for time $0.6/4 = 0.15 \text{ s}$, during which time the $900 \mu\text{F}$ capacitor gets fully discharged and the current in the inductor is fully established. Next switch S_2 is opened and simultaneously switch S_1 is closed for time $0.2/4 = 0.05 \text{ s}$ during which the current in the inductor disappears and the $100 \mu\text{F}$ capacitor gets fully charged.

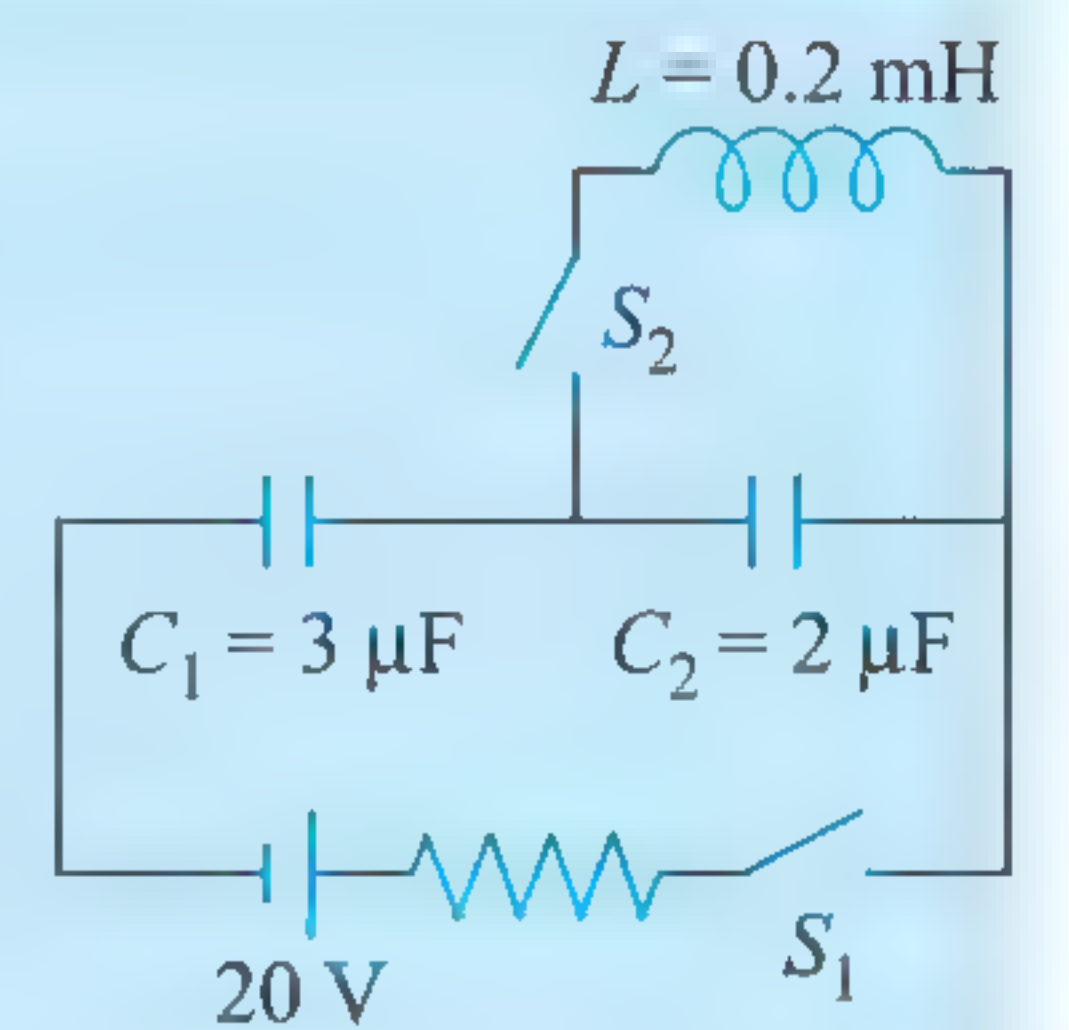
After this time, switch S_1 is also opened. The $100 \mu\text{F}$ capacitor is now charged to 300 V .

Thus, $t_1 = 0.15 \text{ s}$

and $t_2 = 0.05 \text{ s}$

ILLUSTRATION 5.28

The circuit shown in figure is in the steady state with switch S_1 is closed. At $t = 0$, S_1 is opened and switch S_2 is closed.



- Derive an expression for the charge on capacitor C_2 as a function of time.
- Determine the first instant t , when the energy in the inductor becomes one-third of that in the capacitor.

Sol.

- (a) In the steady state, C_1 and C_2 are in series arrangement, their equivalent is

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = 1.2 \mu\text{F}$$

Charge on the capacitor C_2 , $Q_0 = C_{\text{eq}} V = 1.2 \times 20 = 24 \mu\text{C}$. When S_1 is opened, S_2 is closed. Capacitor C_2 starts discharging through the inductor and let at any time t , charge on the capacitor be Q . Then we know that

$$Q = Q_0 \cos \omega t$$

$$U_E + U_B = \frac{Q_0^2}{2C_2}$$

$$\text{where } \omega = \frac{1}{\sqrt{LC_2}} = \frac{1}{\sqrt{0.2 \times 10^{-3} \times 2 \times 10^{-6}}} = 50,000 \text{ rad s}^{-1}$$

- (b) At the time $t = t_1$, $U_B = \frac{1}{3} U_E$, but $U_B + U_E = \frac{Q_0^2}{2C_2}$

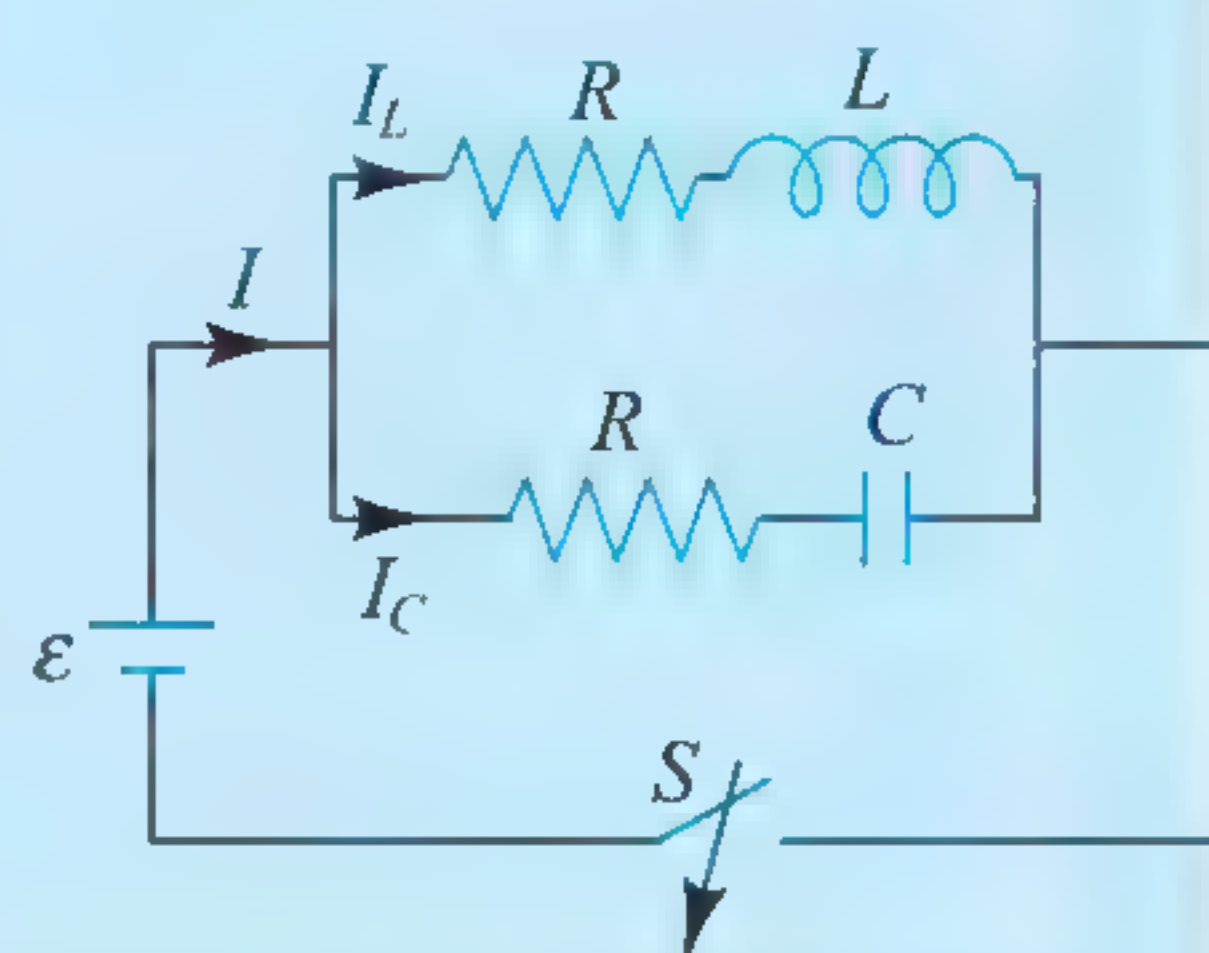
$$\text{Hence, } U_E = \frac{3}{4} \left(\frac{1}{2} \frac{Q_0^2}{C_2} \right) \Rightarrow \frac{1}{2} \frac{Q^2}{C_2} = \frac{3}{4} \left(\frac{1}{2} \frac{Q_0^2}{C_2} \right)$$

$$Q = \frac{\sqrt{3}}{2} Q_0 \quad \text{or} \quad Q_0 \cos \omega t_1 = \frac{\sqrt{3}}{2} Q_0$$

$$\omega t_1 = \frac{\pi}{6} \quad \text{or} \quad t_1 = \frac{\pi}{6\omega} = 1.05 \times 10^{-5} = 10.5 \mu\text{s}$$

ILLUSTRATION 5.29

In the circuit shown in figure, the battery has negligible internal resistance. Show that the current in the circuit through the battery rises instantly to its steady state value E/R when the switch is closed, provided that resistance R is $\sqrt{L/C}$.



Sol. Let the currents through inductive branch and capacitive branch be I_L and I_C , respectively. Then the current through the battery, from KCL, is $I = I_L + I_C$.

Since the battery is connected in parallel to the RL and RC branches of the circuit, the current in the RL branch is unaffected by the presence of the RC branch, so

$$I_L = \frac{E}{R} [1 - e^{-(R/L)t}] \quad \text{and} \quad I_C = \frac{E}{R} e^{-(t/RC)}$$

Hence, the current through battery is $I = \frac{E}{R} [1 - e^{-(R/L)t} + e^{-(t/RC)}]$

If the current has to reach its final value E/R instantaneously, the exponential terms must cancel out, i.e.,

$e^{-t/\tau_L} = e^{-t/\tau_C}$, which is possible if $\tau_L = \tau_C$

$$L/R = RC, R = \sqrt{L/C}$$

For all $t > 0$, this is the desired result.

ILLUSTRATION 5.30

An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of charge Q is 200 μC .

- When $Q = 100 \mu\text{C}$, what is the value of $\left| \frac{dI}{dt} \right|$?
- When $Q = 200 \mu\text{C}$, what is the value of I ?
- Find the maximum value of I .
- When I is equal to one-half its maximum value, what is the value of $|Q|$?

Sol. $L = 2.0 \text{ mH} = 2.0 \times 10^{-3} \text{ H}$
 $C = 5.0 \mu\text{F} = 5.0 \times 10^{-6} \text{ F}$
 $Q_{\text{max}} = 200 \mu\text{C} = 200 \times 10^{-6} \text{ C}$

In an LC circuit, energy transfer continues from inductance to capacitance and vice versa.

- By Kirchhoff's law in an LC circuit

$$\frac{Q}{C} + L \frac{dI}{dt} = 0 \Rightarrow \frac{dI}{dt} = -\frac{Q}{LC}$$

$$\therefore \left| \frac{dI}{dt} \right| = \frac{Q}{LC} = \frac{100 \times 10^{-6}}{2.0 \times 10^{-3} \times 5.0 \times 10^{-6}} = 10^4 \text{ A s}^{-1}$$

- When $Q = 200 \mu\text{C}$, the entire energy of circuit resides in capacitance. That is, no energy is stored in inductance.

$$\therefore \frac{1}{2} LI^2 = 0 \Rightarrow I = 0$$

- Maximum value of I is given by

$$\frac{1}{2} LI_{\text{max}}^2 = \frac{Q_{\text{max}}^2}{2C} \Rightarrow I_{\text{max}} = \frac{1}{\sqrt{LC}} Q_{\text{max}}$$

$$\text{or } I_{\text{max}} = \frac{1}{\sqrt{(2.0 \times 10^{-3}) \times (5.0 \times 10^{-6})}} \times 200 \times 10^{-6} = 2 \text{ A}$$

- Given $I = \frac{I_{\text{max}}}{2} = \frac{2}{2} = 1 \text{ A}$

Then again from the conservation of energy,

$$\frac{1}{2} LI^2 + \frac{Q^2}{2C} = \frac{1}{2} LI_{\text{max}}^2; \frac{Q^2}{2C} = \frac{1}{2} L(I_{\text{max}}^2 - I^2)$$

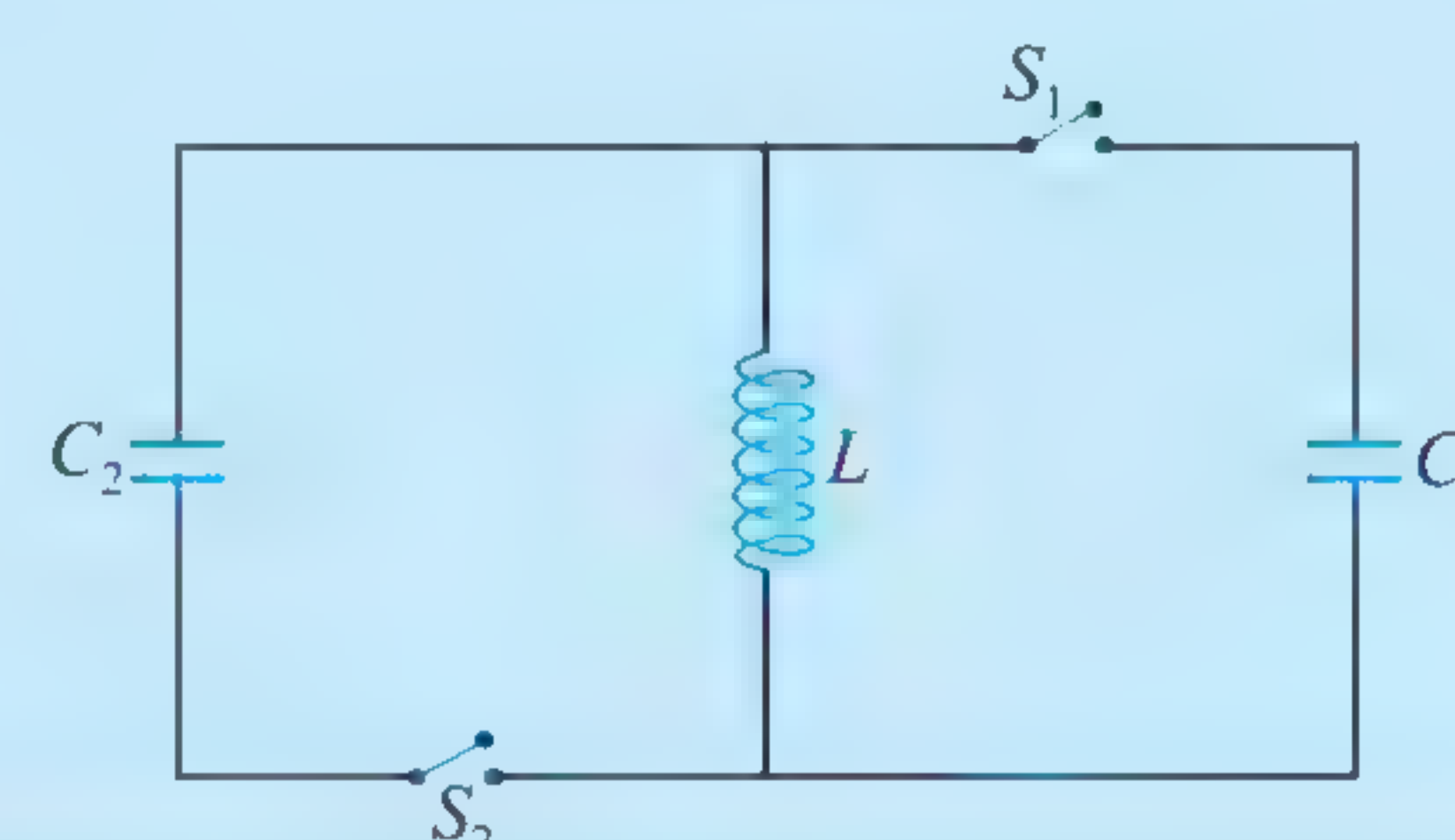
$$Q = \sqrt{LC(I_{\text{max}}^2 - I^2)}$$

$$= \sqrt{(2.0 \times 10^{-3} \times 5.0 \times 10^{-6})(2^2 - 1^2)}$$

$$= 10^{-4} \sqrt{3} \text{ C} = 1.732 \times 10^{-4} \text{ C} = 1.732 \mu\text{C}$$

ILLUSTRATION 5.31

In the circuit shown in the figure a capacitor C_1 is used to transfer energy to other capacitor C_2 . Initially capacitor of capacitance $C_1 = C_0$ is charged to a potential difference of V_0 . Switch S_1 is closed at time $t = 0$. After some time S_1 is opened and S_2 is closed simultaneously. At time $t = T$, S_2 was opened and it was found that the potential difference across capacitor of capacitance $C_2 = \frac{C_0}{9}$ was $3V_0$. What is the smallest possible value of time T ? The coil has inductance L . Assume no resistance.



Sol. Initial energy stored in C_1 is $U_1 = \frac{1}{2} C_0 V_0^2$

Final energy stored in C_2 is $U_2 = \frac{1}{2} \frac{C_0}{9} (3V_0)^2 = \frac{1}{2} C_0 V_0^2$

It means, complete energy of C_1 gets transferred to C_2 . The time period of LC circuit consisting of L and C_1 is

$$T_1 = 2\pi \sqrt{LC_0}$$

The entire energy of the capacitor can get transferred to the inductor in smallest interval of $t_1 = \frac{T_1}{4} = \frac{\pi}{2} \sqrt{LC_0}$

At time t_1 the current through L is maximum and switch S_1 is opened and S_2 is closed.

Now the time period of LC circuit having L and C_2 is

$$T_2 = 2\pi \sqrt{\frac{LC_0}{9}} = \frac{2\pi}{3} \sqrt{LC_0}$$

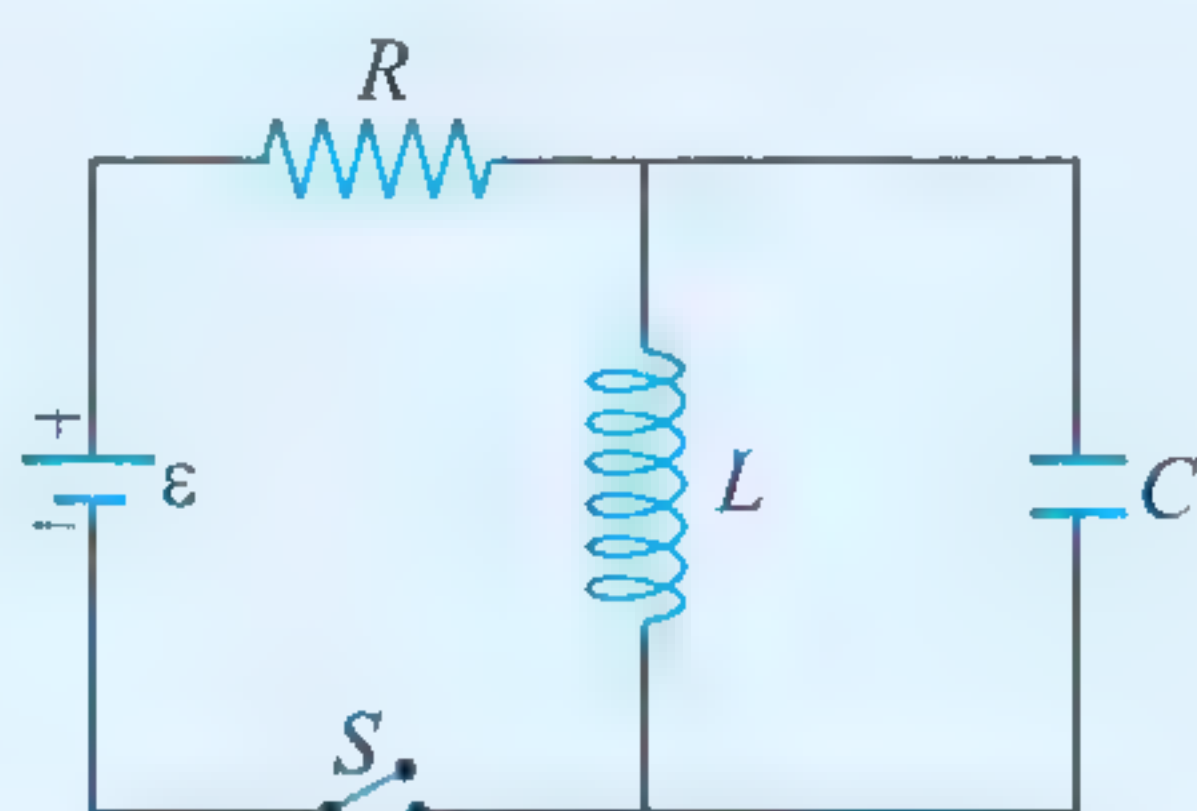
The inductor will transfer its entire energy to C_2 in smallest time given by

$$t_2 = \frac{T_2}{4} = \frac{\pi}{6} \sqrt{LC_0}$$

$$\therefore T_{\min} = t_1 + t_2 = \frac{2\pi}{3} \sqrt{LC_0}$$

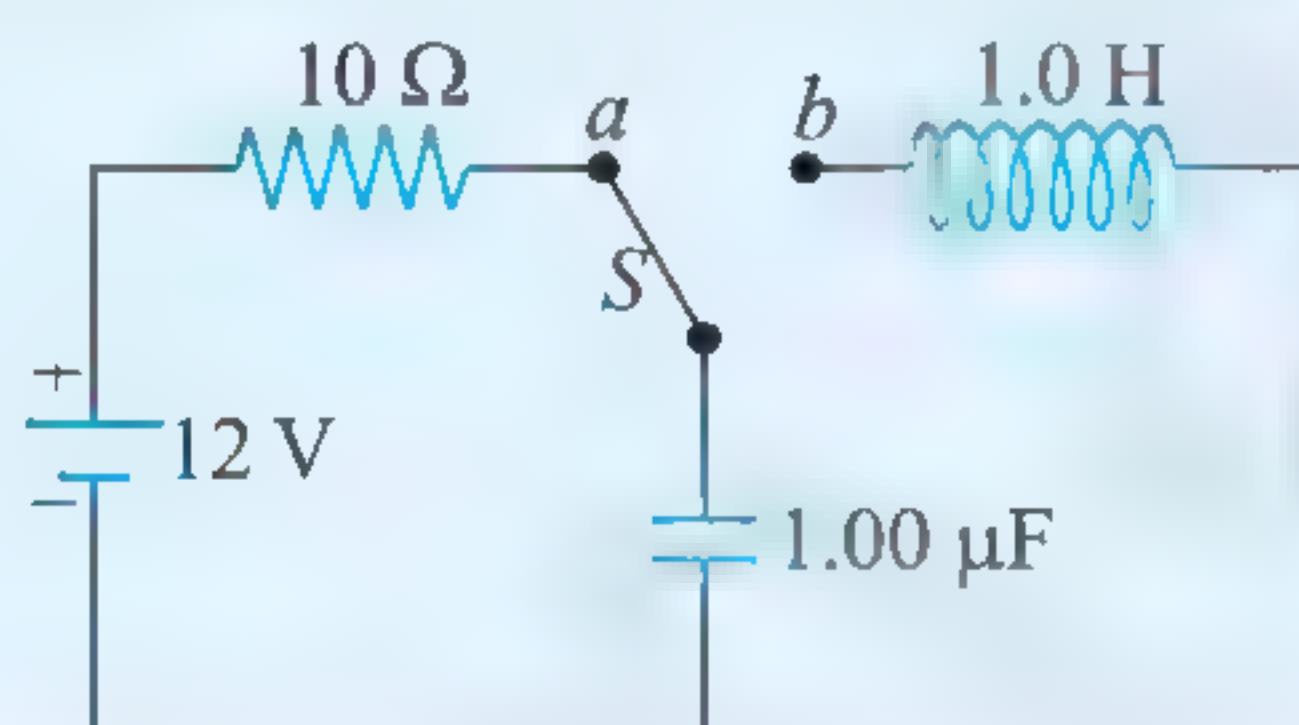
CONCEPT APPLICATION EXERCISE 5.4

1. A $1.0 \mu\text{F}$ capacitor is charged by a 40.0 V power supply. The fully charged capacitor is then discharged through a 10.0 mH inductor. Find the maximum current in the resulting oscillations.
2. In the circuit of figure, the battery emf is 50 V , the resistance is 200Ω , and the capacitance is $1.0 \mu\text{F}$. The switch S is closed for a long time interval, and zero potential difference is measured across the capacitor.



After the switch is opened, the potential difference across the capacitor reaches a maximum value of 100 V . What is the value of the inductance?

3. An LC circuit consists of a 20.0 mH inductor and a $0.500 \mu\text{F}$ capacitor. If the maximum instantaneous current is 0.100 A , what is the greatest potential difference across the capacitor?
4. The switch in figure is connected to position a for a long time interval. At $t = 0$, the switch is thrown to position b . After this time, what are (a) the frequency of oscillation of the LC circuit, (b) the maximum charge that appears on the capacitor, (c) the maximum current in the inductor, and (d) the total energy the circuit possesses at $t = 3.00 \text{ s}$?



5. An inductor having inductance L and a capacitor having capacitance C are connected in series. The current in the circuit increases linearly in time as described by $i = Kt$, where K is a constant. The capacitor is initially uncharged. Determine (a) the voltage across the inductor as a function of time, (b) the voltage across the capacitor as a function of time, and (c) the time when the energy stored in the capacitor first exceeds that in the inductor.
6. A capacitor in a series LC circuit has an initial charge Q and is being discharged. When the charge on the capacitor is $Q/2$, find the flux through each of the N turns in the coil of the inductor in terms of Q , N , L , and C .
7. A coil with an inductance of 2.0 H and a resistance of 10Ω is suddenly connected to an ideal battery with $\mathcal{E} = 100 \text{ V}$. At 0.10 s after the connection is made, what is the rate at which (a) energy is being stored in the magnetic field, (b) thermal energy is appearing in the resistance, and (c) energy is being delivered by the battery?
8. At $t = 0$, a battery is connected to a series arrangement of a resistor and an inductor. If the inductive time constant is 40.0 ms , at what time is the rate at which energy is dissipated in the resistor equal to the rate at which energy is stored in the inductor's magnetic field?

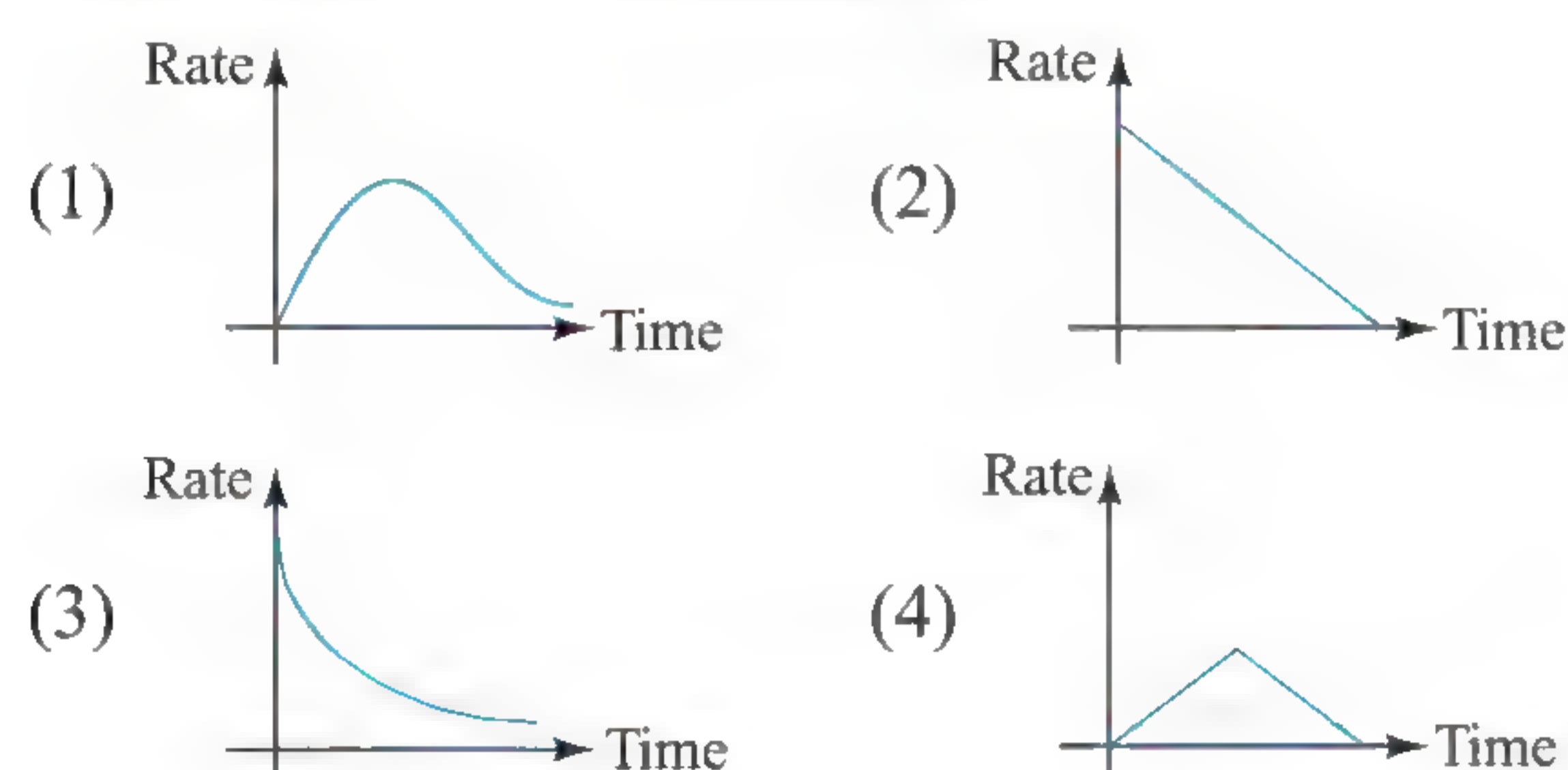
ANSWERS

1. 400 mA
2. 160 mH
3. 20.0 V
4. (a) $\frac{500}{\pi} \text{ Hz}$ (b) $12.0 \mu\text{C}$ (c) 12.0 mA (d) $72.0 \mu\text{C}$
5. (a) $-LK$ (b) $-\frac{Kt^2}{2C}$ (c) $2\sqrt{LC}$
6. $\frac{q}{2N} \sqrt{\frac{3L}{C}}$
7. (a) $2.4 \times 10^2 \text{ W}$; (b) $1.5 \times 10^2 \text{ W}$; (c) $3.9 \times 10^2 \text{ W}$
8. $40.0 \ln 2 \text{ ms}$

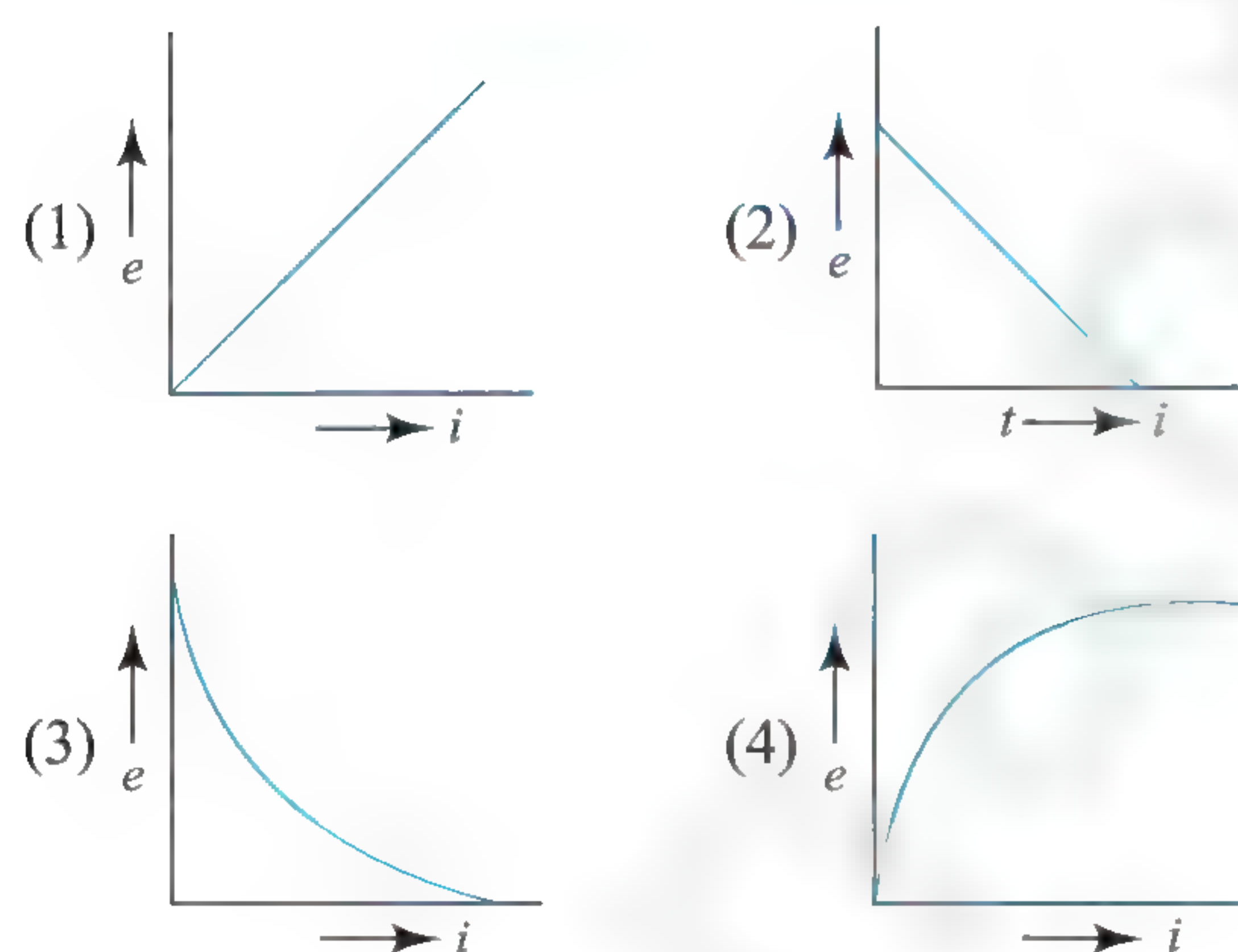
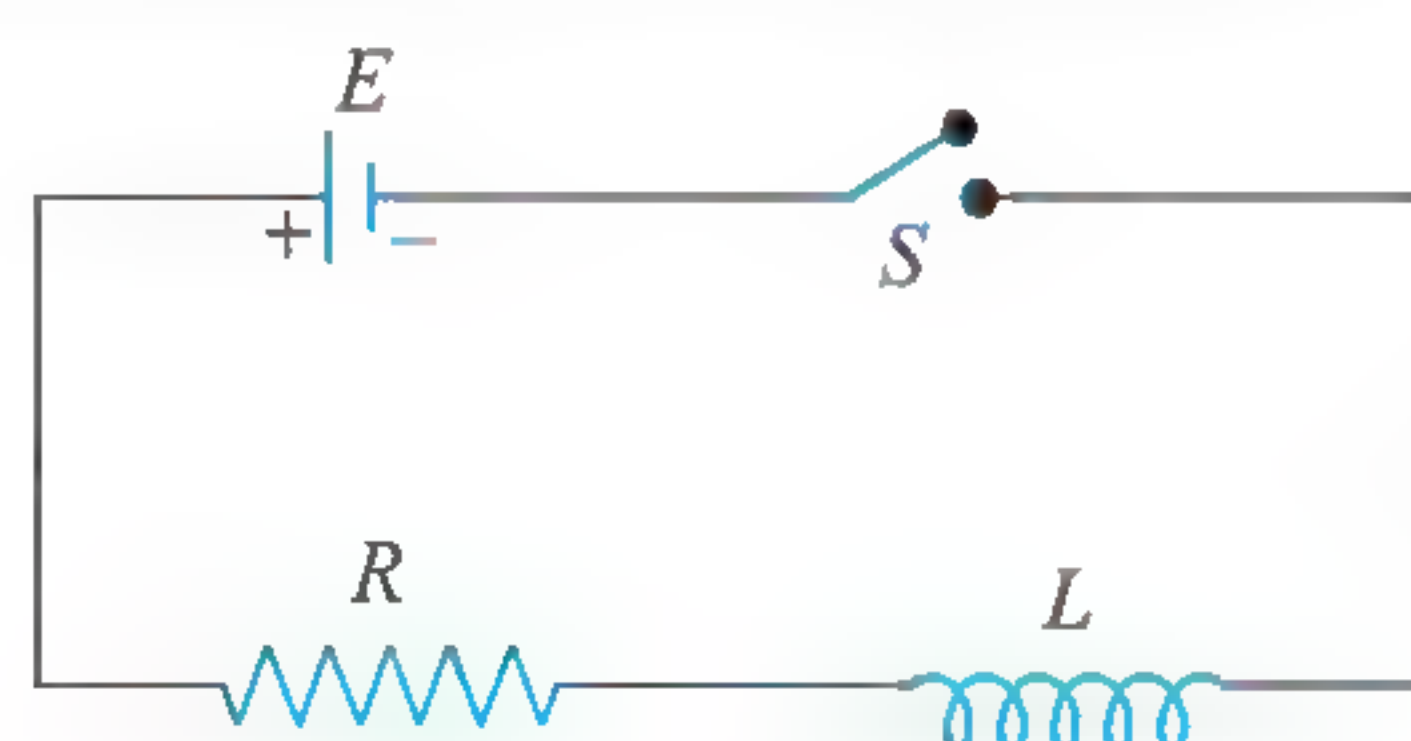
Exercises

Single Correct Answer Type

1. In an LR circuit connected to a battery, the rate at which energy is stored in the inductor is plotted against time during the growth of current in the circuit. Which of the following best represents the resulting curve?



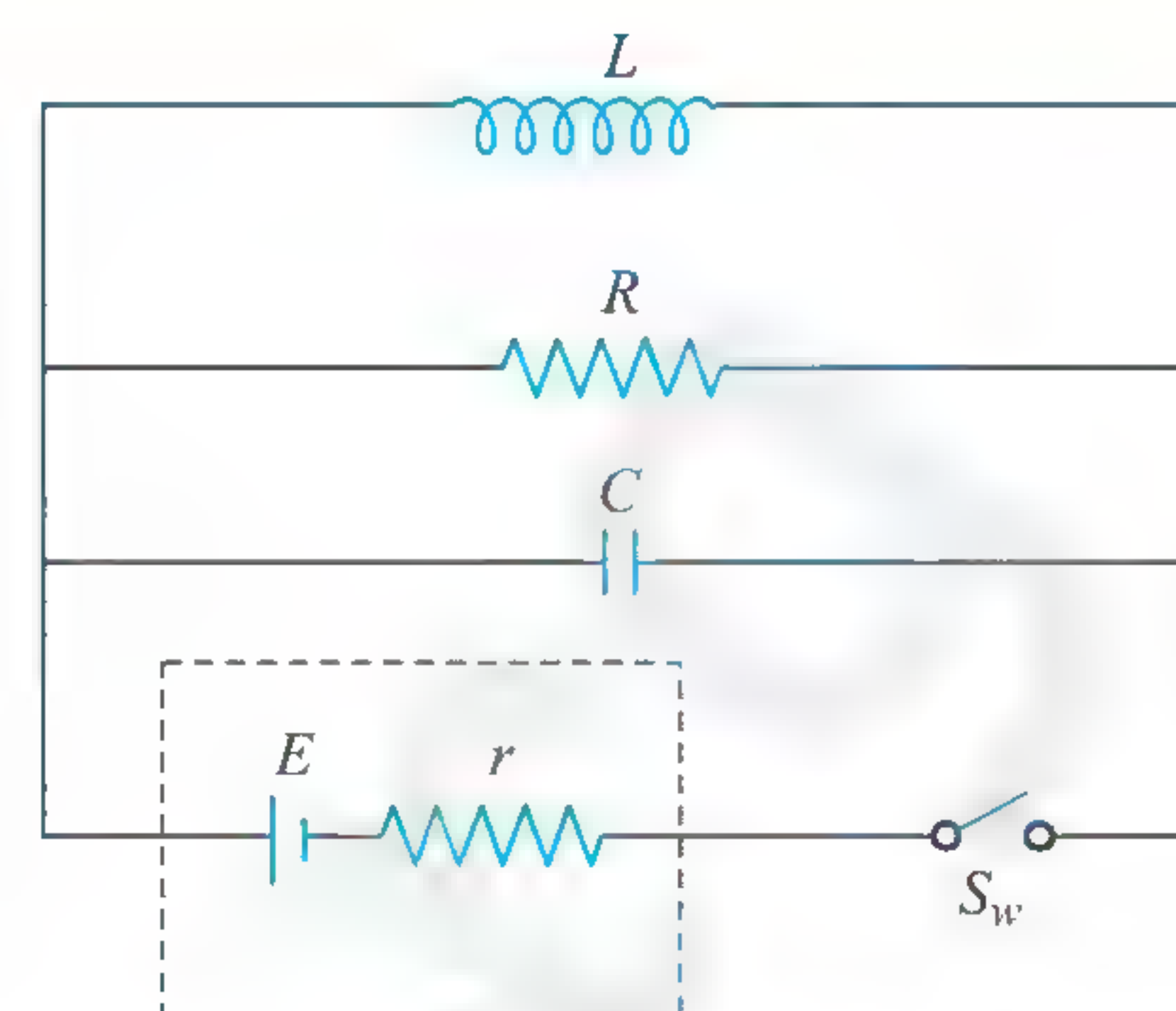
2. Switch S of the circuit shown in figure is closed at $t = 0$. If e denotes the induced emf in L and i the current flowing through the circuit at time t , then which of the following graphs correctly represents the variation of e with i ?



3. A mutual inductor consists of two coils X and Y as shown in figure in which one-quarter of the magnetic flux produced by X links with Y , giving a mutual inductance M . What will be the mutual inductance when Y is used as the primary?

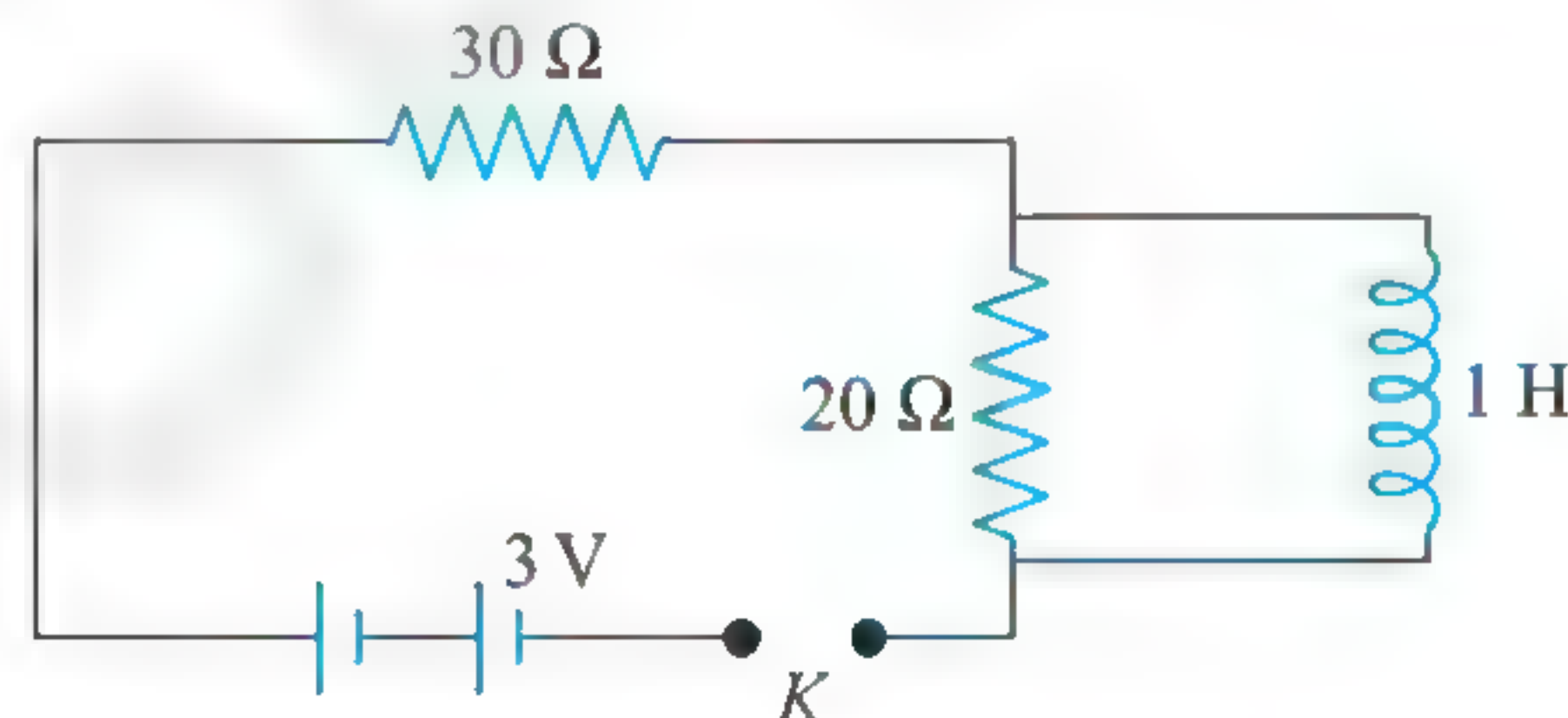


- (1) $M/4$ (2) $M/2$
(3) M (4) $2M$
4. A pure inductor L , a capacitor C and a resistance R are connected across a battery of emf E and internal resistance r as shown in figure. Switch S_w is closed at $t = 0$, select the correct alternative(s).

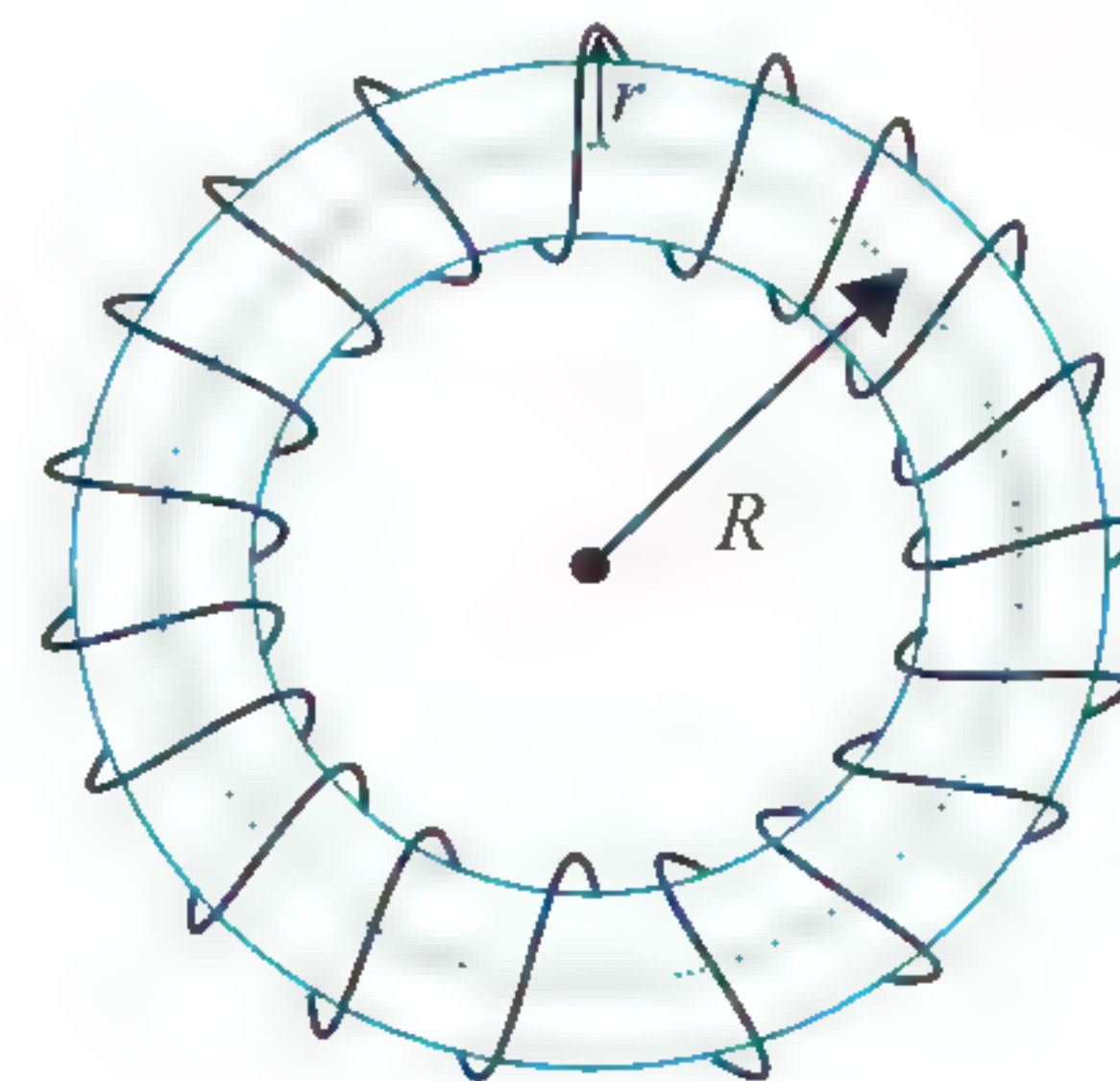


- (1) current through resistance R is zero all the time
(2) current through resistance R is zero at $t = 0$ and $t \rightarrow \infty$
(3) maximum charge stored in the capacitor is CE
(4) maximum energy stored in the inductor is equal to the maximum energy stored in the capacitor
5. A coil carrying a steady current is short-circuited. The current in it decreases α times in time t_0 . The time constant of the circuit is
- (1) $\tau = t_0 \ln \alpha$ (2) $\tau = \frac{t_0}{\ln \alpha}$
(3) $\tau = \frac{t_0}{\alpha}$ (4) $\tau = \frac{t_0}{\alpha - 1}$
6. A solenoid has 2000 turns wound over a length of 0.3 m. Its cross-sectional area is equal to $1.2 \times 10^{-3} \text{ m}^2$. Around its central cross-section a coil of 300 turns is wound. If an initial current of 2 A flowing in the solenoid is reversed in 0.25 s, the emf induced in the coil is
- (1) 0.6 mV (2) 60 mV
(3) 48 mV (4) 0.48 mV
7. The time constant of an inductance coil is $2 \times 10^{-3} \text{ s}$. When a $90\text{-}\Omega$ resistance is joined in series, the same constant becomes $0.5 \times 10^{-3} \text{ s}$. The inductance and resistance of the coil are
- (1) 30 mH; 30 Ω (2) 60 mH; 30 Ω
(3) 30 mH; 60 Ω (4) 60 mH; 60 Ω
8. The coefficient of mutual inductance of two circuits A and B is 3 mH and their respective resistances are 10 and 4 Ω . How much current should change in 0.02 s in circuit A , so that the induced current in B should be 0.0060 A?
- (1) 0.24 A (2) 1.6 A
(3) 0.18 A (4) 0.16 A
9. An emf of 15 V is applied in a circuit containing 5 H inductance and 10 Ω resistance. The ratio of the currents at time $t = \infty$ and $t = 1 \text{ s}$ is
- (1) $\frac{e^{1/2}}{e^{1/2} - 1}$ (2) $\frac{e^2}{e^2 - 1}$
(3) $1 - e^{-1}$ (4) e^{-1}
10. A coil of inductance 0.20 H is connected in series with a switch and a cell of emf 1.6 V. The total resistance of the

- circuit is $4.0 \, \Omega$. What is the initial rate of growth of the current when the switch is closed?
- (1) $0.050 \, \text{As}^{-1}$ (2) $0.40 \, \text{As}^{-1}$
(3) $0.13 \, \text{As}^{-1}$ (4) $8.0 \, \text{As}^{-1}$
11. A straight solenoid of length $1 \, \text{m}$ has 5000 turns in the primary and 200 turns in the secondary. If the area of cross section is $4 \, \text{cm}^2$, the mutual inductance will be
- (1) $503 \, \text{H}$ (2) $503 \, \text{mH}$
(3) $503 \, \mu\text{H}$ (4) $5.03 \, \text{H}$
12. The approximate formula expressing the formula of mutual inductance of two thin coaxial loops of the same radius a when their centers are separated by a distance l with $l \gg a$ is
- (1) $\frac{1}{2} \frac{\mu_0 \pi a^4}{l^3}$ (2) $\frac{1}{2} \frac{\mu_0 a^4}{l^2}$
(3) $\frac{\mu_0 \pi a^4}{4\pi l^2}$ (4) $\frac{\mu_0 a^4}{\pi l^3}$
13. The length of a thin wire required to manufacture a solenoid of length $l = 100 \, \text{cm}$ and inductance $L = 1 \, \text{mH}$, if the solenoid's cross-sectional diameter is considerably less than its length is
- (1) $1 \, \text{km}$ (2) $0.10 \, \text{km}$
(3) $0.010 \, \text{km}$ (4) $10 \, \text{km}$
14. Current in a coil of self-inductance $2.0 \, \text{H}$ is increasing as $i = 2 \sin t^2$. The amount of energy spent during the period when the current changes from 0 to $2 \, \text{A}$ is
- (1) $1 \, \text{J}$ (2) $2 \, \text{J}$
(3) $3 \, \text{J}$ (4) $4 \, \text{J}$
15. A long solenoid having 200 turns per centimeter carries a current of $1.5 \, \text{A}$. At the center of the solenoid is placed a coil of 100 turns of cross-sectional area $3.14 \times 10^{-4} \, \text{m}^2$ having its axis parallel to the field produced by the solenoid. When the direction of current in the solenoid is reversed within $0.05 \, \text{s}$, the induced emf in the coil is
- (1) $0.48 \, \text{V}$ (2) $0.048 \, \text{V}$
(3) $0.0048 \, \text{V}$ (4) $48 \, \text{V}$
16. Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate of $15.0 \, \text{As}^{-1}$, the emf in coil 1 is $25 \, \text{mV}$, when coil 2 has no current and coil 1 has a current of $3.6 \, \text{A}$, the flux linkage in coil 2 is
- (1) $16 \, \text{mWb}$ (2) $10 \, \text{mWb}$
(3) $4.00 \, \text{mWb}$ (4) $6.00 \, \text{mWb}$
17. A current i_0 is flowing through an L - R circuit of time constant t_0 . The source of the current is switched off at time $t = 0$. Let r be the value of $(-di/dt)$ at time $t = 0$. Assuming this rate to be constant, the current will reduce to zero in a time interval of
- (1) t_0 (2) et_0
(3) $\frac{t_0}{e}$ (4) $\left(1 - \frac{1}{e}\right)t_0$
18. A small coil of radius r is placed at the center of a large coil of radius R , where $R \gg r$. The two coils are coplanar. The mutual inductance between the coils is proportional to
- (1) r/R (2) r^2/R
(3) r^2/R^2 (4) r/R^2
19. The length of a wire required to manufacture a solenoid of length l and self-induction L is (cross-sectional area is negligible)
- (1) $\sqrt{\frac{2\pi Ll}{\mu_0}}$ (2) $\sqrt{\frac{\mu_0 Ll}{4\pi}}$
(3) $\sqrt{\frac{4\pi Ll}{\mu_0}}$ (4) $\sqrt{\frac{\mu_0 Ll}{2\pi}}$
20. The total heat produced in resistor R in an RL circuit when the current in the inductor decreases from I_0 to 0 is
- (1) LI_0^2 (2) $\frac{1}{2}LI_0^2$
(3) $\frac{3}{2}LI_0^2$ (4) $\frac{1}{3}LI_0^2$
21. In the circuit (figure), the final current through $30 \, \Omega$ resistance when circuit is completed is

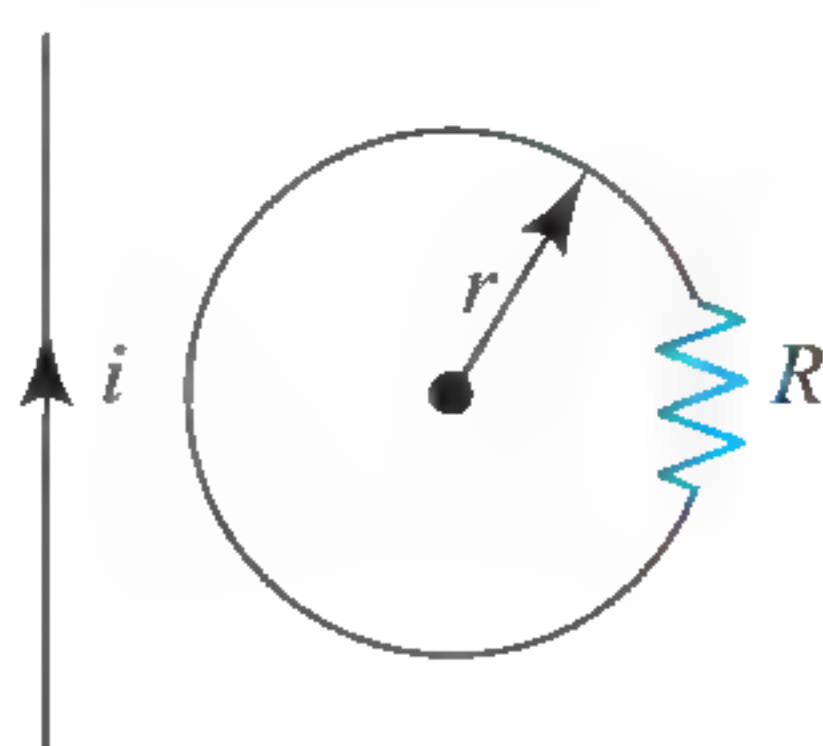


- (1) $3 \, \text{A}$ (2) $0.1 \, \text{A}$
(3) $5 \, \text{A}$ (4) $0.5 \, \text{A}$
22. The inductance L of a solenoid of length l , whose windings are made of material of density D and resistivity ρ , is (the winding resistance is R)
- (1) $\frac{\mu_0}{4\pi l} \frac{Rm}{\rho D}$ (2) $\frac{\mu_0}{4\pi R} \frac{lm}{\rho D}$
(3) $\frac{\mu_0}{4\pi l} \frac{R^2 m}{\rho D}$ (4) $\frac{\mu_0}{2\pi R} \frac{lm}{\rho D}$
23. The capacitance in an oscillatory LC circuit is increased by 1% . The change in inductance required to restore its frequency of oscillation is to
- (1) decrease it by 0.5% (2) increase it by 1%
(3) decrease it by 1% (4) decrease it by 2%
24. A toroid is wound over a circular core. Radius of each turn is r and radius of toroid is R ($\gg r$). The coefficient of self-inductance of the toroid is given by



- (1) $L = \frac{\mu_0 N r^2}{2R}$ (2) $L = \frac{\mu_0 N r}{2R}$
(3) $L = \frac{\mu_0 N r^2}{R}$ (4) $L = \frac{\mu_0 N^2 r^2}{2R}$

25. In figure, the mutual inductance of a coil and a very long straight wire is M , the coil has resistance R and self-inductance L . The current in the wire varies according to the law $i = at$, where a is a constant and t is the time, the time dependence of current in the coil is

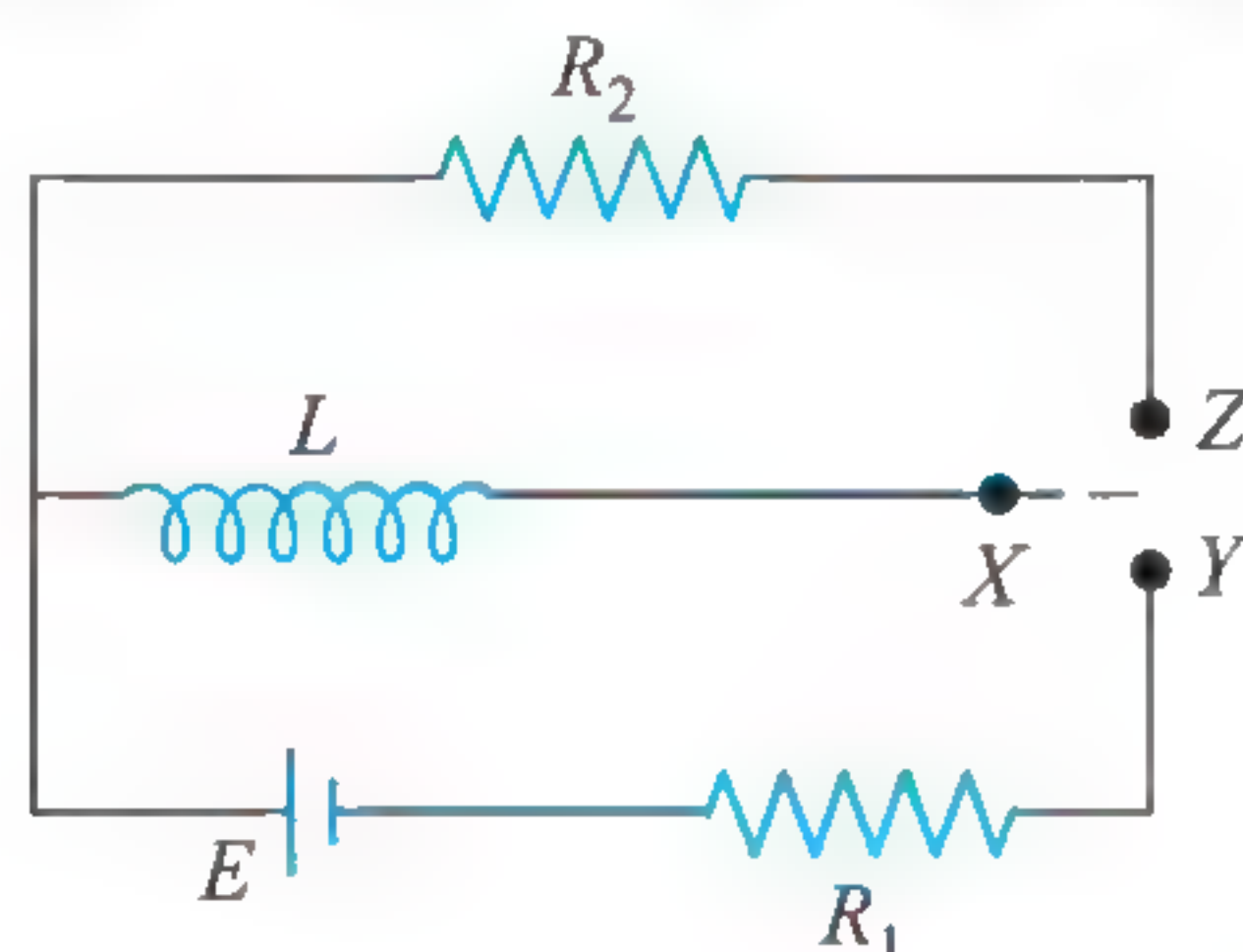


- (1) $\frac{M}{aR}$ (2) $MaR e^{-Rt/L}$
 (3) $\frac{M}{R} e^{-tR/L}$ (4) $\frac{Ma}{R} (1 - e^{-tR/L})$

26. A closed circuit consists of a resistor R ; inductor of inductance L and a source of emf E are connected in series. If the inductance of the coil is abruptly decreased to $L/4$ (by removing its magnetic core), the new current immediately after this moment is (before decreasing the inductance the circuit is in steady state)

- (1) zero (2) $\frac{E}{R}$
 (3) $4\frac{E}{R}$ (4) $\frac{E}{4R}$

27. In the circuit shown (figure), X is joined to Y for a long time and then X is joined to Z . The total heat produced in R_2 is

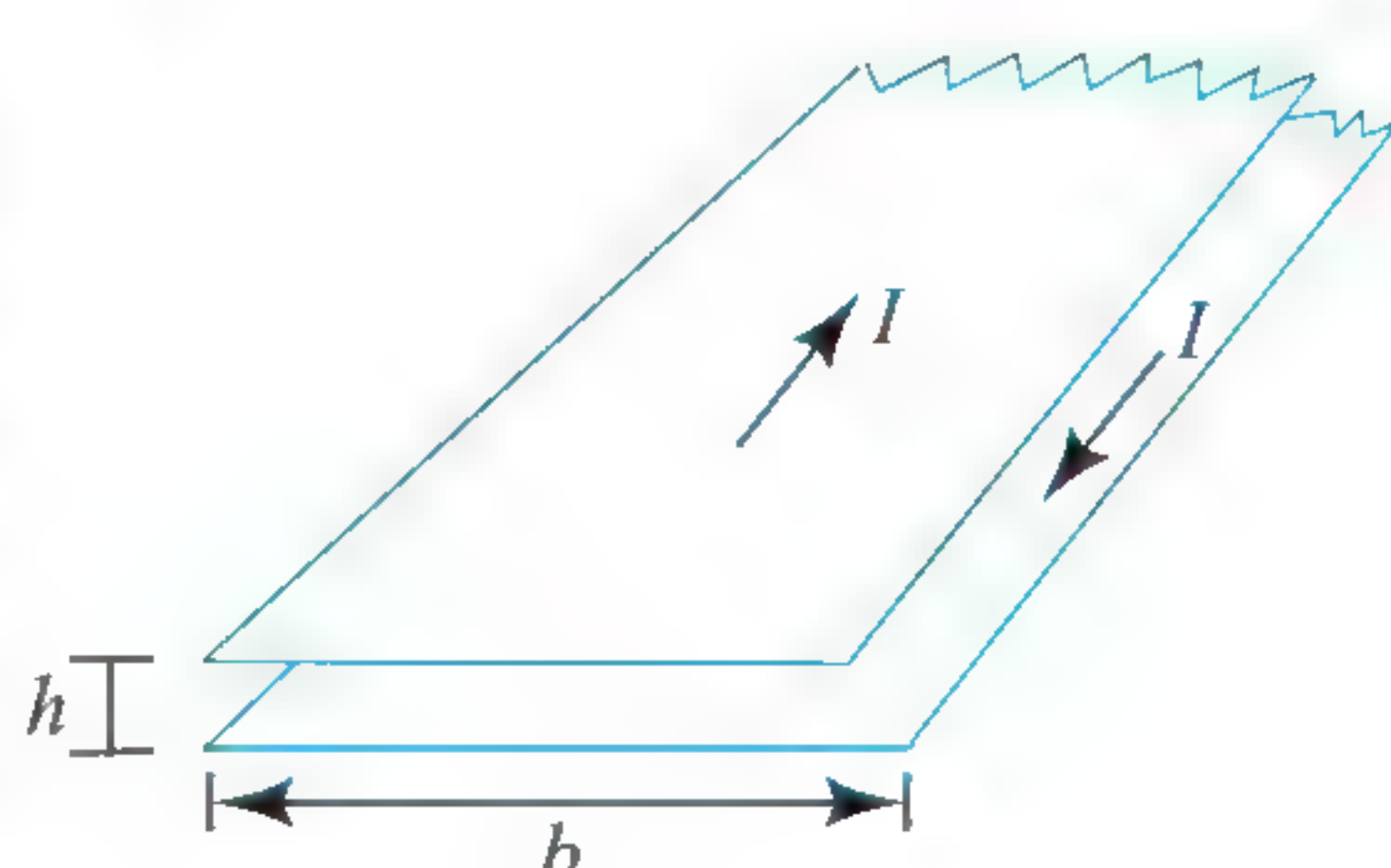


- (1) $\frac{LE^2}{2R_1^2}$ (2) $\frac{LE^2}{2R_2^2}$
 (3) $\frac{LE^2}{2R_1R_2}$ (4) $\frac{LE^2R_2}{2R_1^3}$

28. A horizontal ring of radius $r = \frac{1}{2}$ m is kept in a vertical constant magnetic field 1 T. The ring is collapsed from maximum area to zero area in 1 s. Then the emf induced in the ring is

- (1) 1 V (2) $(\pi/4)$ V
 (3) $(\pi/2)$ V (4) π V

29. Calculate the inductance of a unit length of a double tape line as shown in figure if the tapes are separated by a distance h which is considerably less than their width b .



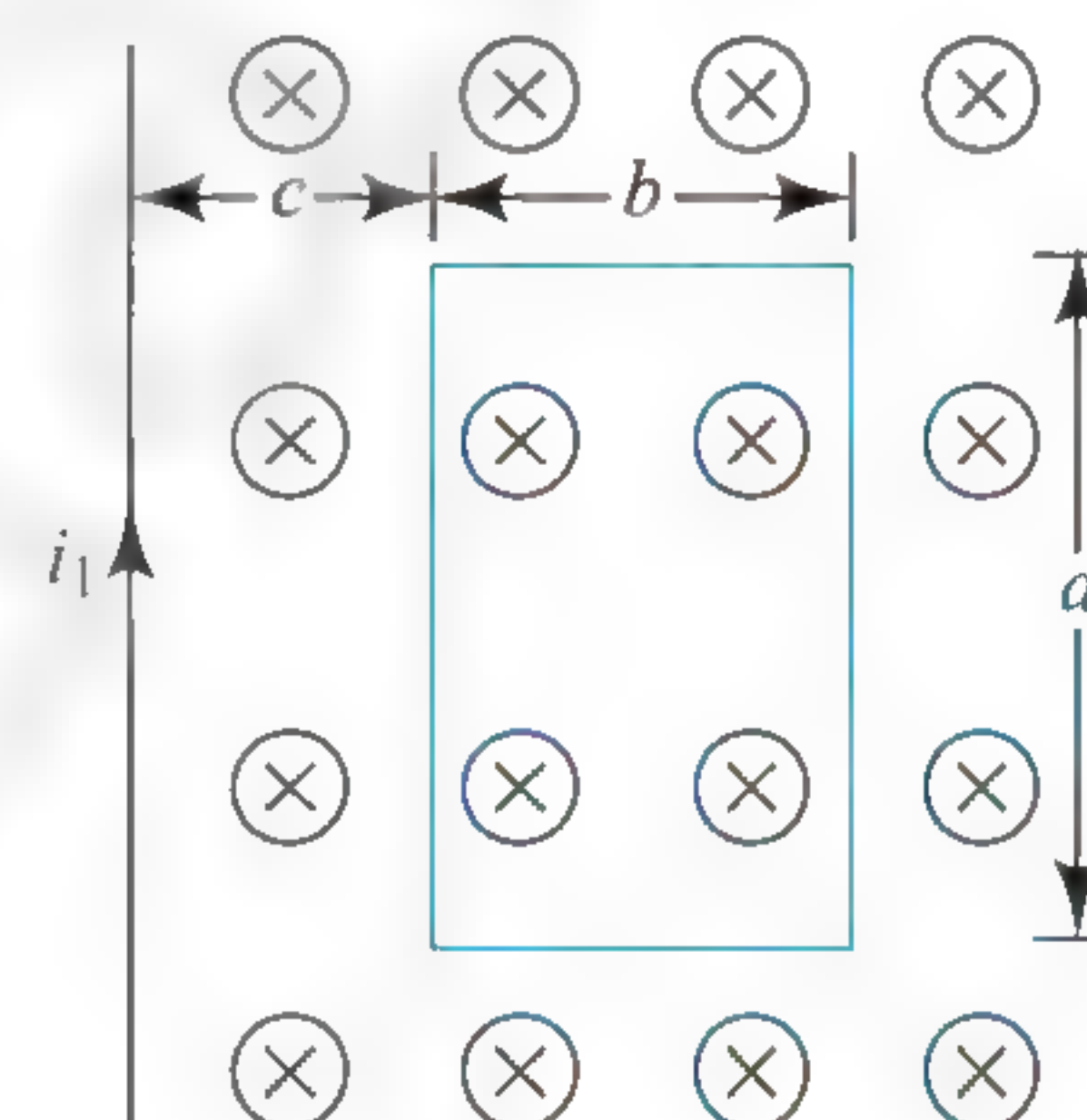
- (1) $\frac{\mu_0 h}{b}$ (2) $\frac{\mu_0 h}{2b}$

- (3) $\frac{2\mu_0 h}{b}$ (4) $\frac{\sqrt{2}\mu_0 h}{b}$

30. Find the inductance of a unit length of two parallel wires, each of radius a , whose centers are a distance d apart and carry equal currents in opposite directions. Neglect the flux within the wire.

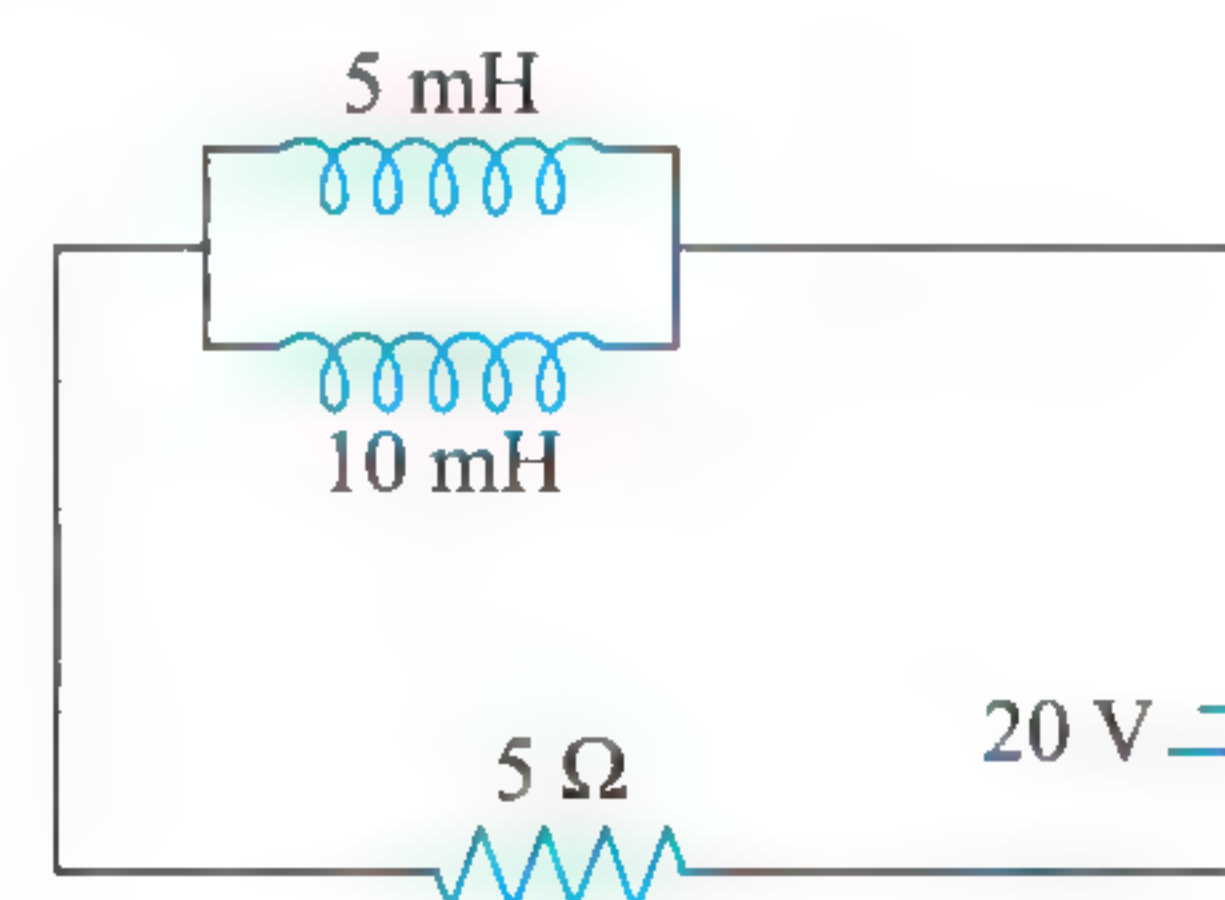
- (1) $\frac{\mu_0}{2\pi} \ln\left(\frac{d-a}{a}\right)$ (2) $\frac{\mu_0}{\pi} \ln\left(\frac{d-a}{a}\right)$
 (3) $\frac{3\mu_0}{\pi} \ln\left(\frac{d-a}{a}\right)$ (4) $\frac{\mu_0}{3\pi} \ln\left(\frac{d-a}{a}\right)$

31. Figure shows a rectangular coil near a long wire. The mutual inductance of the combination is



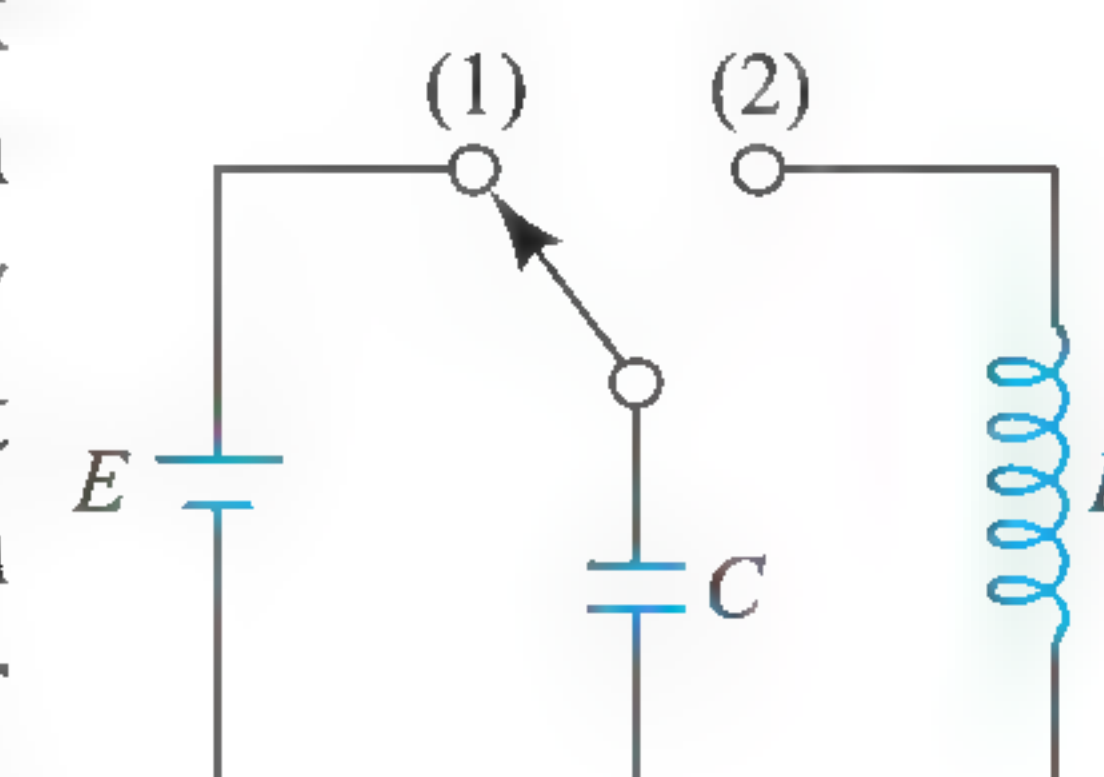
- (1) $\frac{\mu_0 a}{2\pi} \ln\left(1 - \frac{b}{c}\right)$ (2) $\frac{\mu_0 a}{2\pi} \ln\left(1 + \frac{b}{c}\right)$
 (3) $\frac{\mu_0 a}{\pi} \ln\left(1 + \frac{b}{c}\right)$ (4) $\frac{\mu_0 a}{\sqrt{2}\pi} \ln\left(1 + \frac{b}{c}\right)$

32. In the given circuit (figure), current through the 5 mH inductor in steady state is



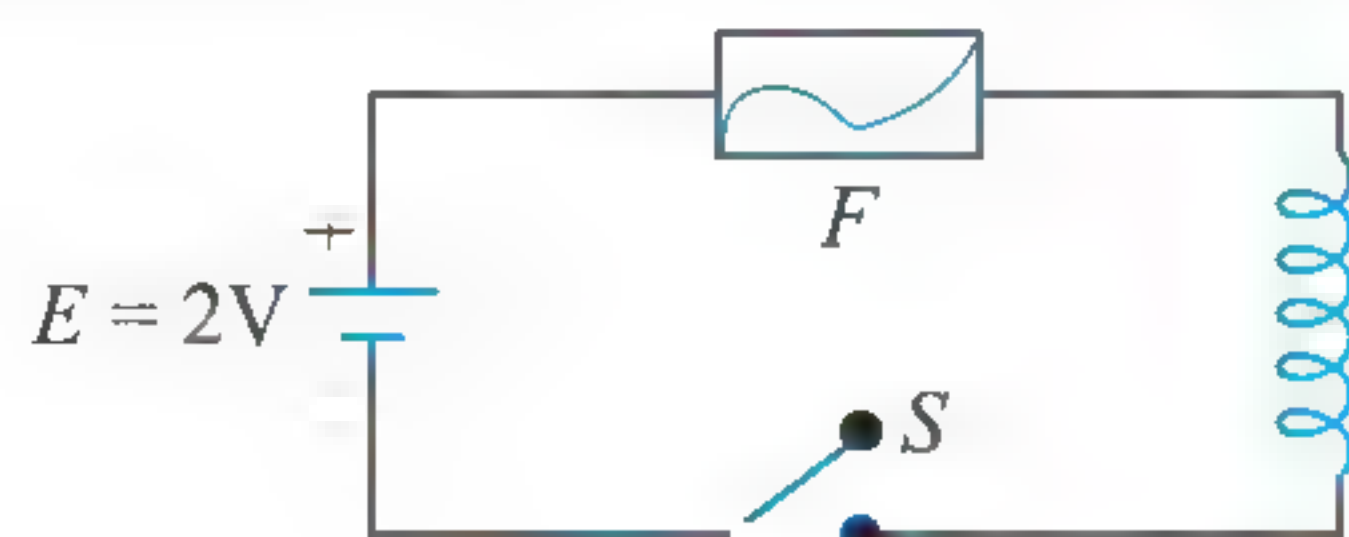
- (1) $\frac{2}{3}$ A (2) $\frac{8}{3}$ A
 (3) $\frac{1}{3}$ A (4) $\frac{2}{3}$ A

33. In the following electrical network at $t < 0$ (figure), key is placed on (1) till the capacitor got fully charged. Key is placed on (2) at $t = 0$. Time when the energy in both the capacitor and the inductor will be same for the first time is

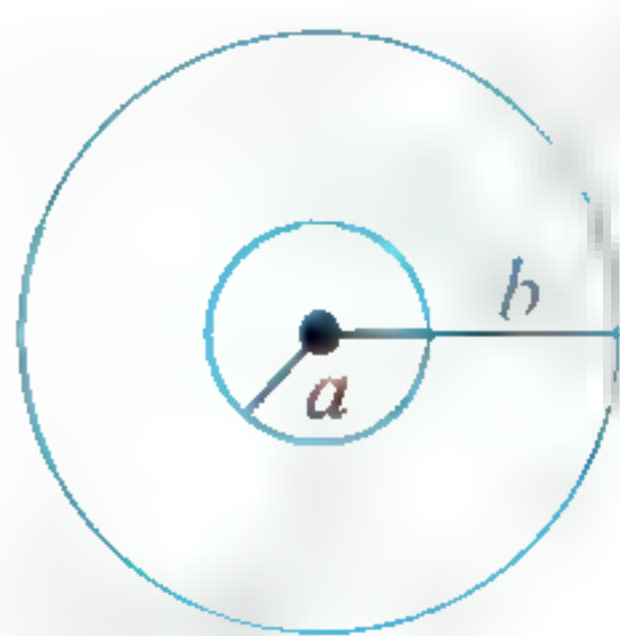
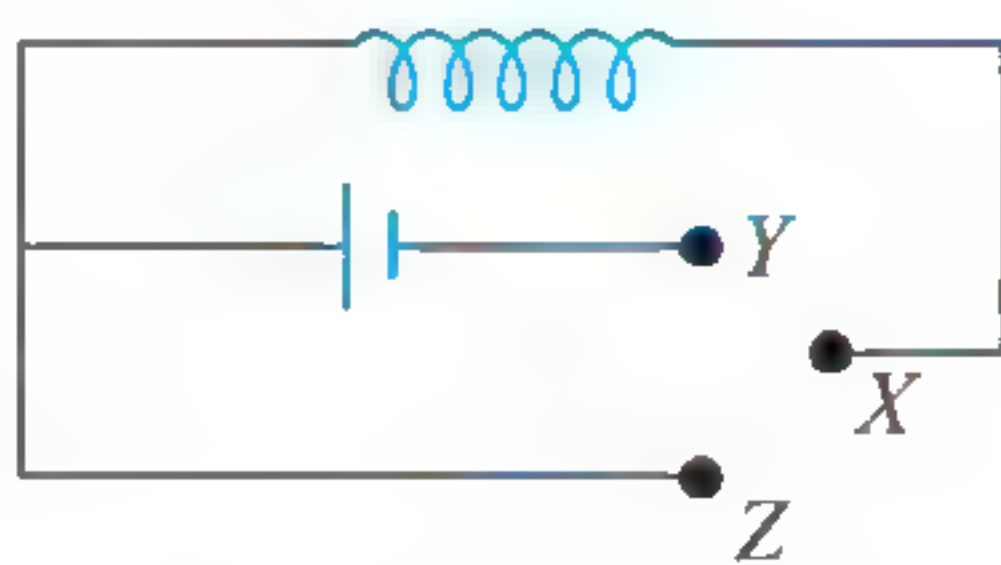


- (1) $\frac{\pi}{4} \sqrt{LC}$ (2) $\frac{3\pi}{4} \sqrt{LC}$
 (3) $\frac{\pi}{3} \sqrt{LC}$ (4) $\frac{2\pi}{3} \sqrt{LC}$

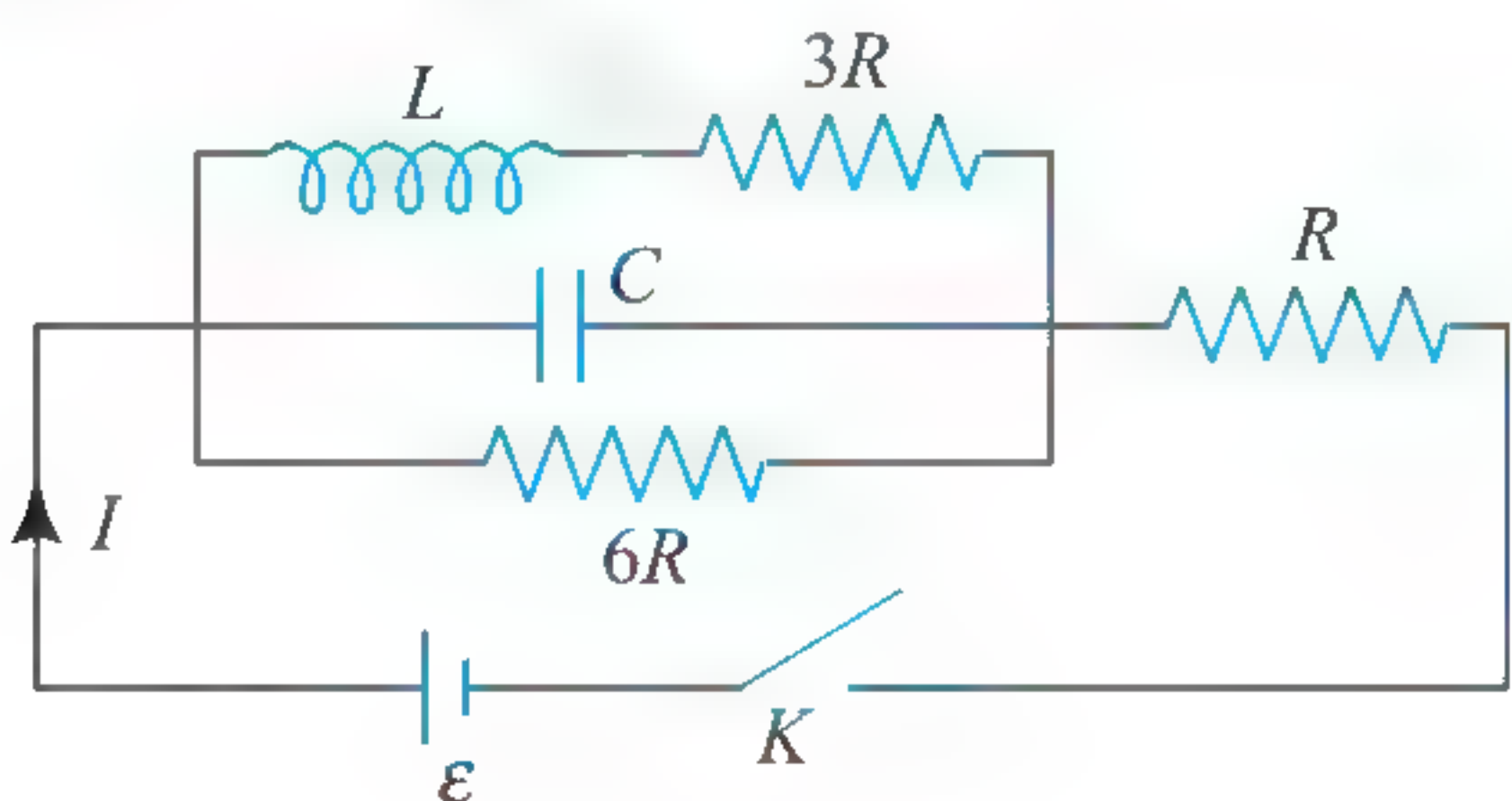
34. In the circuit shown (figure), the cell is ideal. The coil has an inductance of 4 H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5 A. The switch is closed at $t = 0$. The fuse will blow



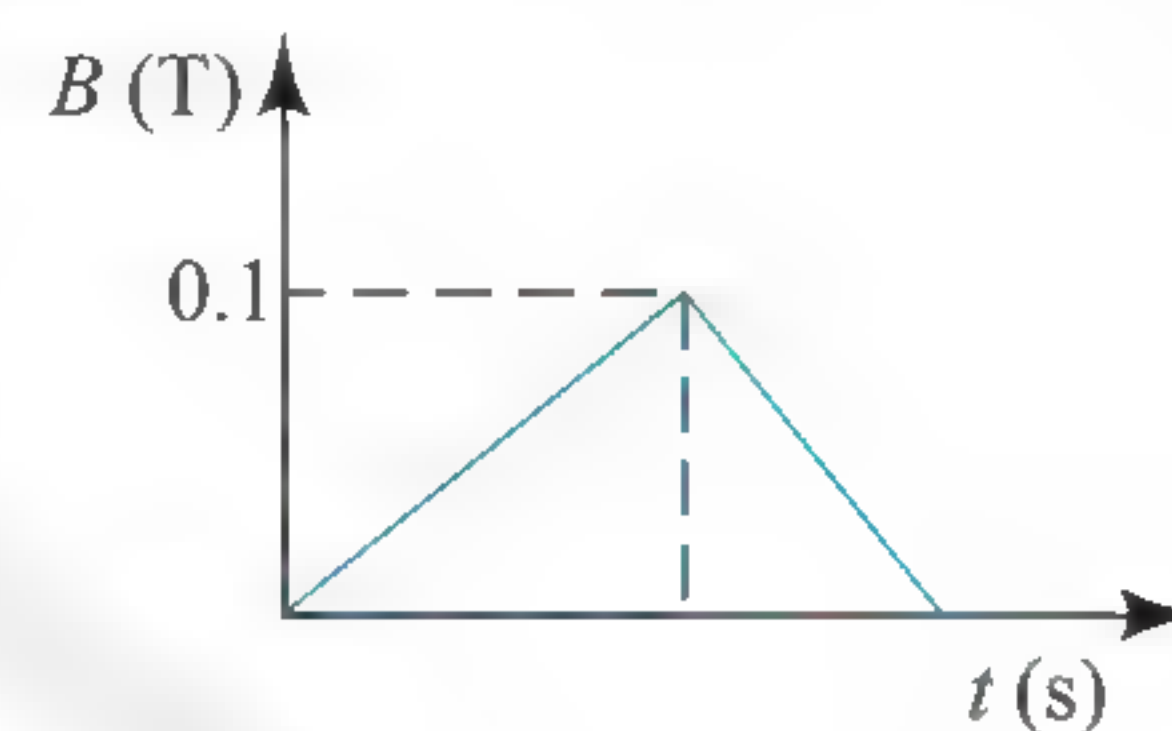
- (1) almost at once (2) after 2 s
(3) after 5 s (4) after 10 s
35. In the circuit shown (figure), the coil has inductance and resistance. When X is joined to Y , the time constant is τ during the growth of current.
- When the steady state is reached, heat is produced in the coil at a rate P . X is now joined to Z . After joining X and Z :
- (1) The total heat produced in the coil is $P\tau$
(2) The total heat produced in the coil is $\frac{1}{2} P\tau$
(3) The total heat produced in the coil is $2P\tau$
(4) The data given are not sufficient to reach a conclusion
36. Two concentric and coplanar coils have radii a and b ($\gg a$) as shown in figure. Resistance of the inner coil is R . Current in the outer coil is increased from 0 to i , then the total charge circulating the inner coil is



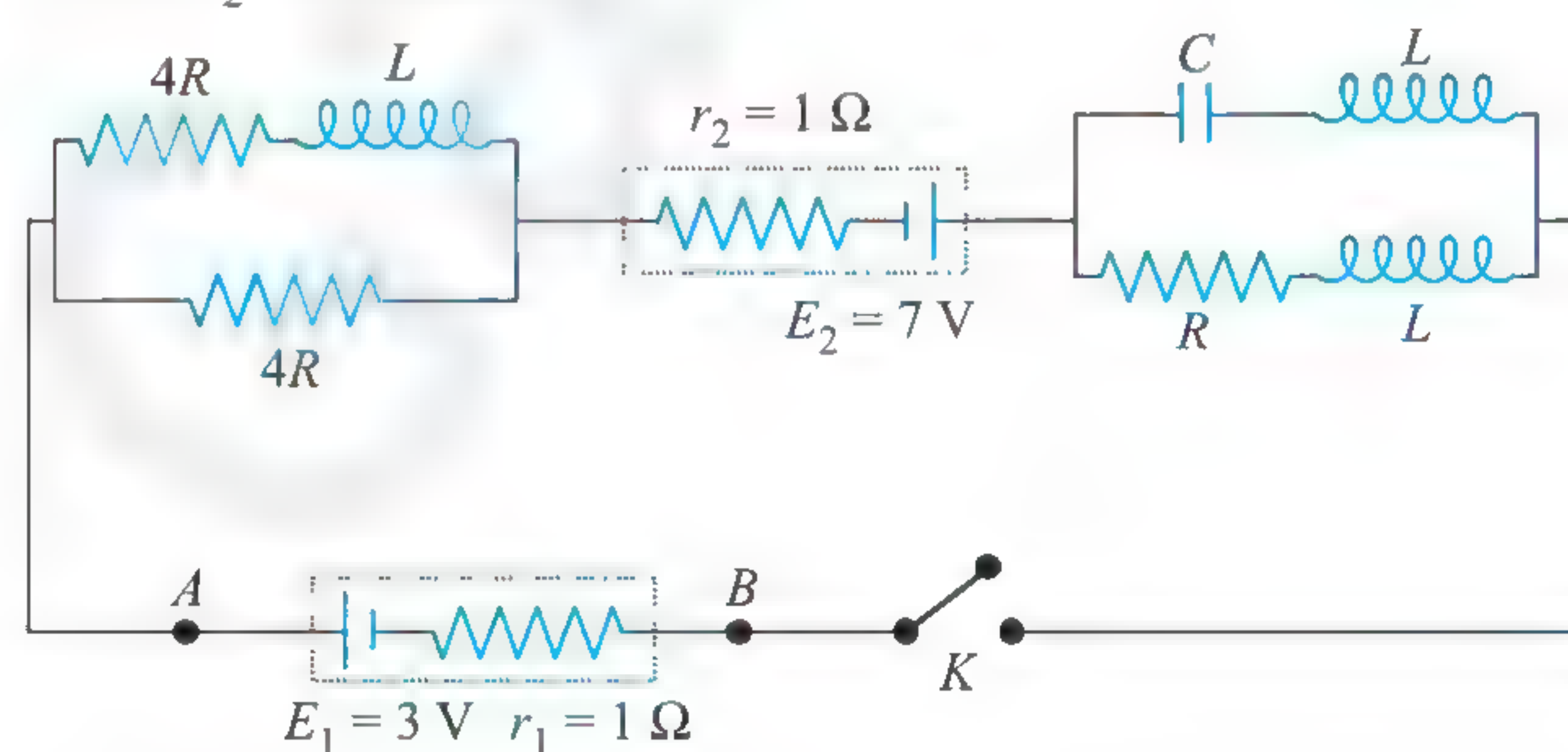
- (1) $\frac{\mu_0 i a^2 \pi}{2Rb}$ (2) $\frac{\mu_0 i ab}{2R}$
(3) $\frac{\mu_0 i ab}{2a} \frac{\pi b^2}{R}$ (4) $\frac{\mu_0 i b}{2\pi R}$
37. In the circuit shown in figure, switch S is closed at time $t = 0$. The charge that passes through the battery in one time constant is
- (1) $\frac{eR^2 E}{L}$ (2) $E \left(\frac{L}{R} \right)$
(3) $\frac{EL}{eR^2}$ (4) $\frac{eL}{ER}$
38. In the given circuit diagram (figure), key K is switched on at $t = 0$. The ratio of current i through the cell at $t = 0$ to that at $t = \infty$ will be



39. A closed loop of cross-sectional area 10^{-2} m^2 which has inductance $L = 10 \text{ mH}$ and negligible resistance is placed in a time-varying magnetic field. Figure shows the variation of B with time for the interval 4 s.



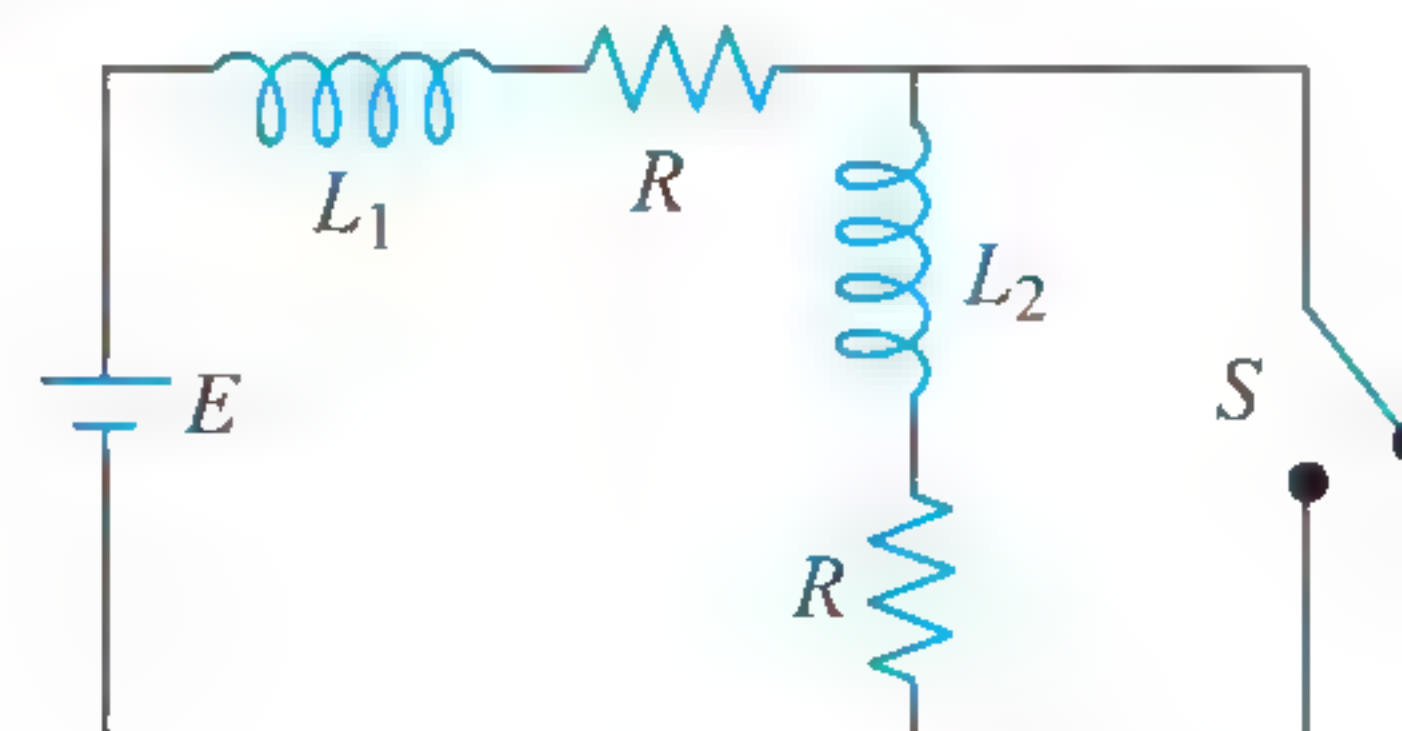
- The field is perpendicular to the plane of the loop (given at $t = 0, B = 0, I = 0$). The value of the maximum current induced in the loop is
- (1) 0.1 mA (2) 10 mA
(3) 100 mA (4) Data insufficient
40. In figure, key K is closed at $t = 0$. After a long time, the potential difference between A and B is zero, the value of R will be [$r_1 = r_2 = 1 \Omega$, $E_1 = 3 \text{ V}$ and $E_2 = 7 \text{ V}$, $C = 2 \mu\text{F}$, $L = 4 \text{ mH}$, where r_1 and r_2 are the internal resistances of cells E_1 and E_2 , respectively].



- (1) $\frac{4}{3} \Omega$ (2) $\frac{4}{9} \Omega$
(3) $\frac{2}{3} \Omega$ (4) 4Ω
41. The cell in the circuit shown in figure is ideal. The coil has an inductance of 4 mH and a resistance of 2 m Ω . The switch is closed at $t = 0$. The amount of energy stored in the inductor at $t = 2 \text{ s}$ is (take $e = 3$)



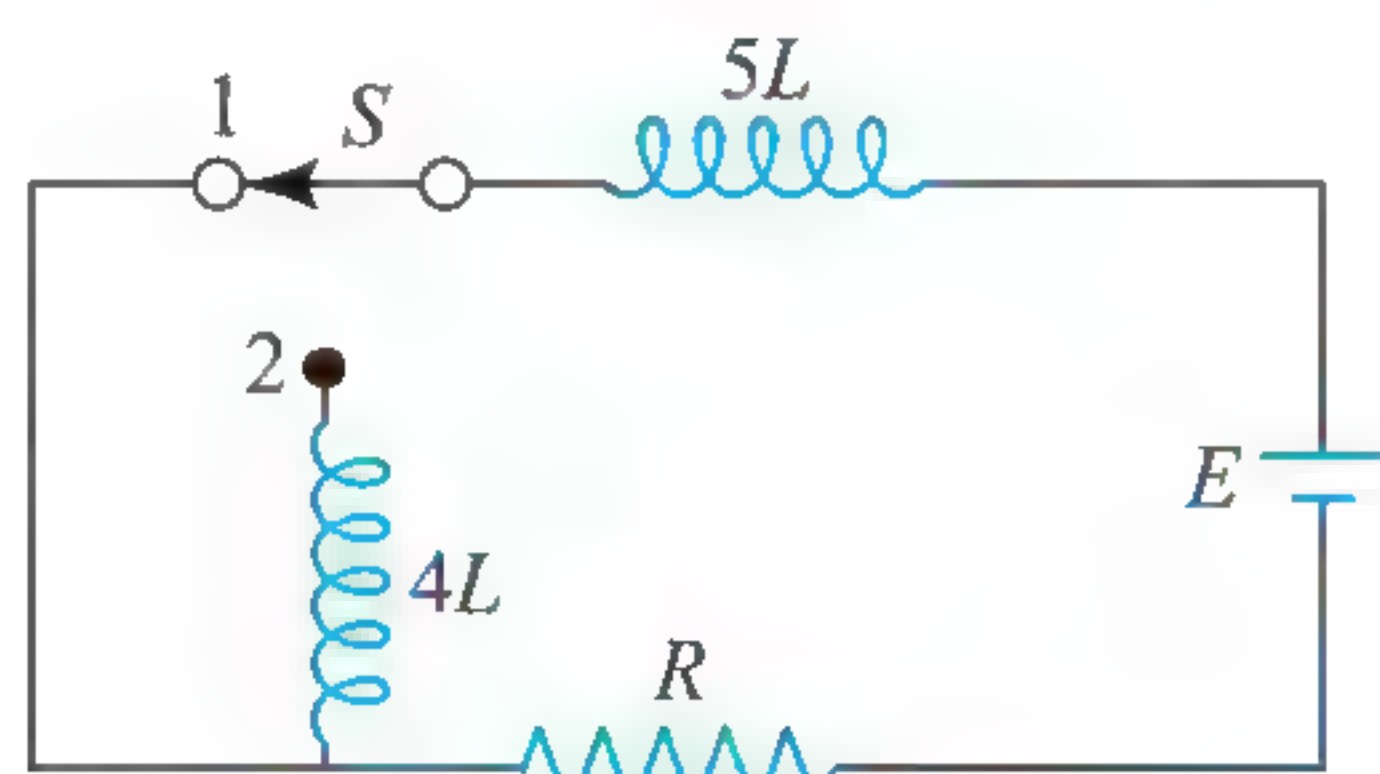
- (1) $\frac{4}{3} \text{ J}$ (2) $\frac{8}{9} \times 10^3 \text{ J}$
(3) $\frac{8}{3} \times 10^{-3} \text{ J}$ (4) $2 \times 10^3 \text{ J}$
42. Switch S shown in figure is closed for $t < 0$ and is opened at $t = 0$. When currents through L_1 and L_2 are equal, their common value is



- (1) $\frac{E}{R}$ (2) $\frac{E(L_2 + L_1)}{RL_1}$
(3) $\frac{EL_1}{R(L_1 + L_2)}$ (4) $\frac{E}{R} \frac{(L_1 + L_2)}{L_2}$

- (1) 3 : 1 (2) 1 : 3
(3) 1 : 2 (4) 2 : 1

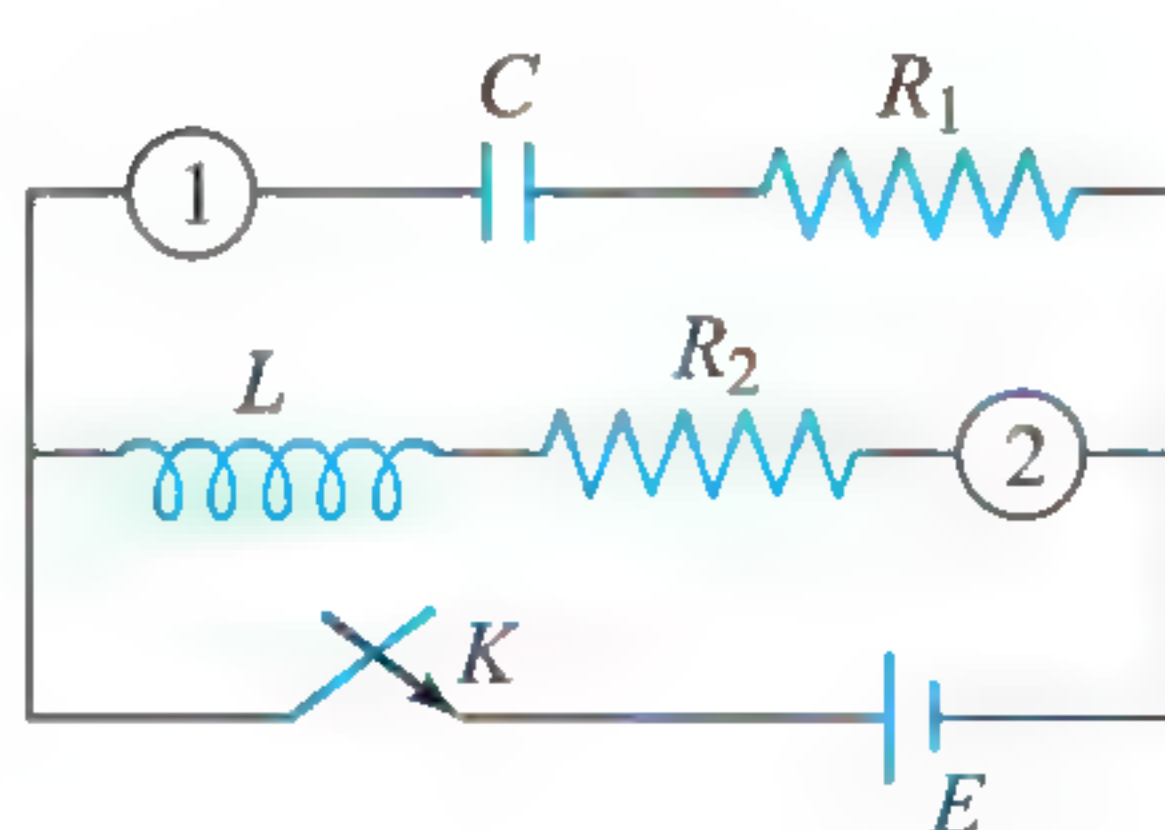
43. In the circuit shown in figure, switch S is shifted to position 2 from position 1 at $t = 0$, having been in position 1 for a long time. The current in the circuit just after shifting of switch will be (battery and both the inductors are ideal)



- (1) $\frac{4}{5} \frac{E}{R}$ (2) $\frac{5}{4} \frac{E}{R}$
 (3) $\frac{5}{9} \frac{E}{R}$ (4) $\frac{E}{R}$

44. In the circuit of figure, (1) and (2) are ammeters. Just after key K is pressed to complete the circuit, the reading is

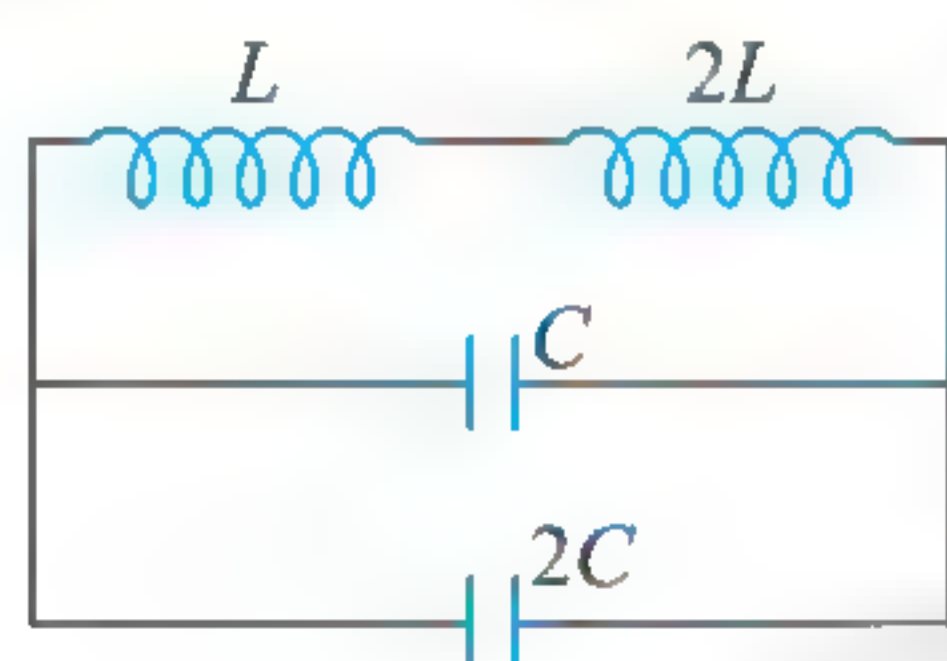
- (1) maximum in both 1 and 2
 (2) zero in both 1 and 2
 (3) zero in 1, minimum in 2
 (4) maximum in 1, zero in 2



45. A wire of fixed length is wound on a solenoid of length l and radius r . Its self-inductance is found to be L . Now, if the same wire is wound on a solenoid of length $l/2$ and radius $r/2$, then the self-inductance will be

- (1) $2L$ (2) L
 (3) $4L$ (4) $8L$

46. The frequency of oscillation of current in the inductance is



- (1) $\frac{1}{3\sqrt{LC}}$ (2) $\frac{1}{6\pi\sqrt{LC}}$
 (3) $\frac{1}{\sqrt{LC}}$ (4) $\frac{1}{2\pi\sqrt{LC}}$

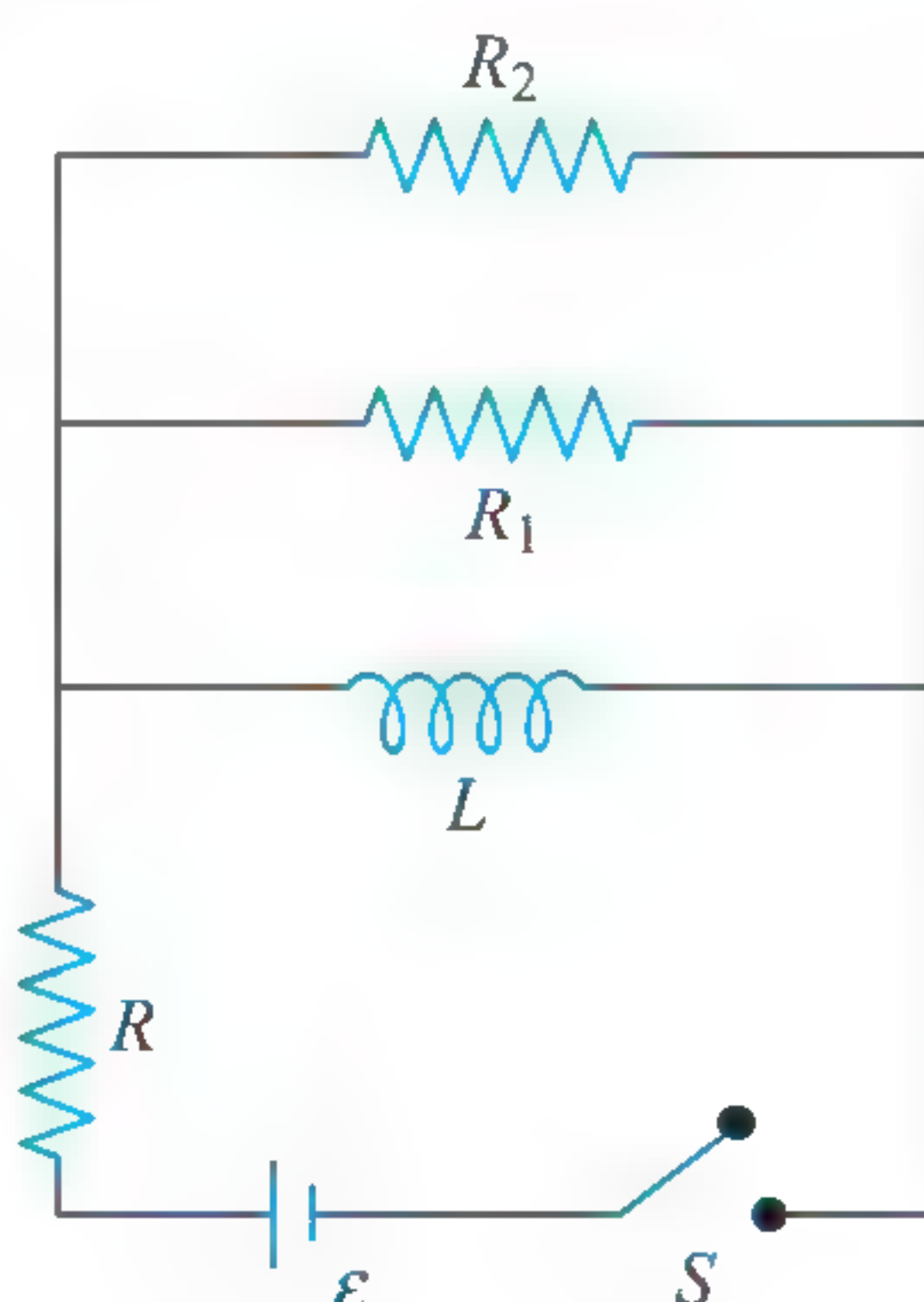
47. In figure switch S is closed for a long time. At $t = 0$, if it is opened, then

- (1) total heat produced in resistor R after opening the switch is

$$\frac{1}{2} \frac{L\epsilon}{R^2}$$

- (2) total heat produced in resistor R_1 after opening the switch is

$$\frac{1}{2} \frac{L\epsilon^2}{R^2} \left(\frac{R_1}{R_1 + R_2} \right)$$

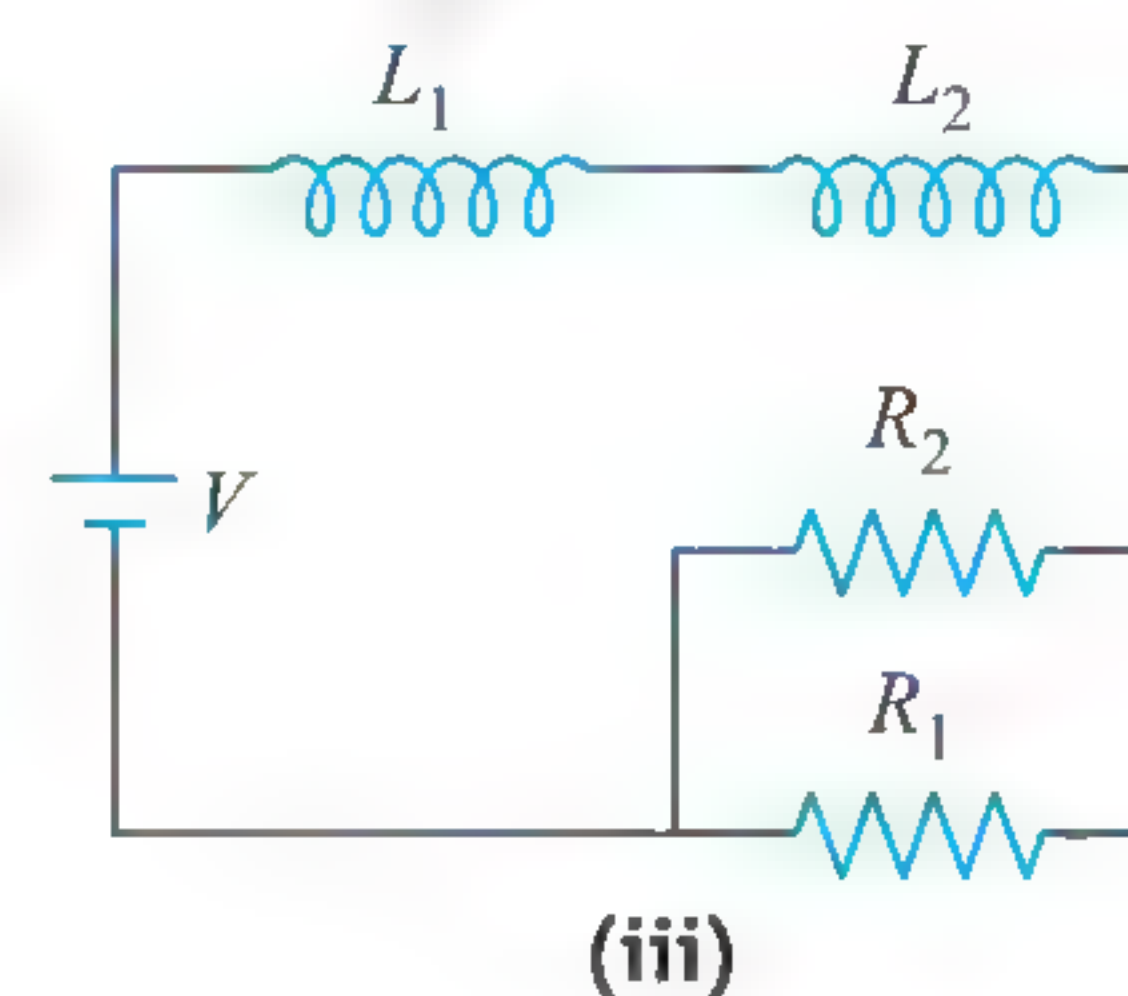
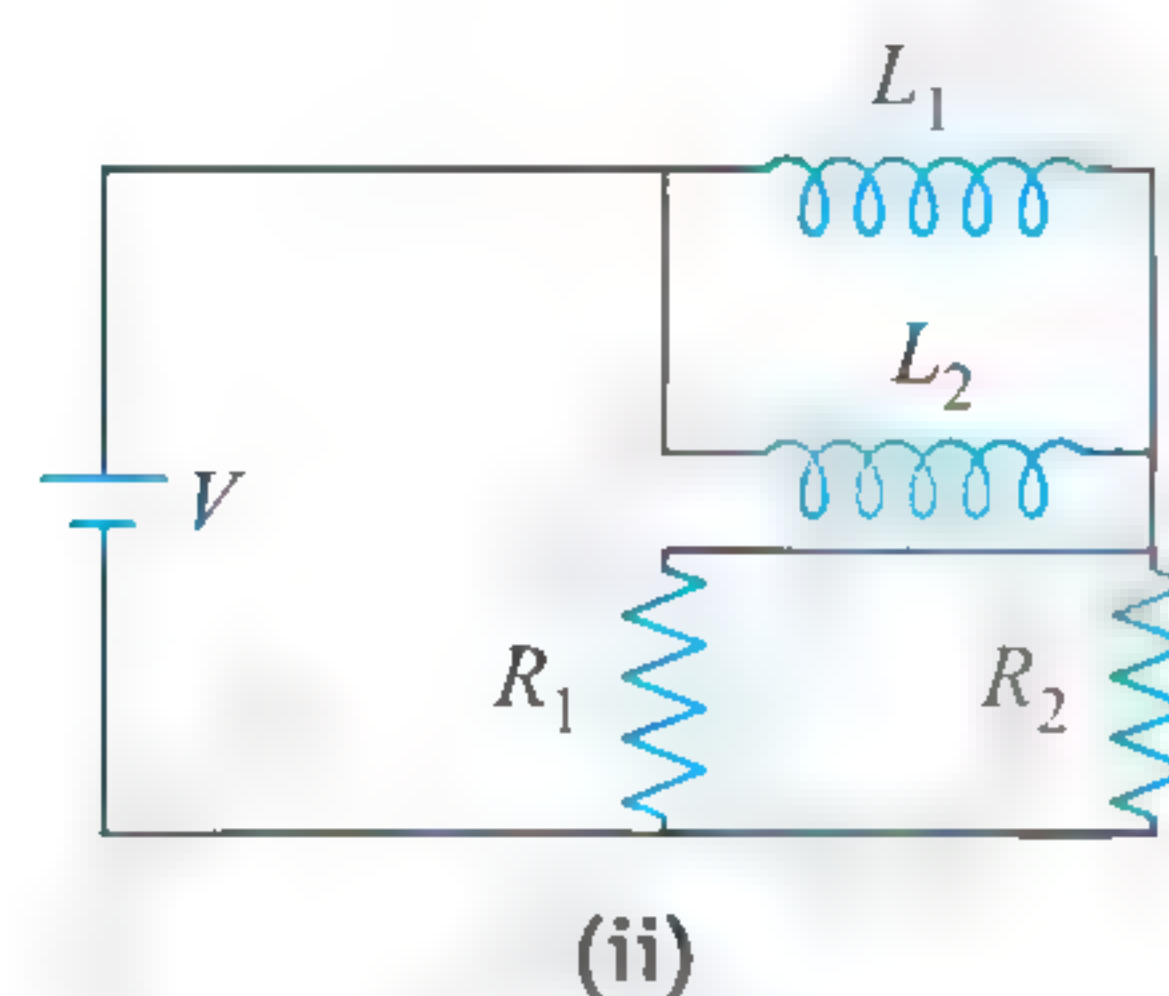
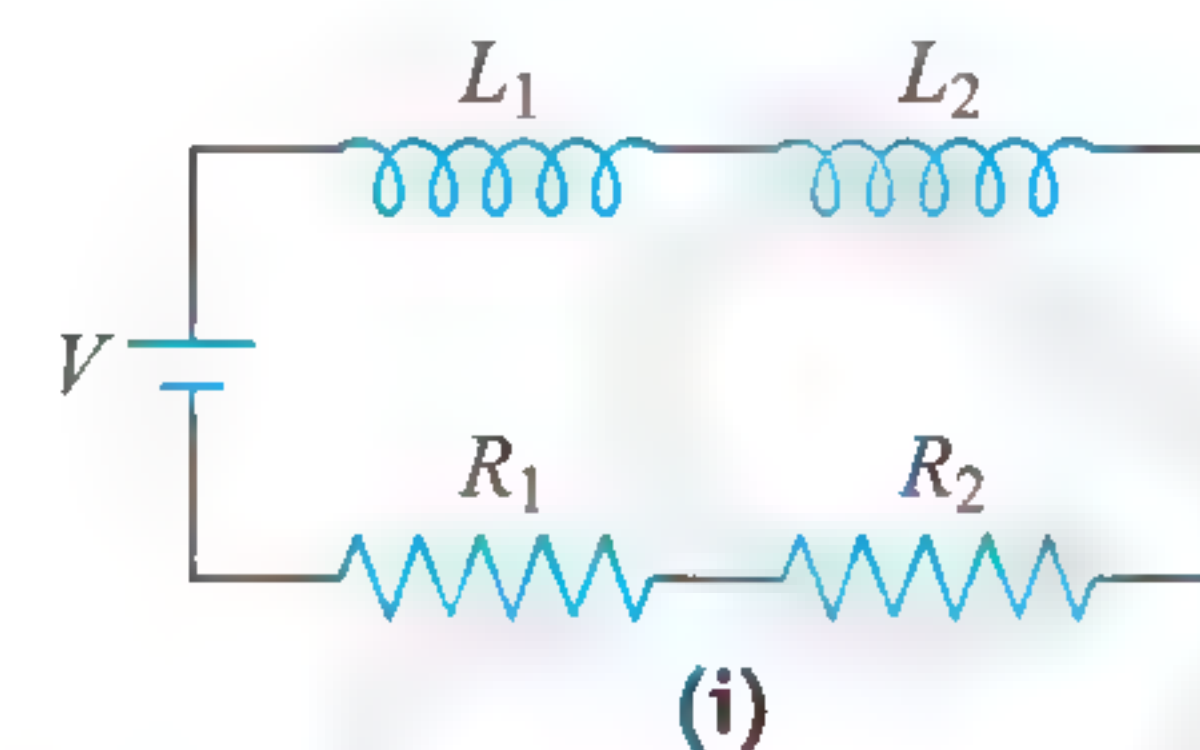


- (3) heat produced in resistor R_1 after opening the switch is

$$\frac{1}{2} \frac{R_2 L \epsilon^2}{(R_1 + R_2) R^2}$$

- (4) no heat will be produced in R_1

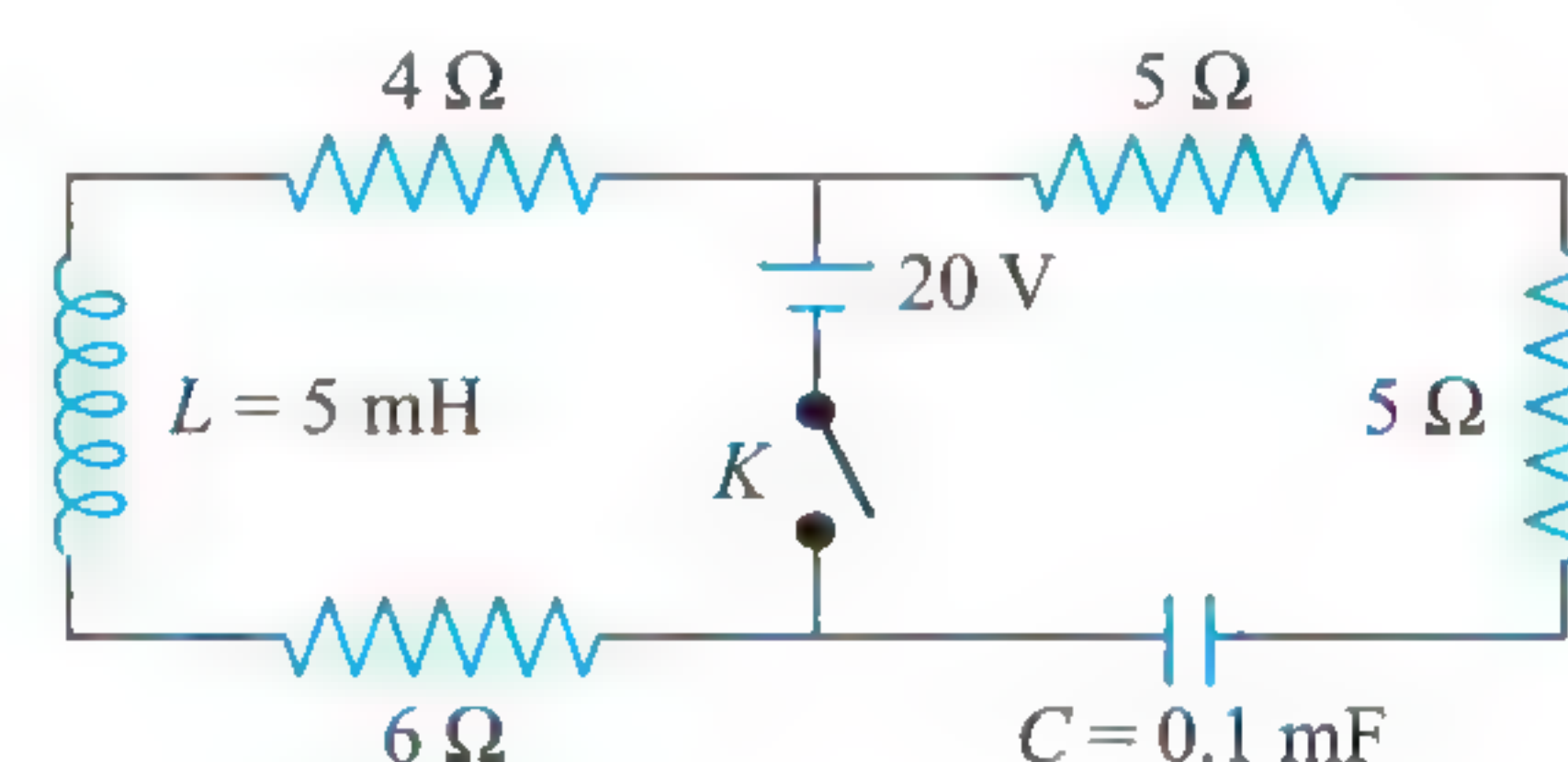
48. Given $L_1 = 1 \text{ mH}$, $R_1 = 1 \Omega$, $L_2 = 2 \text{ mH}$, $R_2 = 2 \Omega$



- Neglecting mutual inductance, the time constants (in ms) for circuits (i), (ii), and (iii) are

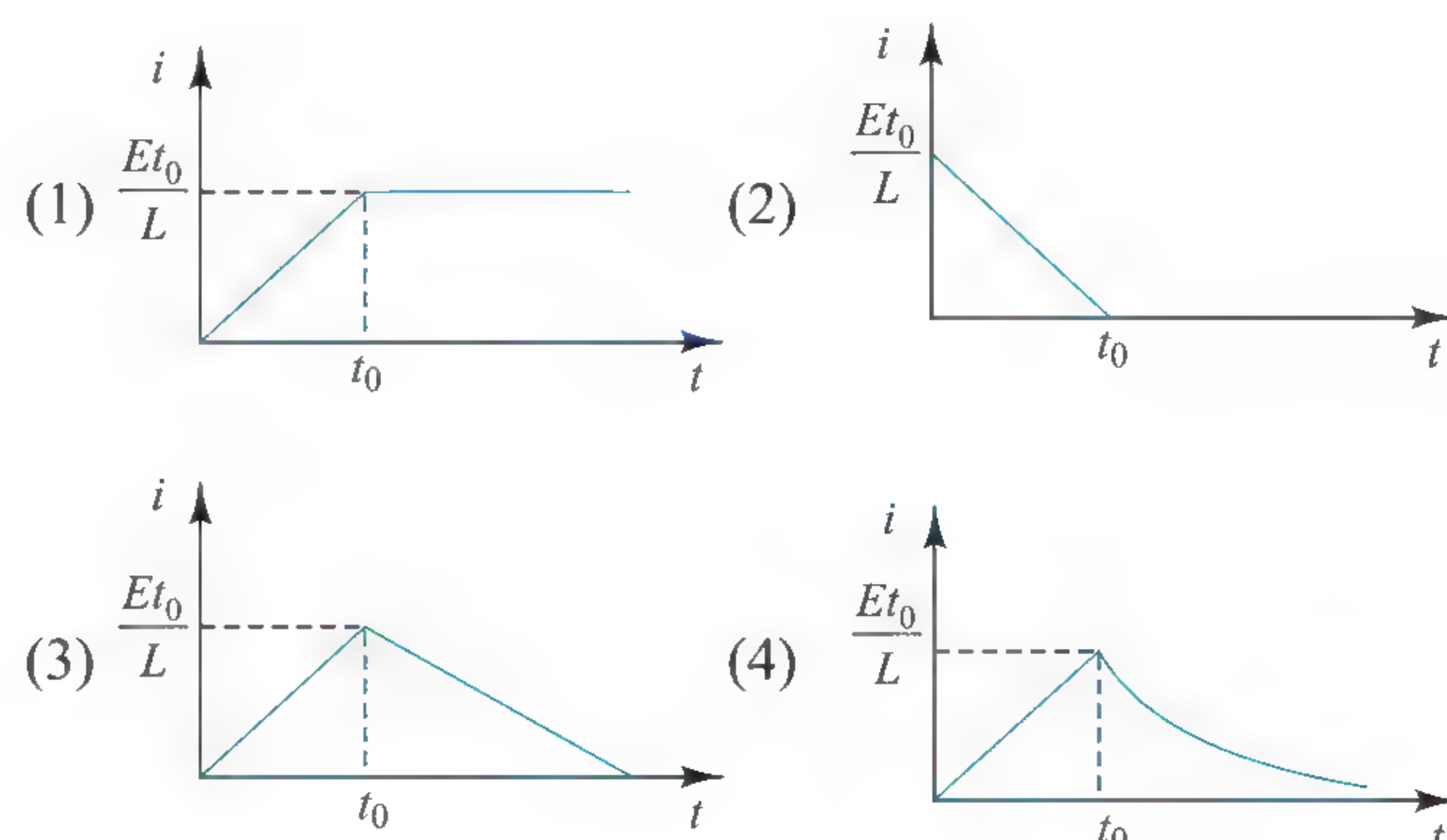
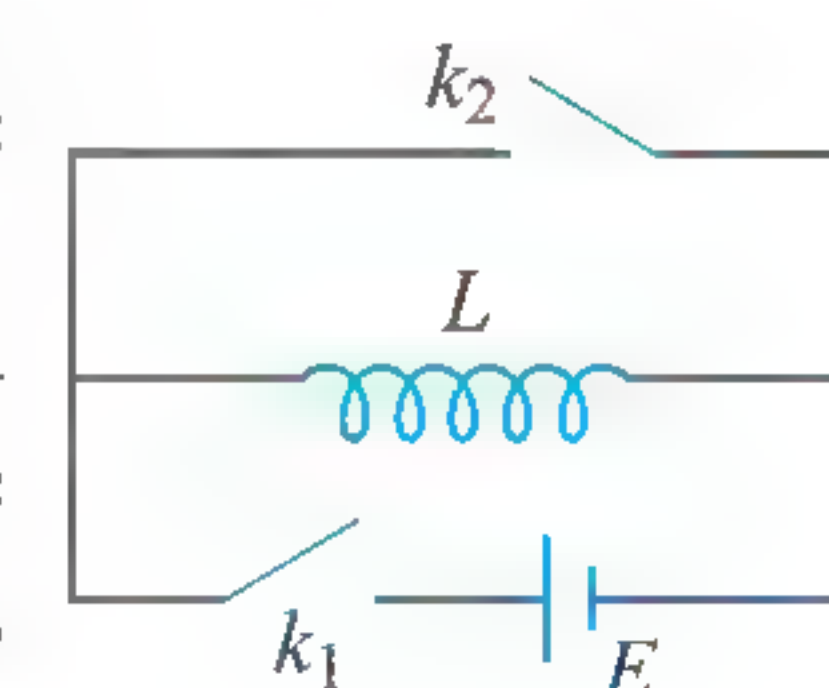
- (1) $1, 1, \frac{9}{2}$ (2) $\frac{9}{4}, 1, 1$
 (3) $1, 1, 1$ (4) $1, \frac{9}{4}, 1$

49. In the circuit shown in figure, key K is closed at $t = 0$, the current through the key at the instant $t = 10^{-3} \ln 2 \text{ s}$ is

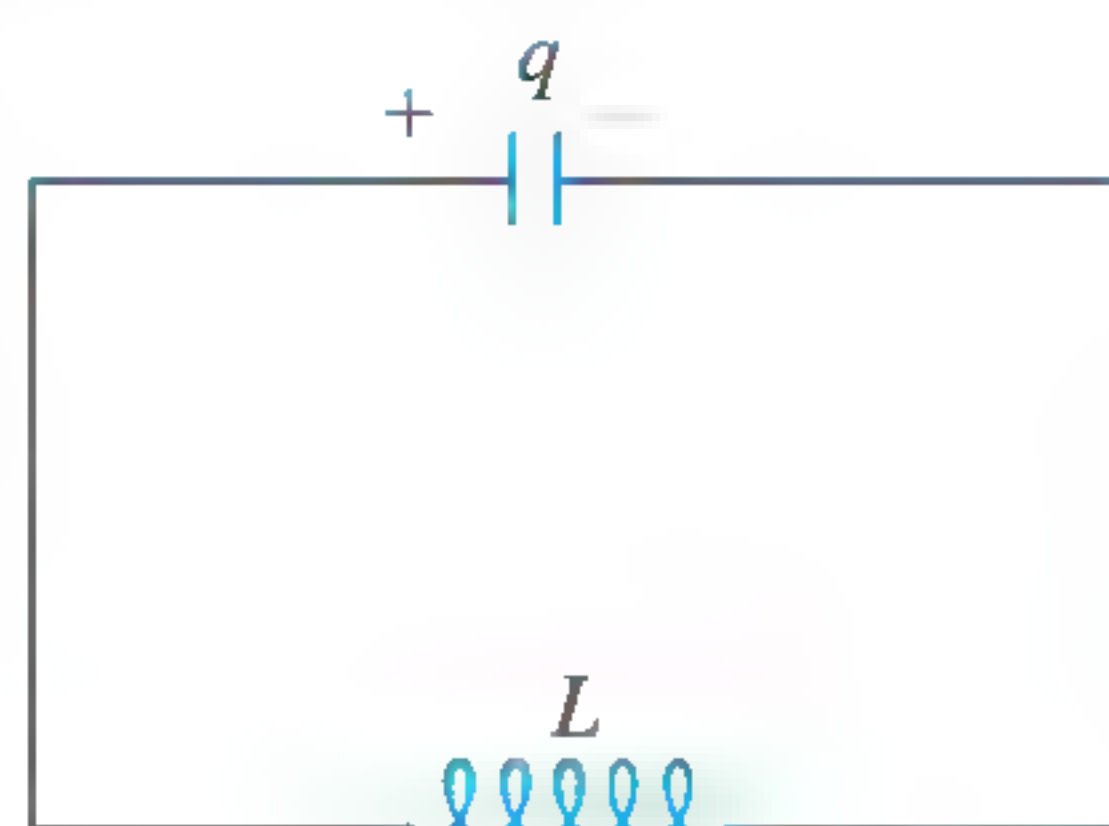


- (1) 2 A (2) 3.5 A
 (3) 2.5 A (4) 0

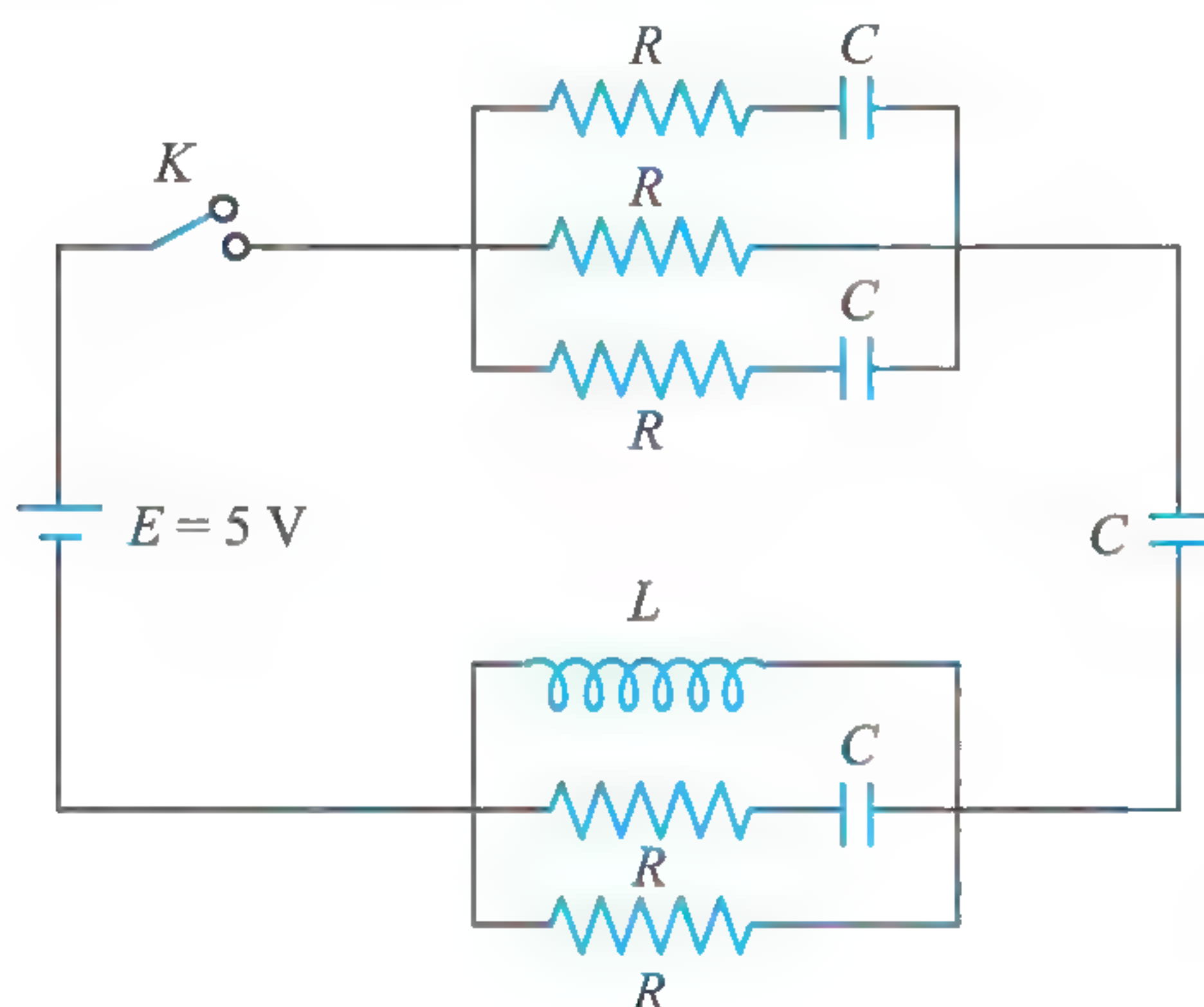
50. In the circuit shown in figure, switch k_2 is open and switch k_1 is closed at $t = 0$. At time $t = t_0$, switch k_1 is opened and switch k_2 is simultaneously closed. The variation of inductor current with time is



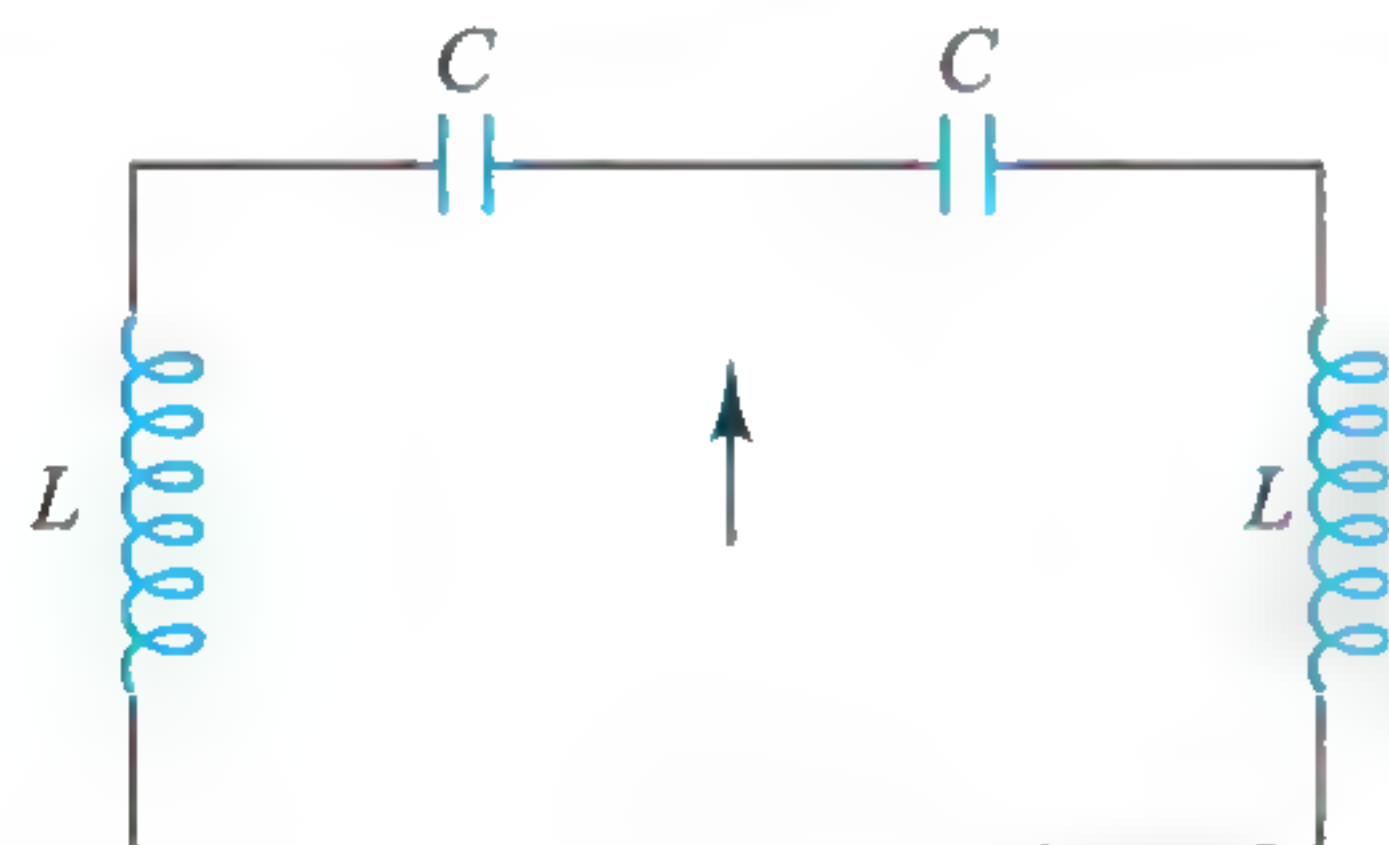
51. In an LC circuit shown in figure, $C = 1 \text{ F}$, $L = 4 \text{ H}$. At time $t = 0$, charge in the capacitor is 4 C and it is decreasing at the rate of $\sqrt{5} \text{ C s}^{-1}$. Choose the correct statement.



- (1) Maximum charge in the capacitor can be 6 C
 (2) Maximum charge in the capacitor can be 8 C
 (3) Charge in the capacitor will be maximum after time $3 \sin^{-1}(2/3) \text{ s}$
 (4) None of these
52. The current passing through the battery immediately after key K is closed [it is given that initially all the capacitors are uncharged (given that $R = 6 \Omega$ and $C = 4 \mu\text{F}$)] is

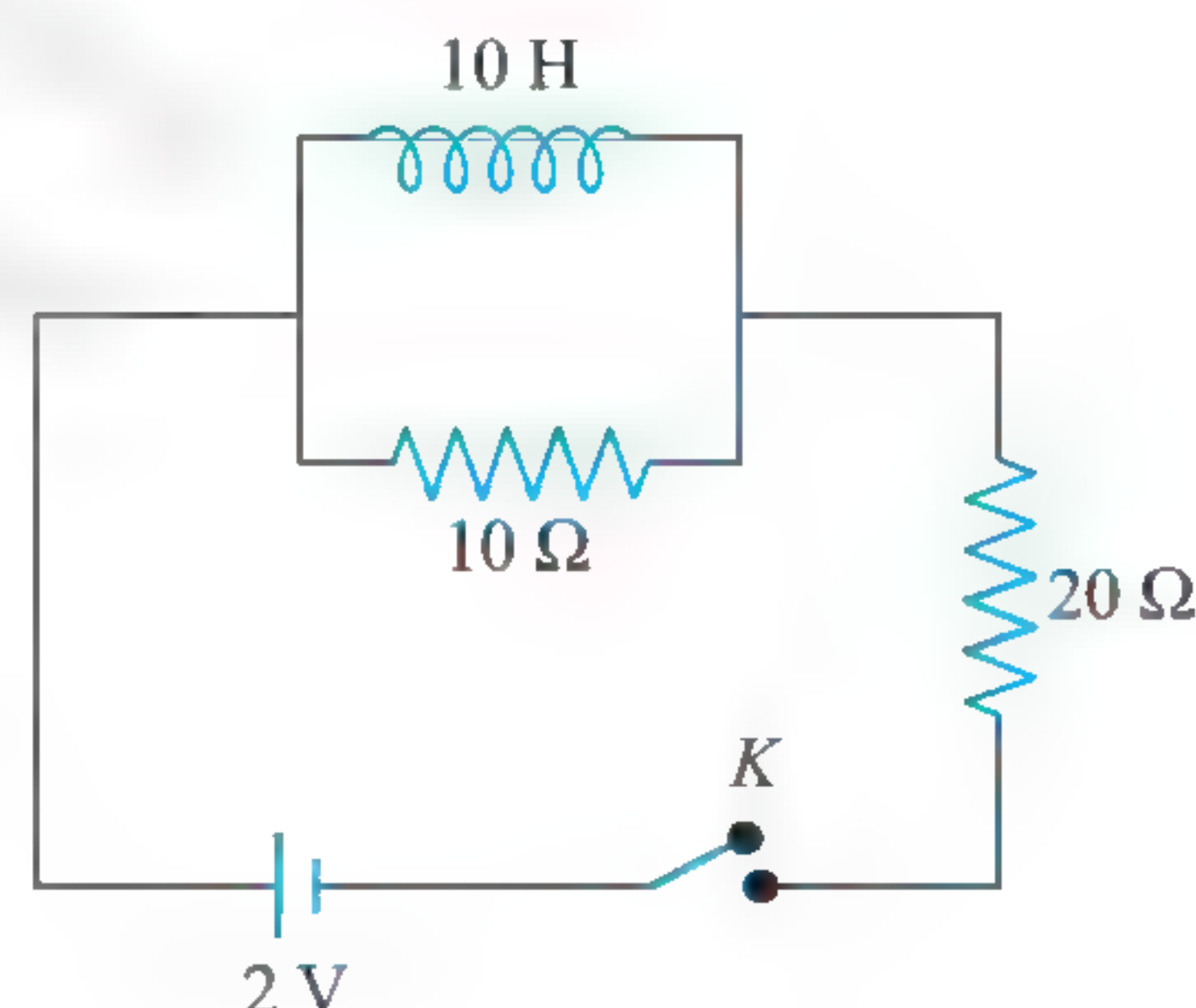


- (1) 1 A (2) 5 A
 (3) 3 A (4) 2 A
53. The natural frequency of the circuit shown in figure is



- (1) $\frac{1}{\sqrt{LC}}$ (2) $\frac{1}{\sqrt{2LC}}$
 (3) $\frac{2}{\sqrt{LC}}$ (4) none of these

54. Two resistors of 10Ω and 20Ω and an ideal inductor of 10 H are connected to a 2 V battery as shown in figure. Key K is inserted at time $t = 0$. The initial ($t = 0$) and final ($t \rightarrow \infty$) currents through the battery are



- (1) $\frac{1}{15} \text{ A}, \frac{1}{10} \text{ A}$ (2) $\frac{1}{10} \text{ A}, \frac{1}{15} \text{ A}$
 (3) $\frac{2}{15} \text{ A}, \frac{1}{10} \text{ A}$ (4) $\frac{1}{15} \text{ A}, \frac{2}{25} \text{ A}$

55. A simple LR circuit is connected to a battery at time $t = 0$. The energy stored in the inductor reaches half its maximum value at time

- (1) $\frac{R}{L} \ln \left[\frac{\sqrt{2}}{\sqrt{2}-1} \right]$ (2) $\frac{L}{R} \ln \left[\frac{\sqrt{2}-1}{\sqrt{2}} \right]$
 (3) $\frac{L}{R} \ln \left[\frac{\sqrt{2}}{\sqrt{2}-1} \right]$ (4) $\frac{R}{L} \ln \left[\frac{\sqrt{2}-1}{\sqrt{2}} \right]$

56. A square conducting loop of side L is situated in gravity-free space. A small conducting circular loop of radius r ($r \ll L$) is placed at the center of the square loop, with its plane perpendicular to the plane of the square loop. The mutual inductance of the two coils is

- (1) $\frac{2\sqrt{2}\mu_0 I}{L} r^2$ (2) $\frac{\sqrt{2}\mu_0 I_0}{L} r^2$
 (3) 0 (4) none of these

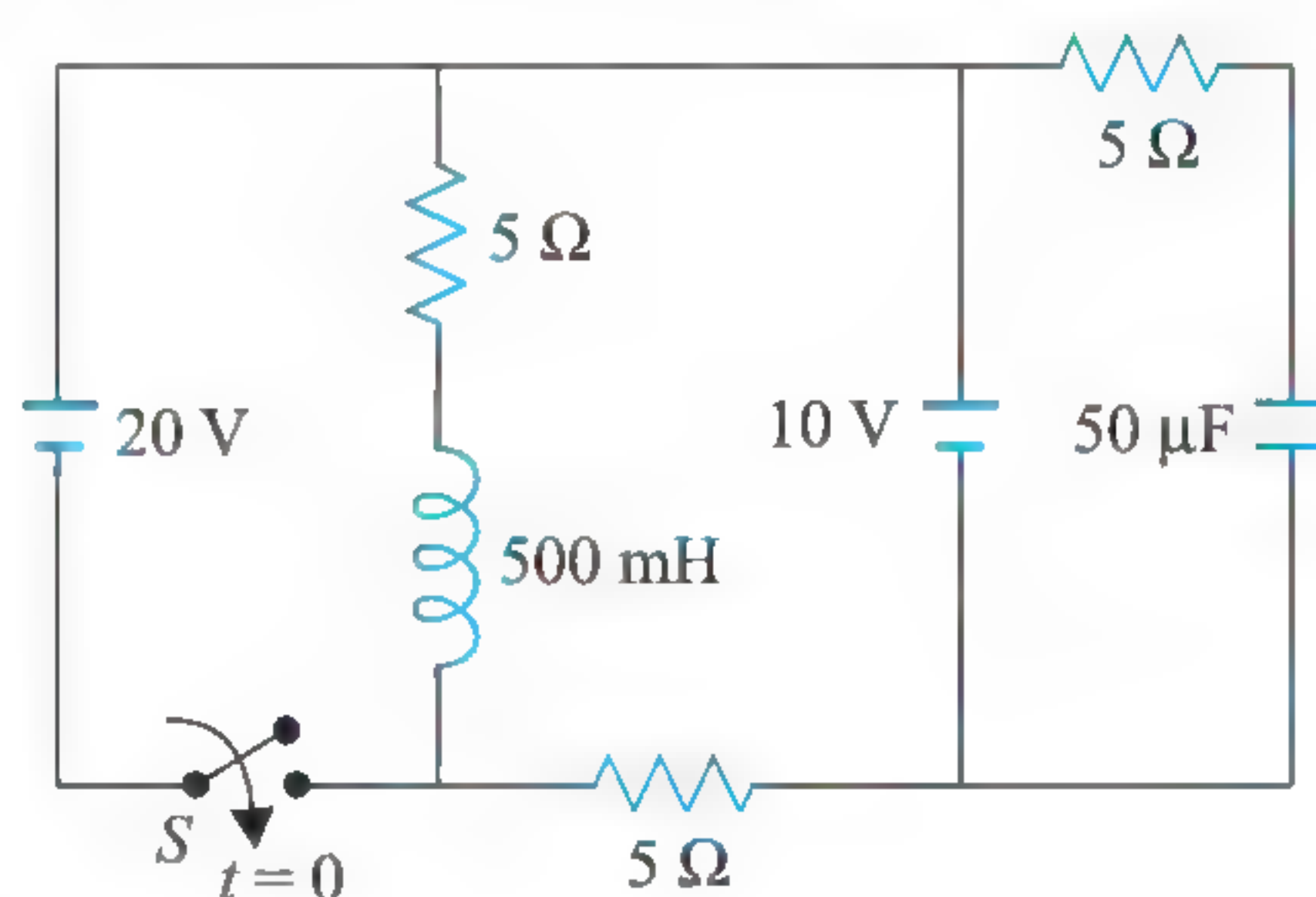
57. There is a conducting ring of radius R . Another ring having current i and radius r ($r \ll R$) is kept on the axis of bigger ring such that its center lies on the axis of bigger ring at a distance x from the center of bigger ring and its plane is perpendicular to that axis. The mutual inductance of the bigger ring due to the smaller ring is

- (1) $\frac{\mu_0 \pi R^2 r^2}{(R^2 + x^2)^{3/2}}$ (2) $\frac{\mu_0 \pi R^2 r^2}{4(R^2 + x^2)^{3/2}}$
 (3) $\frac{\mu_0 \pi R^2 r^2}{16(R^2 + x^2)^{3/2}}$ (4) $\frac{\mu_0 \pi R^2 r^2}{2(R^2 + x^2)^{3/2}}$

58. Two coils of self inductance 100 mH and 400 mH are placed very close to each other. Find the maximum mutual inductance between the two when 4 A current passes through them.

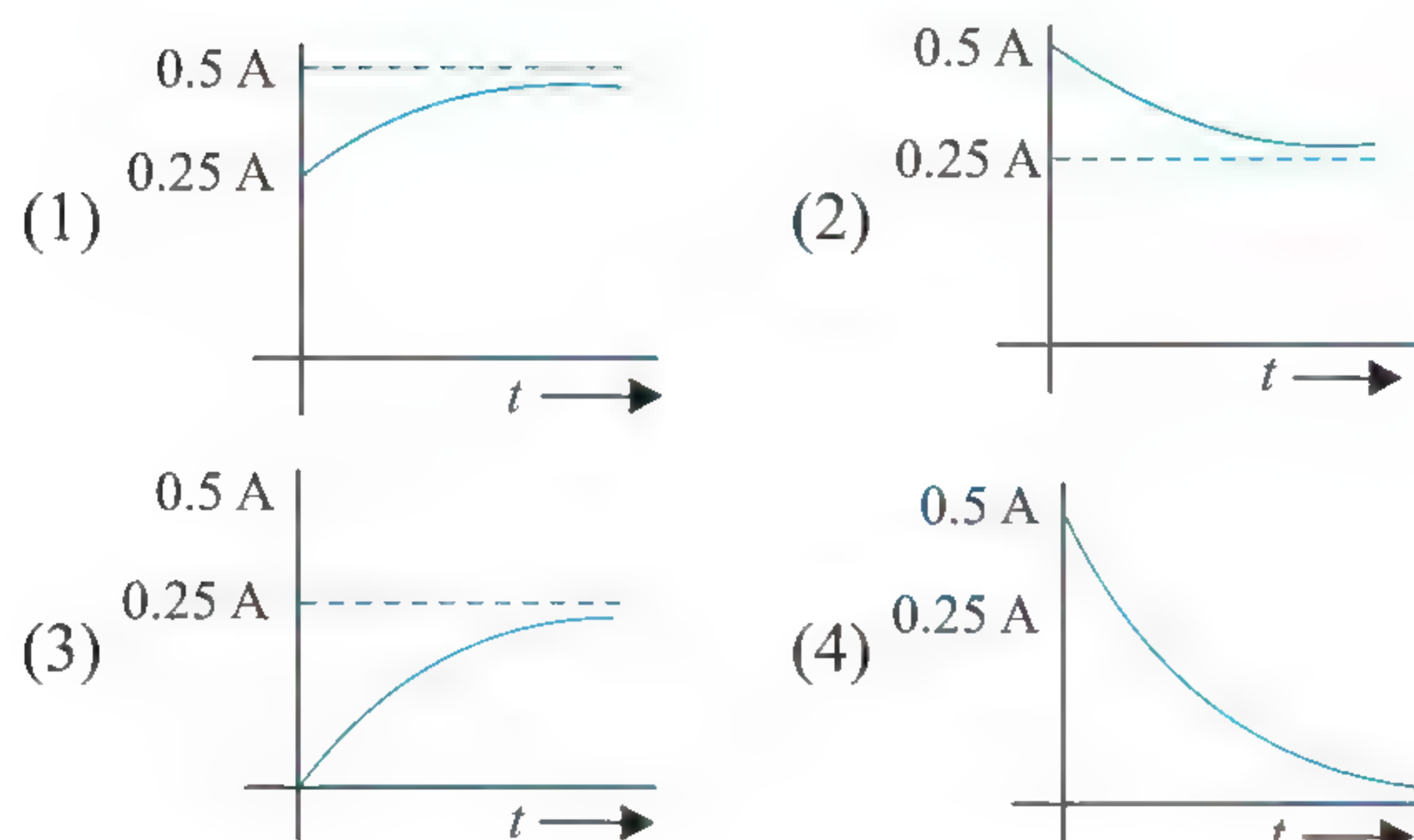
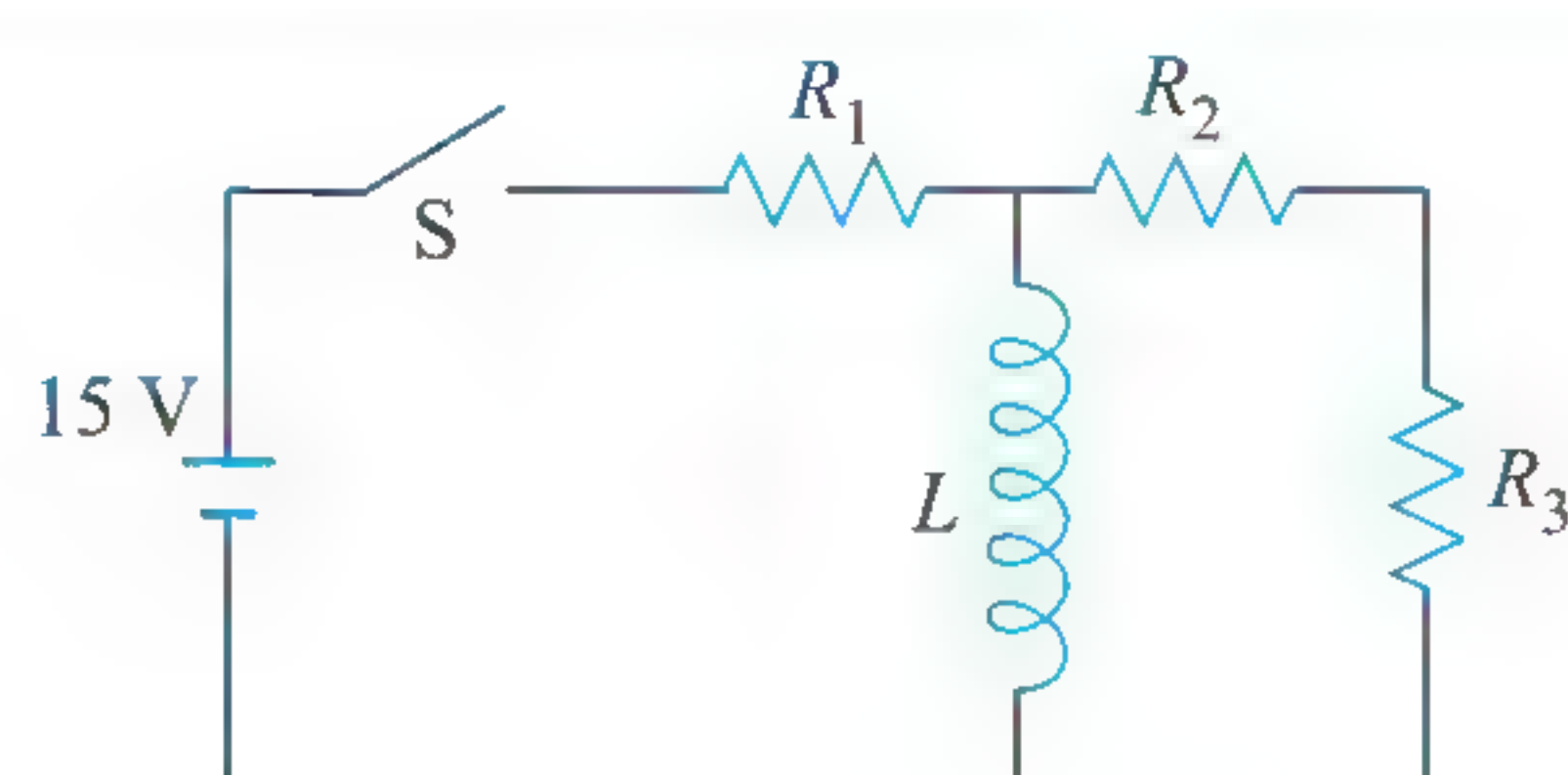
- (1) 200 mH (2) 300 mH
 (3) $100\sqrt{2} \text{ mH}$ (4) none of these

59. Switch S is closed at $t = 0$, in the circuit shown in figure. The change in flux in the inductor ($L = 500 \text{ mH}$) from $t = 0$ to an instant when it reaches steady state is

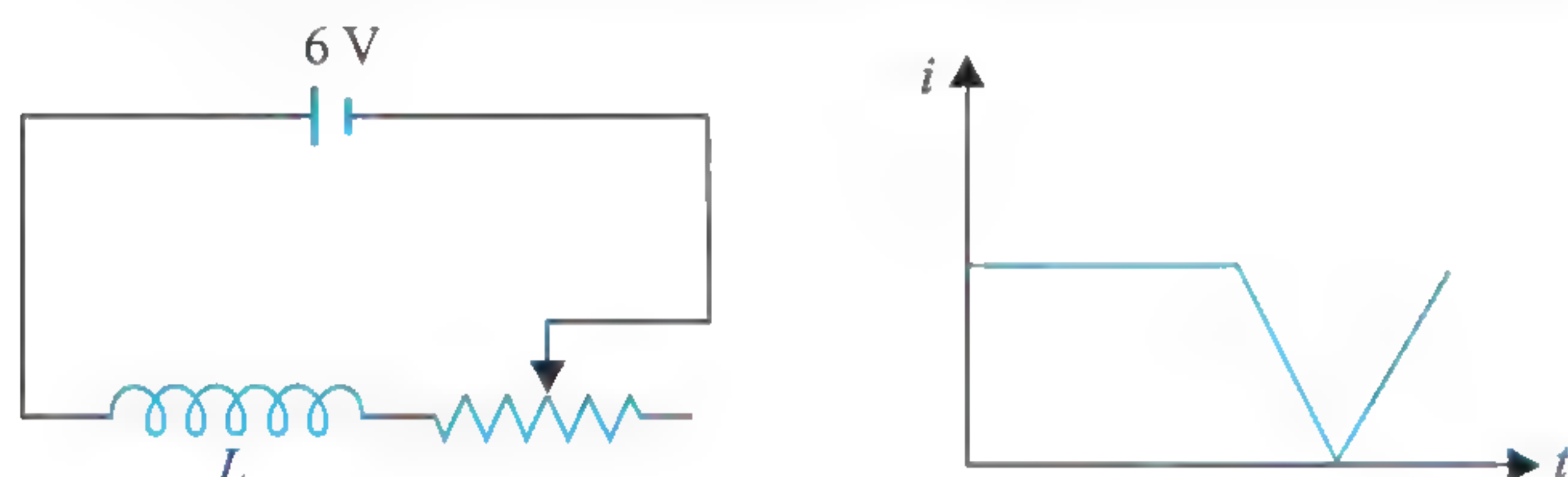


- (1) 2 Wb (2) 1.5 Wb
 (3) 0 Wb (4) None

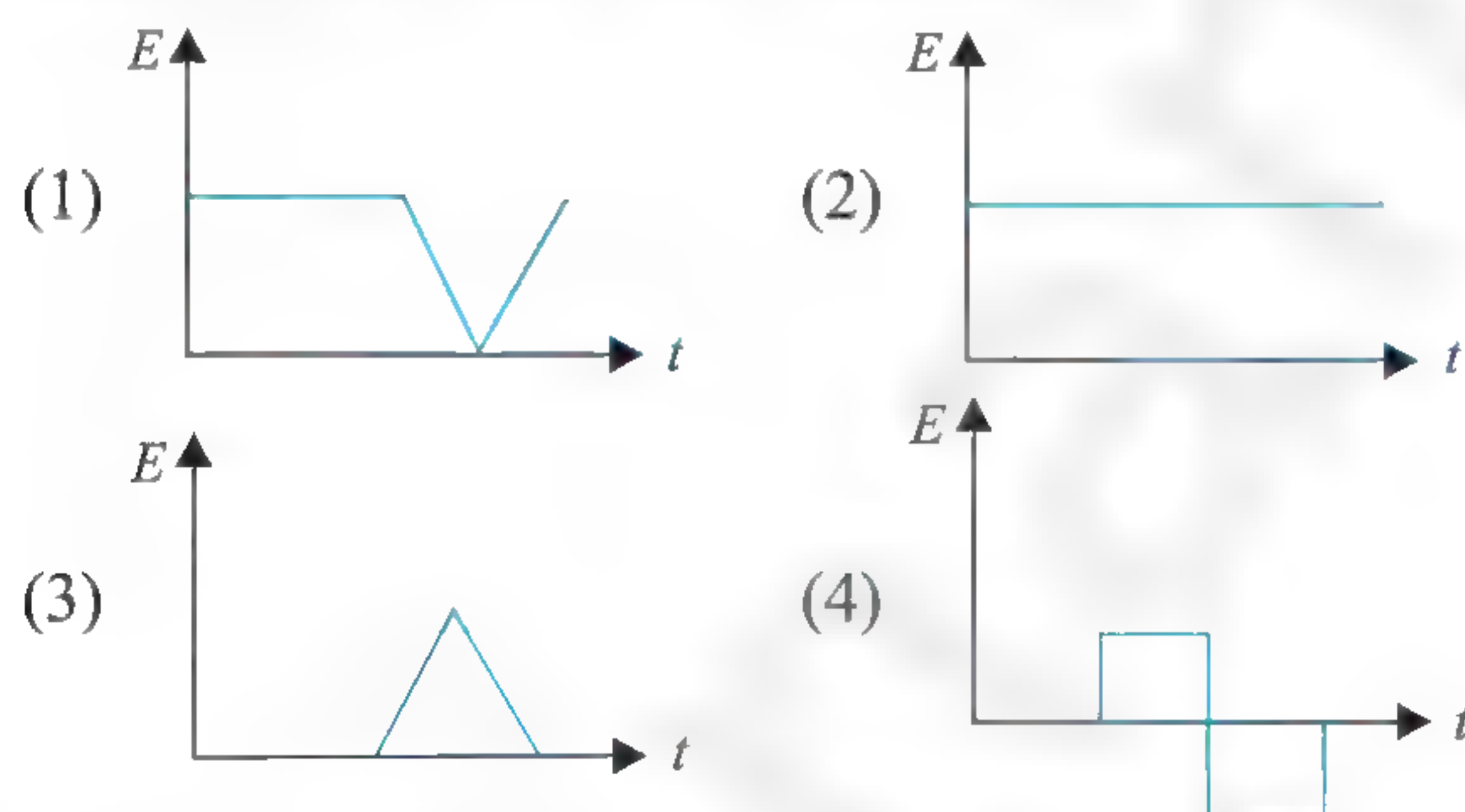
60. The figure below shows a battery with emf 15 V in a circuit with $R_1 = 30 \Omega$, $R_2 = 10 \Omega$, $R_3 = 20 \Omega$ and $L = 3.0 \text{ H}$. The switch S is initially in the open position and is then closed at time $t = 0$. Then the graph which shows the correct variation of current through battery after switch S is closed



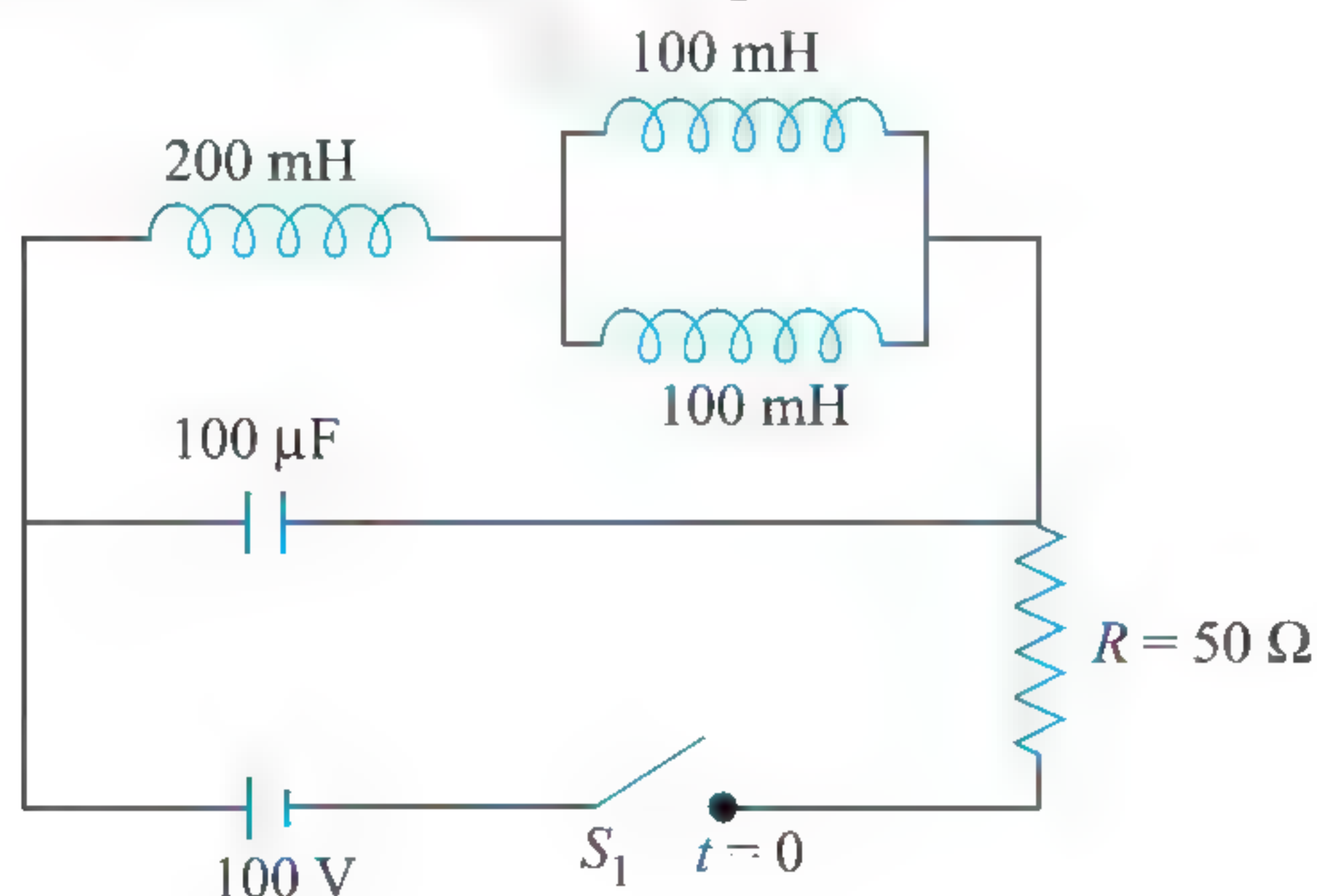
61. In the circuit shown in figure, sliding contact is moving with uniform velocity towards right. Its value at some instance is $12\ \Omega$. The current in the circuit at this instant of time will be



- (1) 0.5 A
(2) More than 0.5 A
(3) Less than 0.5 A
(4) May be less or more than 0.5 A depending on the value of L .
62. The current i in an induction coil varies with time according to the graph shown in figure. Which of the following graphs shows induced emf in the coil with time:

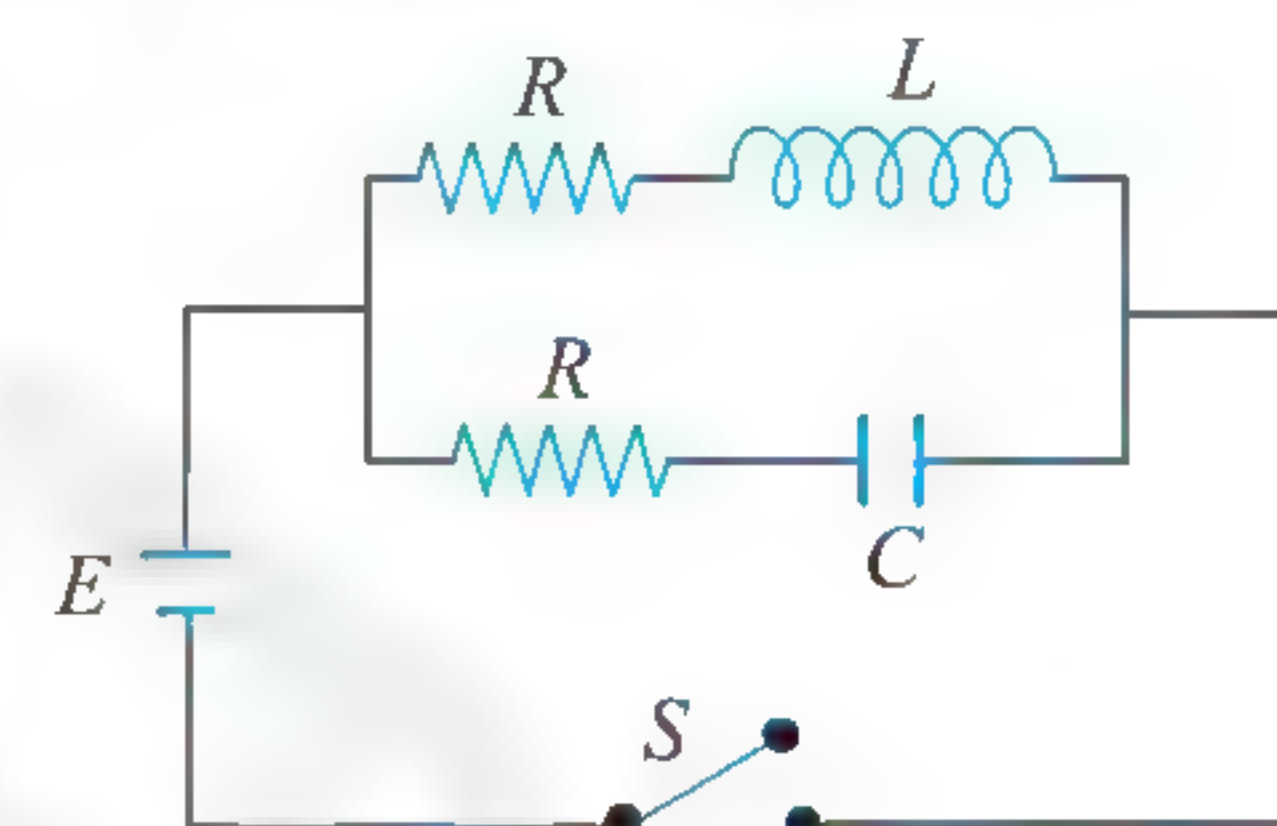


63. In the circuit shown in figure, switch S was closed for a long time. At time $t = 0$, the switch is opened again. The maximum potential difference across the plates of the capacitor after the switch is opened is



- (1) 10 V
(2) 100 V
(3) 20 V
(4) 200 V

64. In the circuit shown in figure, the time constants of the two branches are equal ($= T$). Then if the key S is closed at the instant $t = 0$, the time in which the current in the circuit through the battery will rise to its final value of (E/R) will be, (assume that the internal resistance of the battery and of the connecting wires are negligible)



- (1) Instantly
(2) $t = \frac{T}{2}$
(3) $t = 2T$
(4) $t = \text{infinity}$

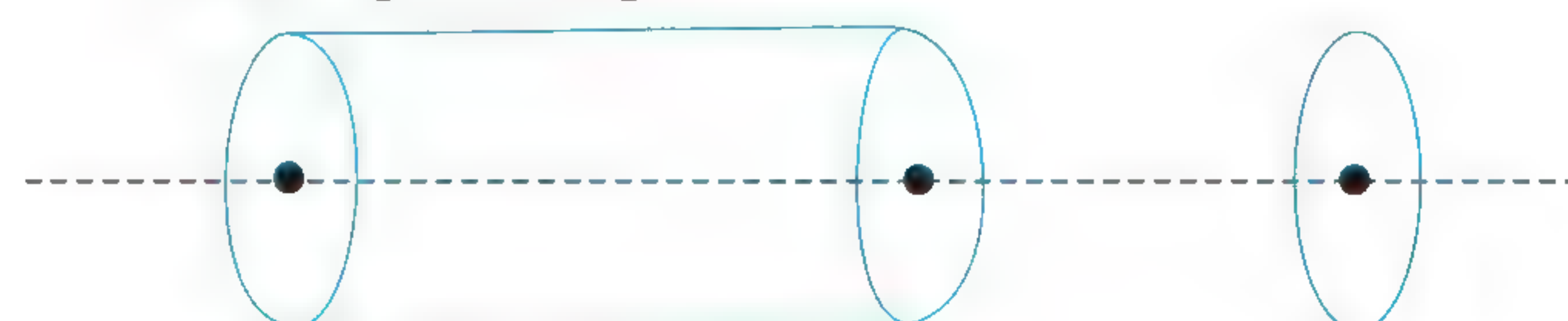
65. A bulb of 100 W is connected in parallel to an ideal inductance of 1 H. This arrangement is connected to a 90 V battery through a switch. On pressing the switch the

- (1) bulb does not glow
(2) bulb glows
(3) bulb glows after a short time and then continues to glow
(4) bulb glows for a short time and then stops glowing.

66. A uniformly wound solenoid coil of self inductance 4 mH and resistance $12\ \Omega$ is broken into two parts. These two coils are connected in parallel across a 12 V battery. The steady current through battery is

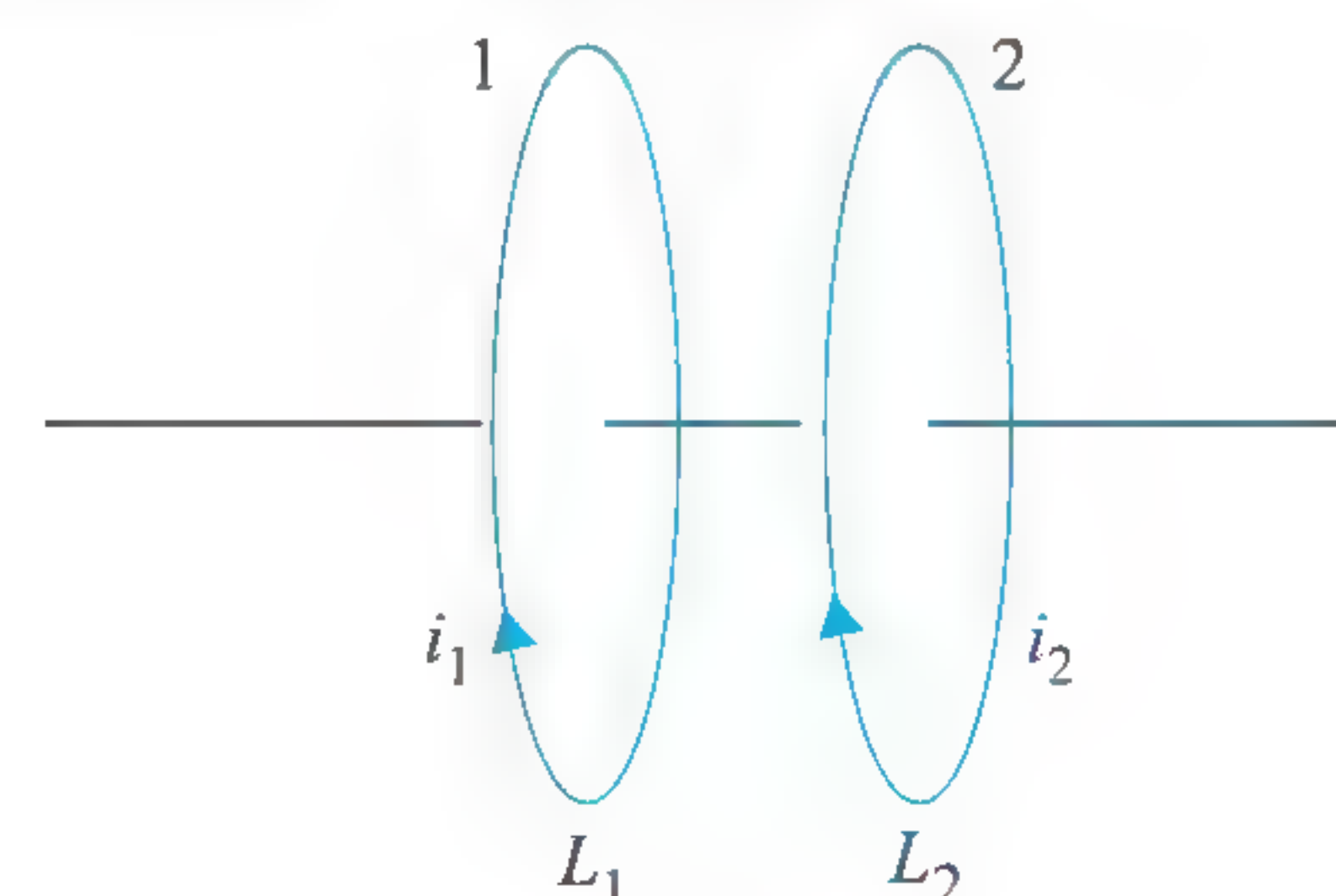
- (1) 1 A
(2) 2 A
(3) 4 A
(4) 8 A

67. The diagram given below shows a solenoid carrying time varying current $i = i_0 t$. On the axis of the solenoid, a ring has been placed. The mutual inductance of the ring and the solenoid is M and the self inductance of the ring is L . If the resistance of the ring is R then maximum current which can flow through the ring is



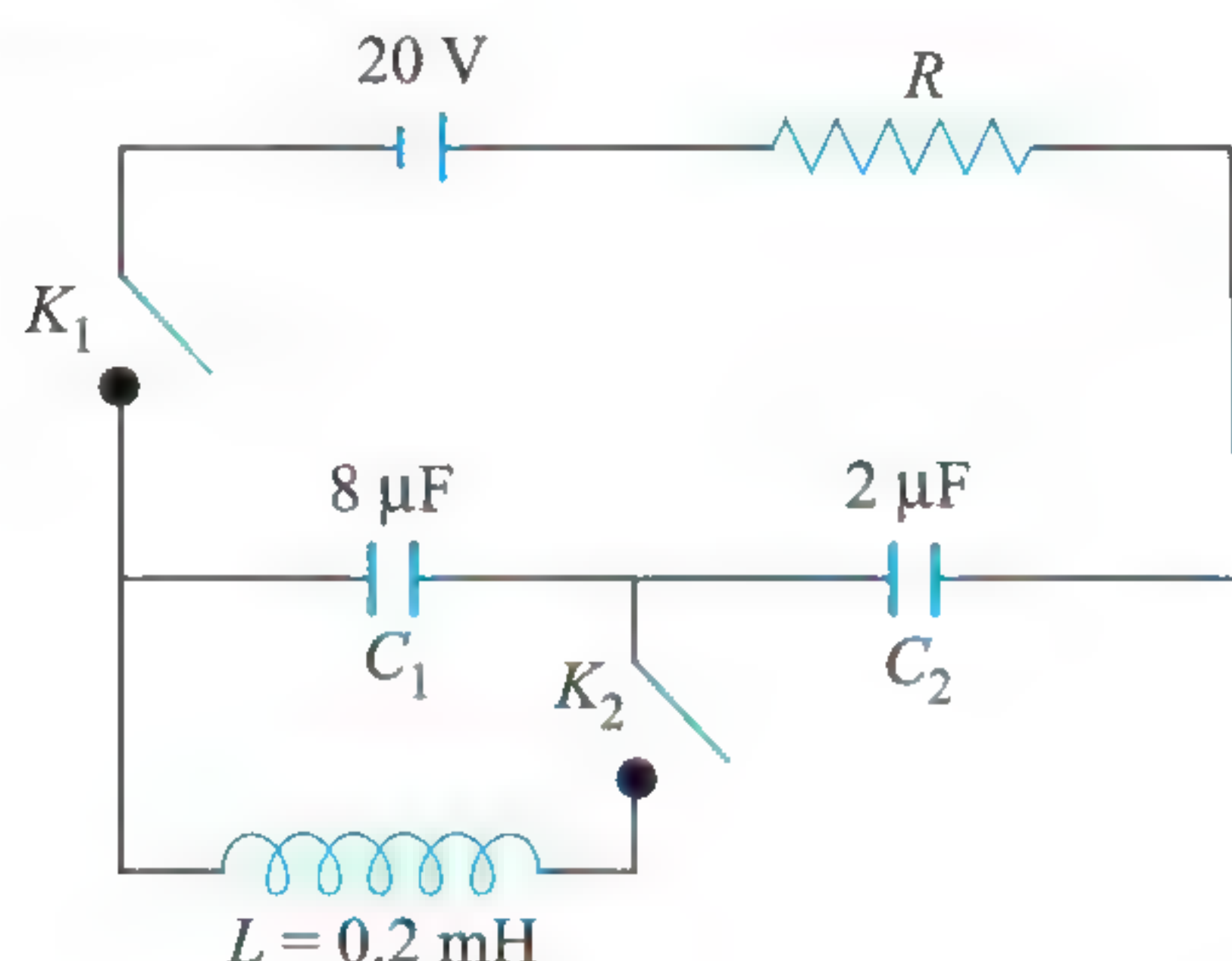
- (1) $\frac{(2M + L)i_0}{R}$
(2) $\frac{Mi_0}{R}$
(3) $\frac{(2M - L)I_0}{R}$
(4) $\frac{(M + L)I_0}{R}$

68. A system consists of two coaxial current carrying circular loops as shown in figure. The self inductance of loop 1 is L_1 and self inductance of loop 2 is L_2 . Magnitude of mutual inductance is M . If at any instant current flowing through loop 1 and loop 2 are i_1 and i_2 respectively, then total magnetic energy of the system is



- (1) $\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - |M| i_1 i_2$ (2) $\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + |M| i_1 i_2$
 (3) $\frac{1}{2}L_1 i_2^2 + \frac{1}{2}L_2 i_1^2 - |M| i_1 i_2$ (4) $\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2$

69. A circuit containing capacitors C_1 and C_2 , shown in figure is in the steady state with key K_1 closed. At the instant $t = 0$, K_1 is opened and K_2 is closed. The angular frequency of oscillation of the circuit and charge across C_1 at $t = 125 \mu\text{s}$ are respectively

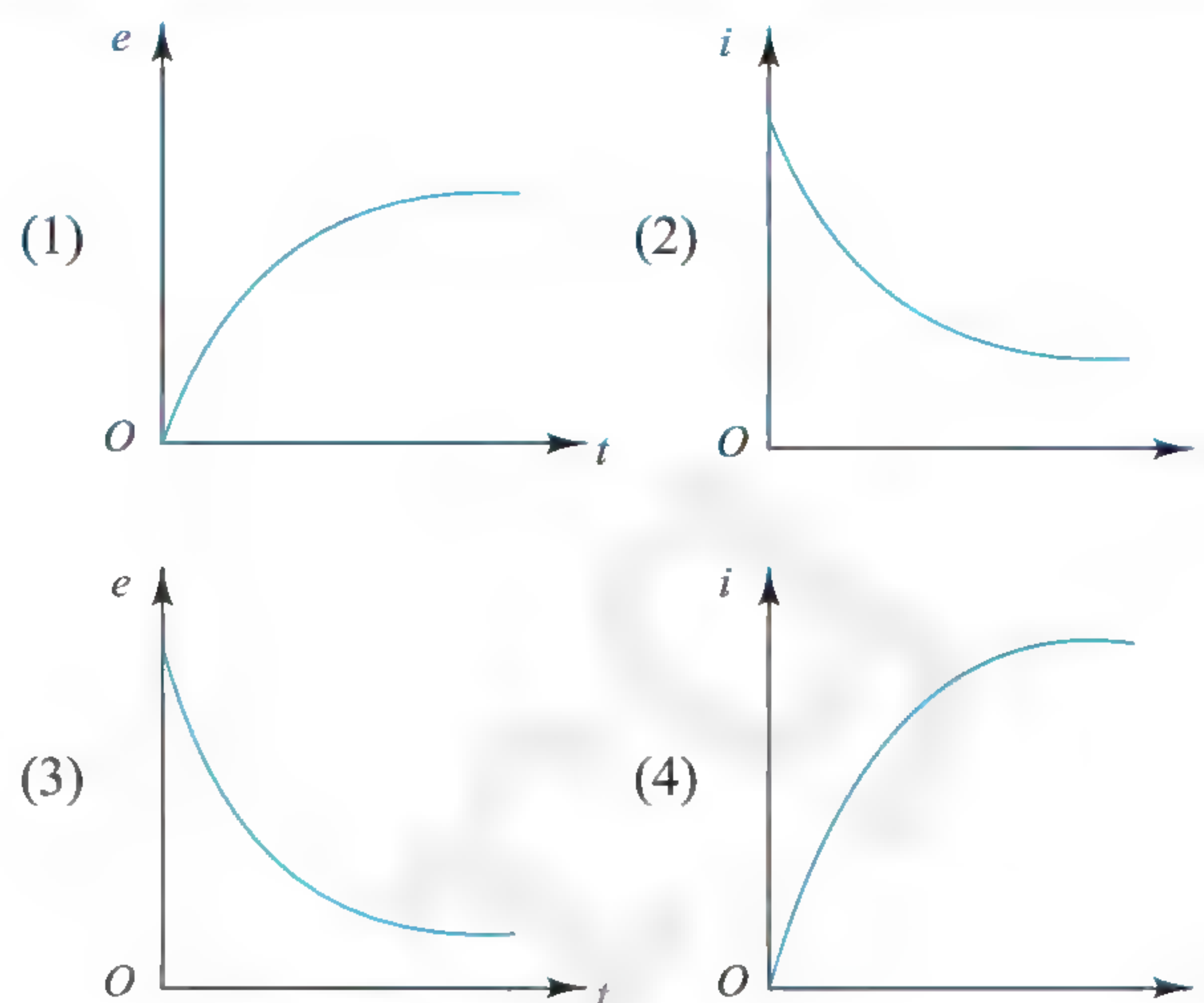
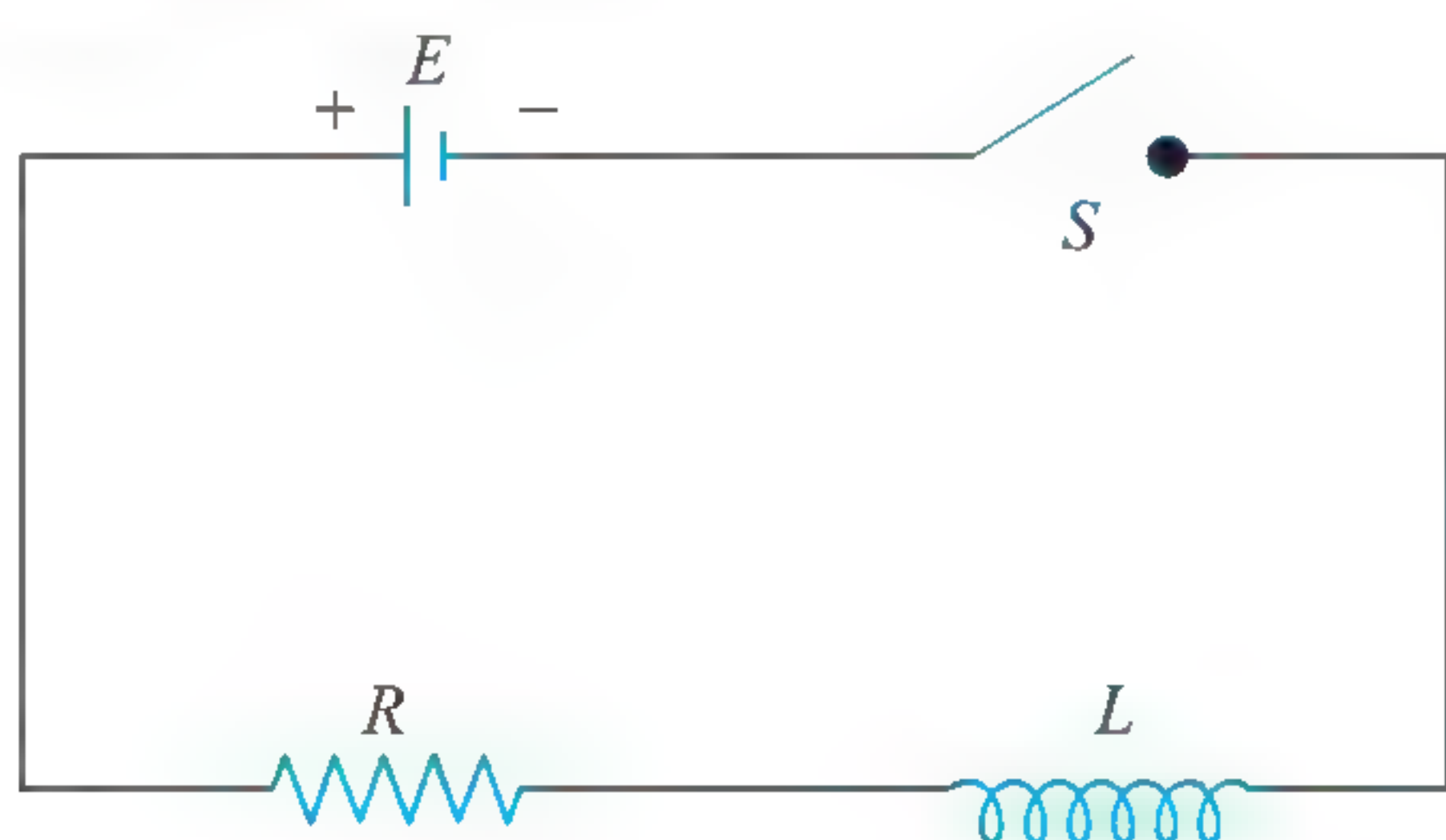


- (1) $25 \times 10^3 \text{ rad s}^{-1}$, $32 \mu\text{C}$ (2) $5 \times 10^4 \text{ rad s}^{-1}$, $16 \mu\text{C}$
 (3) $5 \times 10^4 \text{ rad s}^{-1}$, $-16 \mu\text{C}$ (4) $25 \times 10^3 \text{ rad s}^{-1}$, $-32 \mu\text{C}$
70. An inductor coil stores 32 J of magnetic field energy and dissipates energy as heat at the rate of 320 W when a current of 4 A is passed through it. Find the time constant of the circuit when it is formed across an ideal battery.
- (1) $t = 0.2 \text{ s}$ (2) $t = 0.32 \text{ s}$
 (3) $t = 0.5 \text{ s}$ (4) $t = 1 \text{ s}$
71. A solenoid of inductance 50 mH and resistance 10Ω is connected to a battery of 6 V. Find the time elapsed before the current acquires half of its steady state value.
- (1) 3.5 ms (2) 2.5 ms
 (3) 0.693 ms (4) 2 ms
72. An inductor resistance battery circuit is switched on at $t = 0$. If the emf of the battery is E , find the charge which passes through the battery during one time constant τ .

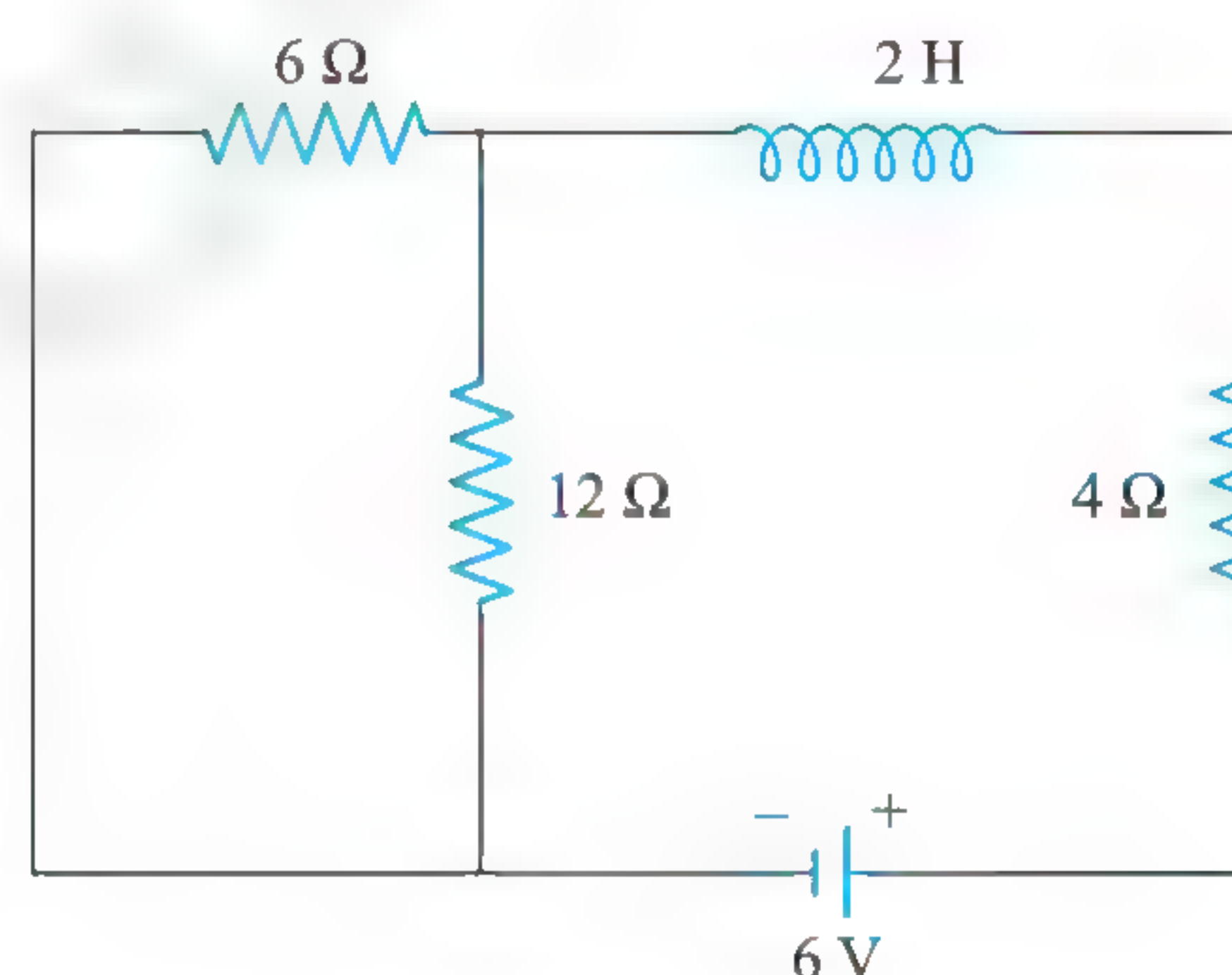
- (1) $\frac{E}{R} \cdot \frac{\tau}{e}$ (2) $\frac{E}{R} \cdot \frac{\tau}{(e-1)}$
 (3) $\frac{E}{R} \cdot \frac{\tau}{(e+1)}$ (4) $\frac{E}{R} \cdot \frac{(e-1)}{\tau}$

Multiple Correct Answers Type

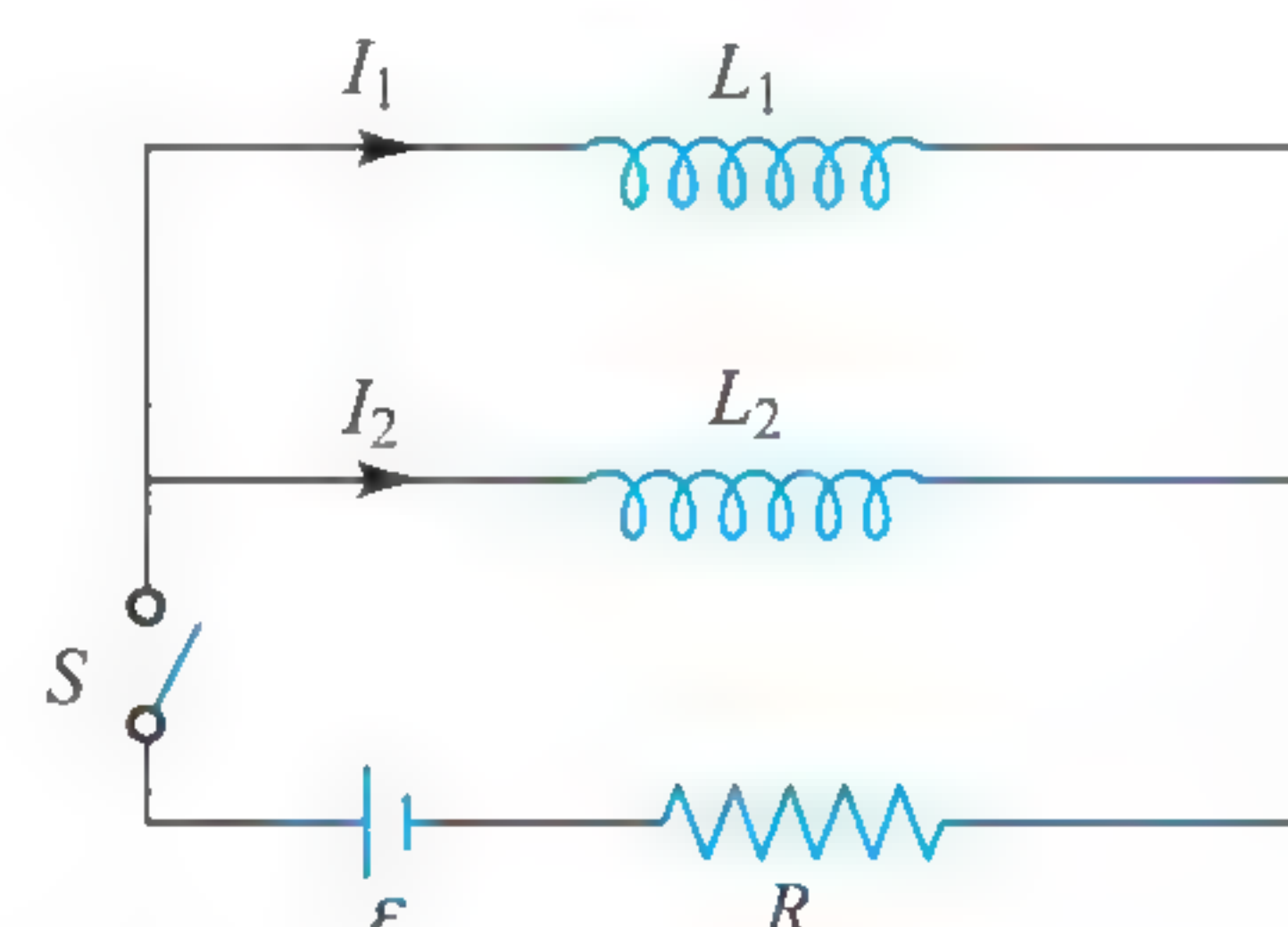
1. Switch S of the circuit shown in figure is closed at $t = 0$. If e denotes the induced emf in L and I is the current flowing through the circuit at time t , which of the following graphs is/are correct?



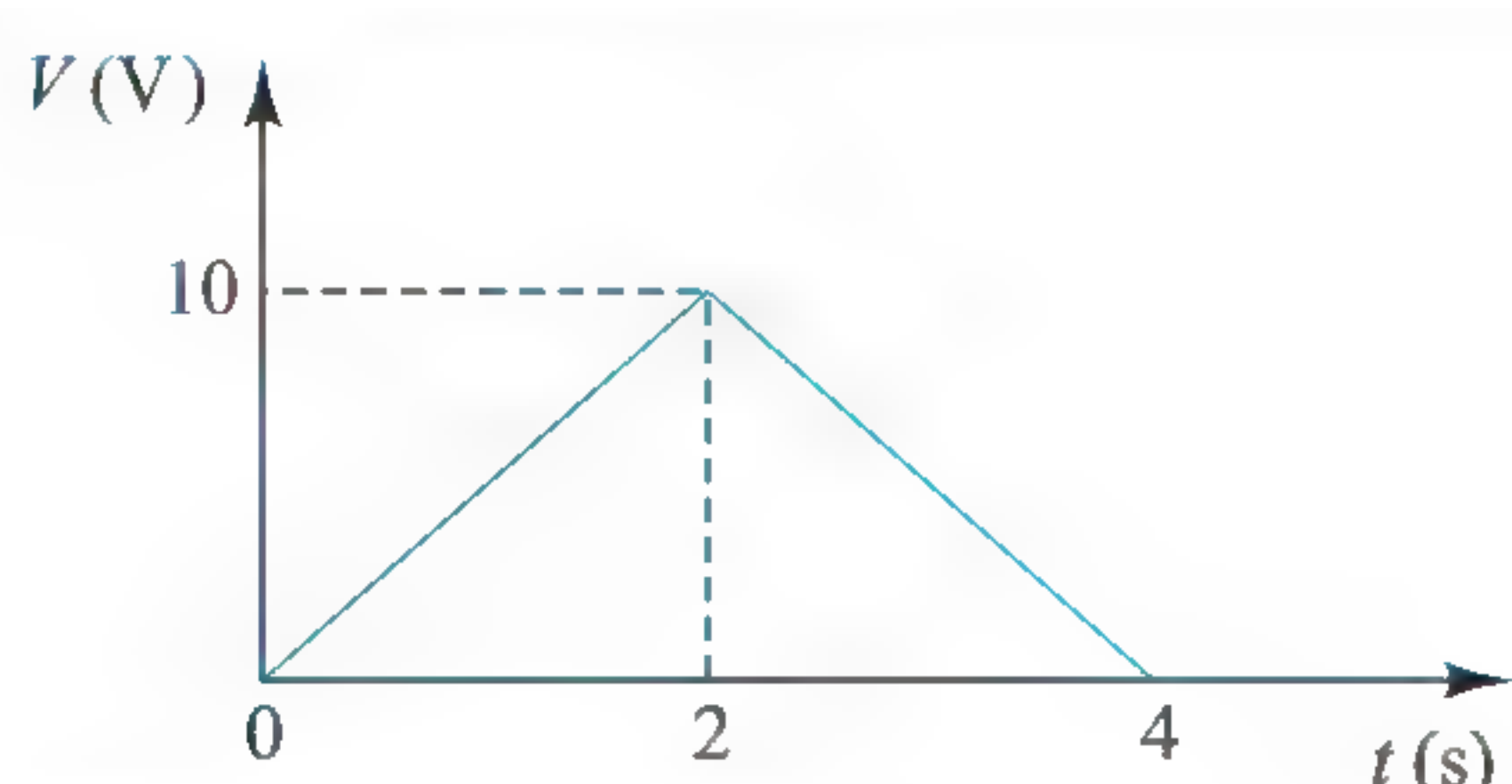
2. For the circuit shown in figure, which of the following statements is/are correct?



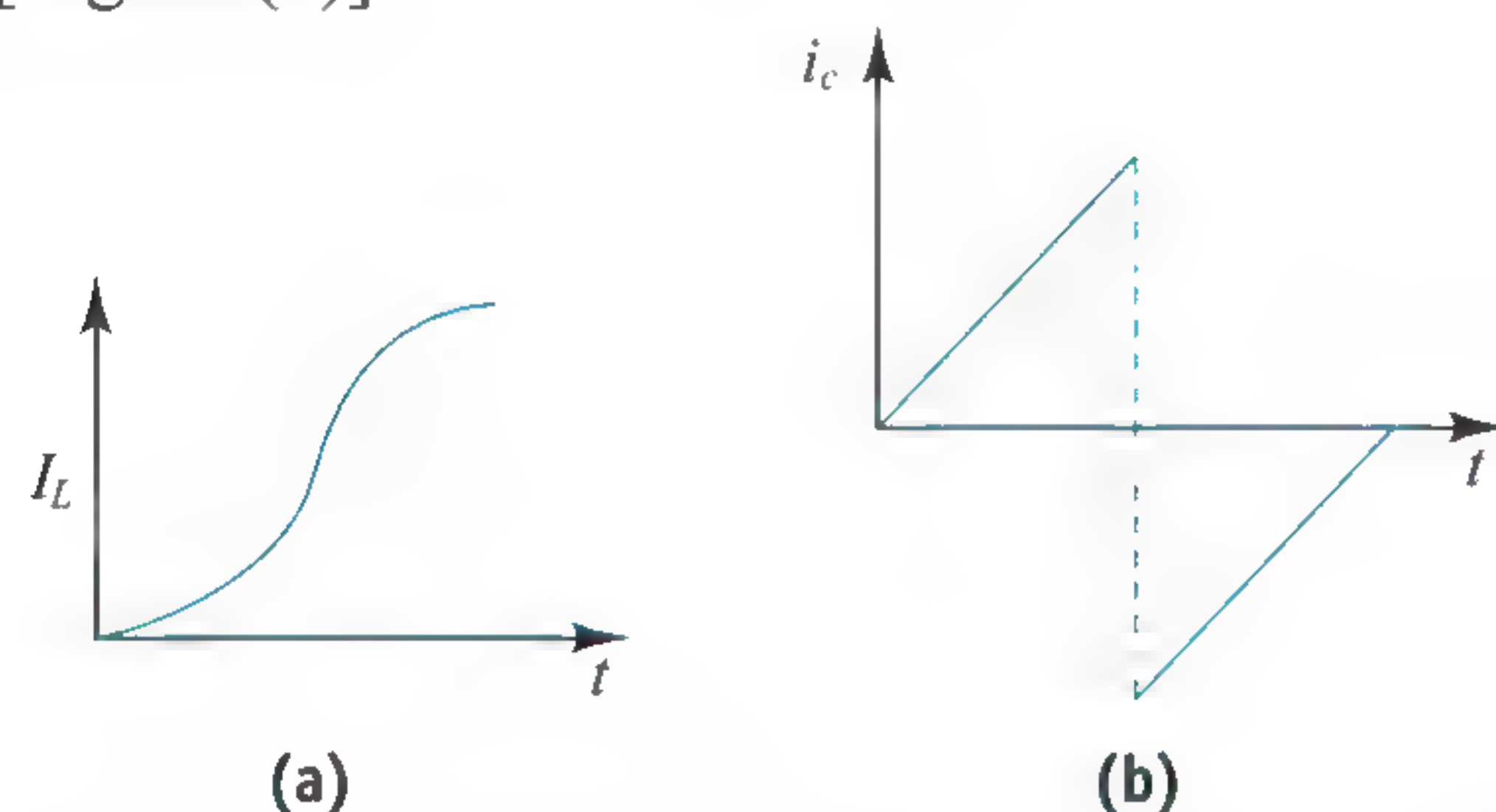
- (1) Its time constant is 0.25 s
 (2) In steady state, current through the inductance will be equal to zero
 (3) In steady state, current through the battery will be equal to 0.75 A
 (4) None of these
3. In the circuit shown in figure, the switch is closed at $t = 0$.



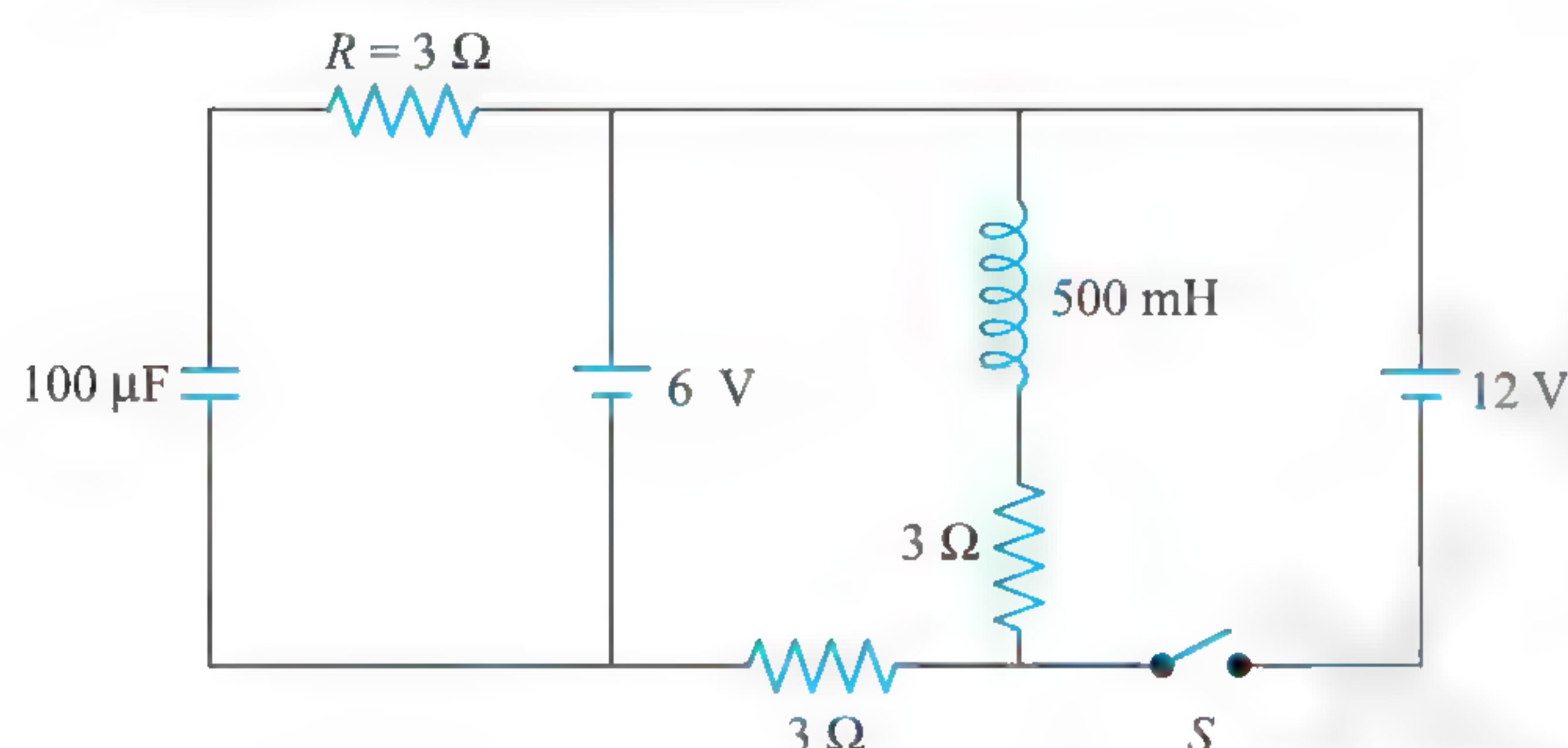
- (1) At $t = 0$, $I_1 = I_2 = 0$
 (2) At any time t , $\frac{I_1}{I_2} = \frac{L_2}{L_1}$
 (3) At any time t , $I_1 + I_2 = \frac{\varepsilon}{R}$
 (4) At $t = \infty$, I_1 and I_2 are independent of L_1 and L_2
4. The potential difference across a 2-H inductor as a function of time is shown in figure. At $t = 0$, current is zero. Choose the correct statement.



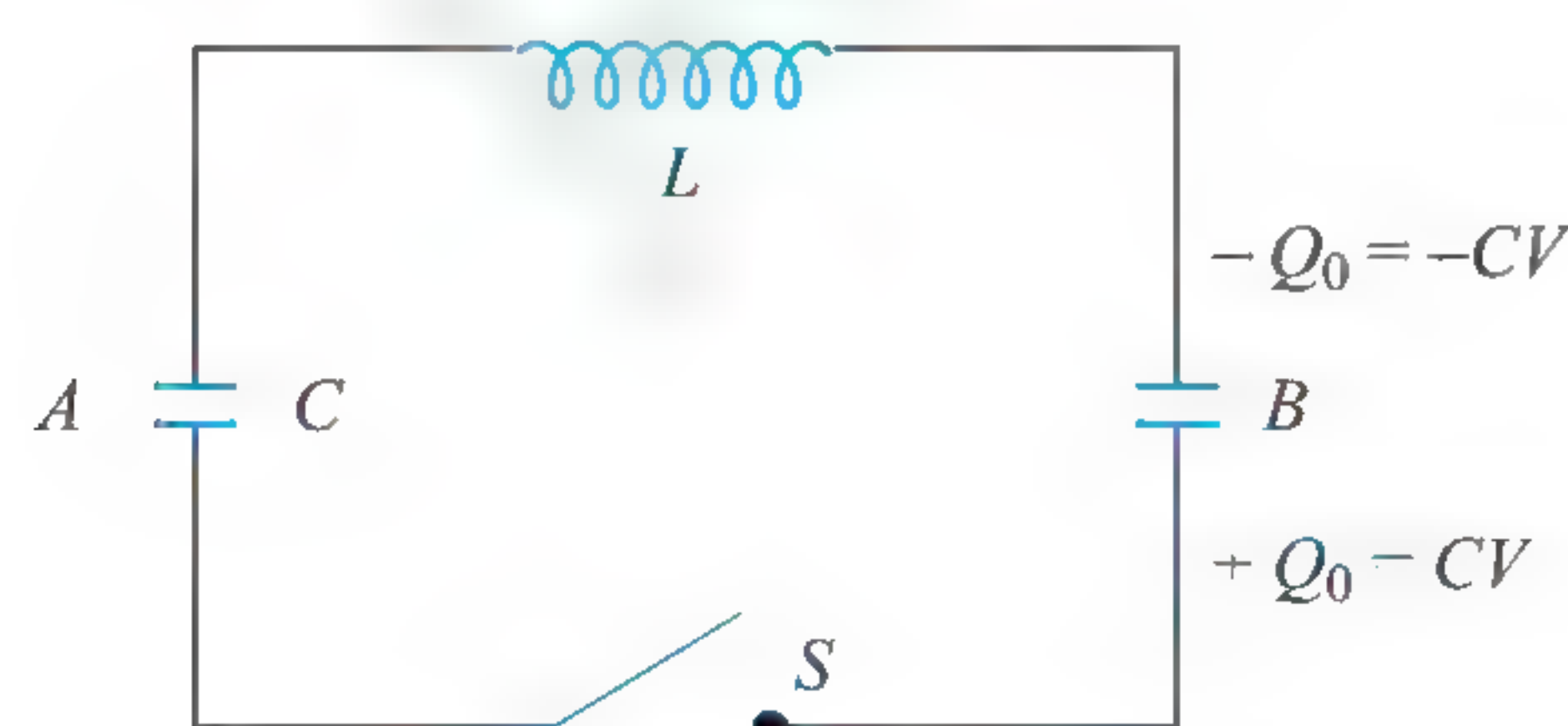
- (1) Current at $t = 2$ s is 5 A
- (2) Current at $t = 2$ s is 10 A
- (3) Current versus time graph across the inductor will be [Figure (a)]
- (4) Current versus time graph across inductor will be [Figure (b)]



5. In the given circuit (figure), the switch is closed at $t = 0$. Choose the correct answers.



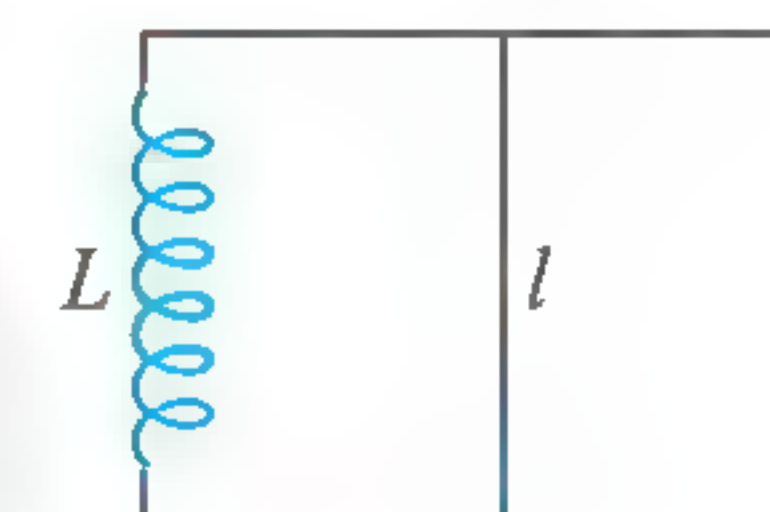
- (1) Current in the inductor when the circuit reaches the steady state is 4 A.
 - (2) The net change in flux in the inductor is 1.5 Wb.
 - (3) The time constant of the circuit after closing S is 555.55 s.
 - (4) The charge stored in the capacitor in steady state is 1.2 mC.
6. An inductor and two capacitors are connected in the circuit as shown in figure. Initially capacitor A has no charge and capacitor B has CV charge. Assume that the circuit has no resistance at all. At $t = 0$, switch S is closed, then [given $LC = \frac{2}{\pi^2 \times 10^4} \text{ s}^2$ and $CV = 100 \text{ mC}$]



- (1) when current in the circuit is maximum, charge on each capacitor is same
- (2) when current in the circuit is maximum, charge on capacitor A is twice the charge on capacitor B

- (3) $q = 50(1 + \cos 100 \pi t) \text{ mC}$, where q is the charge on capacitor B at time t
- (4) $q = 50(1 - \cos 100 \pi t) \text{ mC}$, where q is the charge on capacitor B at time t

7. Two parallel resistanceless rails are connected by an inductor of inductance L at one end as shown in figure. A magnetic field B exists in the space which is perpendicular to the plane of the rails. Now a conductor of length l and mass m is placed transverse on the rail and given an impulse J toward the rightward direction. Then choose the correct option(s).



- (1) Velocity of the conductor is half of the initial velocity

$$\text{after a displacement of the conductor } d = \sqrt{\frac{3J^2 L}{4B^2 l^2 m}}$$

- (2) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is

$$i = \sqrt{\frac{3J^2}{4Lm}}$$

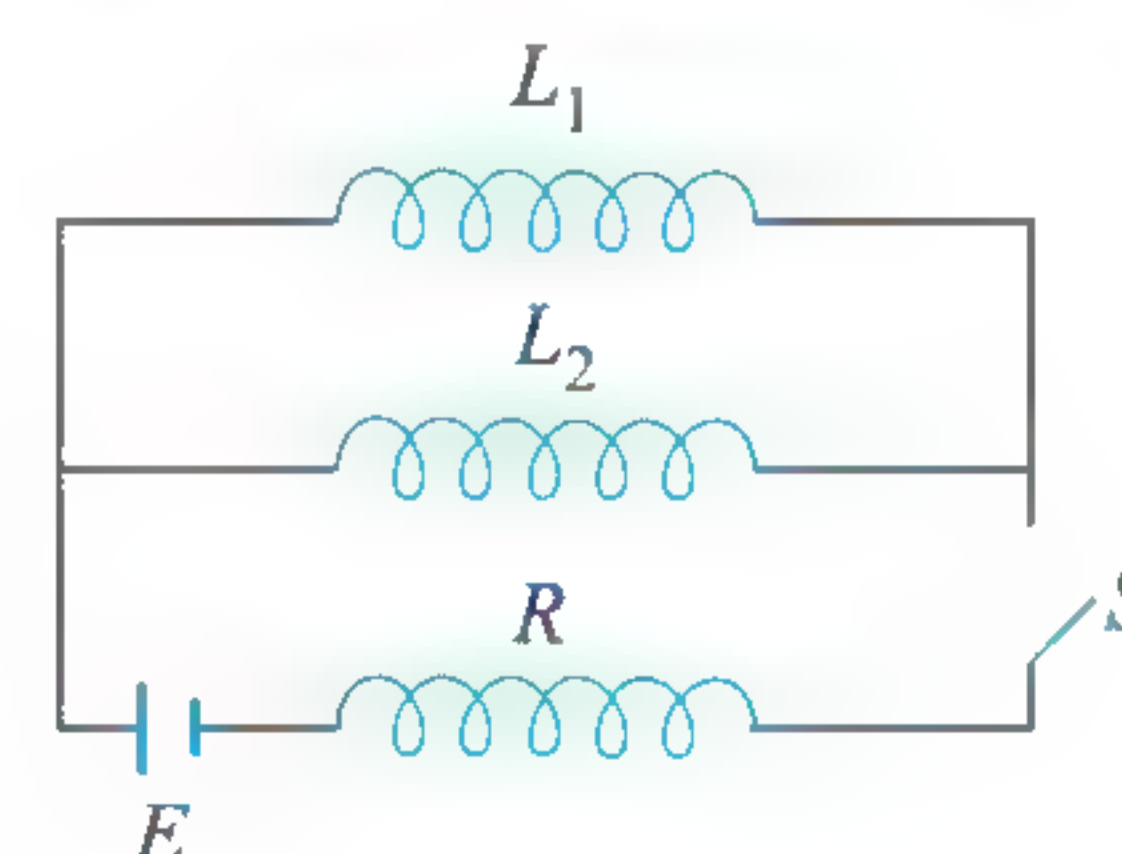
- (3) Velocity of the conductor is half of the initial velocity

$$\text{after a displacement of the conductor } d = \sqrt{\frac{3J^2 L}{B^2 l^2 m}}$$

- (4) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is

$$i = \sqrt{\frac{3J^2}{mL}}$$

8. In the circuit, a battery of emf E , a resistance R and inductance coil L_1 and L_2 and switch S are connected as shown. Initially the switch is open



- (1) The time constant of the circuit is $\frac{1}{R} \left(\frac{L_1 L_2}{L_1 + L_2} \right)$

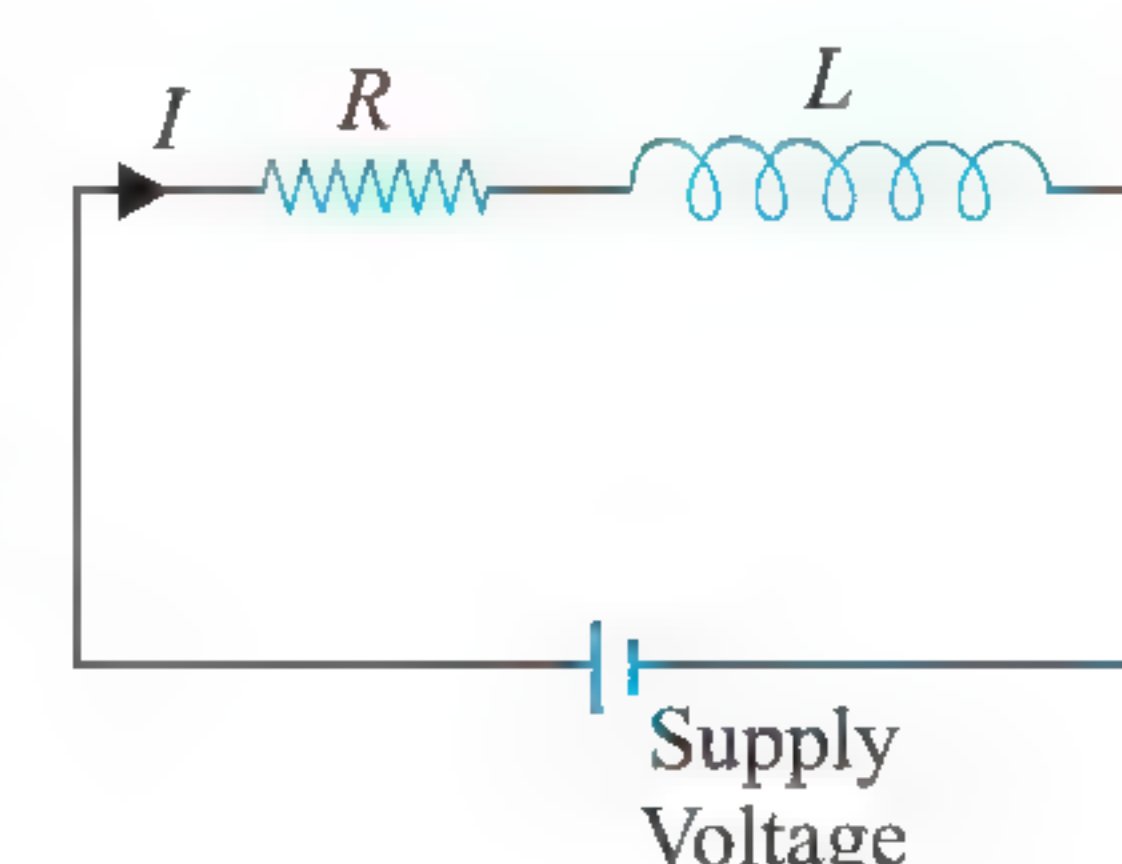
- (2) Steady state current in the inductor L_1 is $\frac{EL_2}{R(L_1 + L_2)}$

- (3) Steady state, current in the inductor L_2 is $\frac{EL_1}{R(L_1 + L_2)}$

- (4) In steady state the total energy stored in the inductor coils

$$\text{is } \frac{1}{2} \left(\frac{L_1 L_2}{L_1 + L_2} \right) \frac{E^2}{R^2}$$

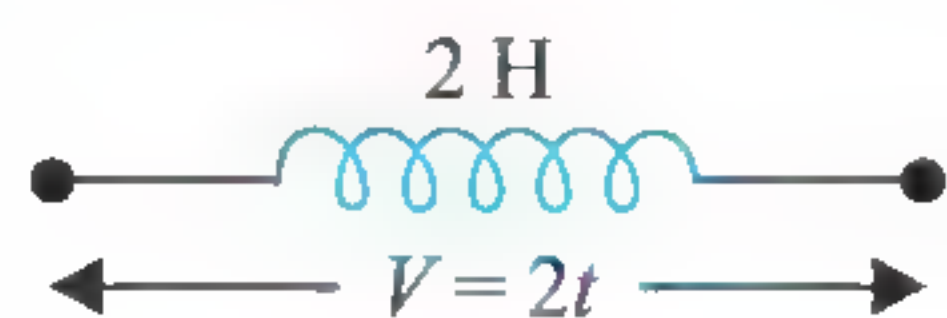
9. When current (I) in R - L series circuit becomes constant, where L is a pure inductor, which of the following given statements is/are correct?



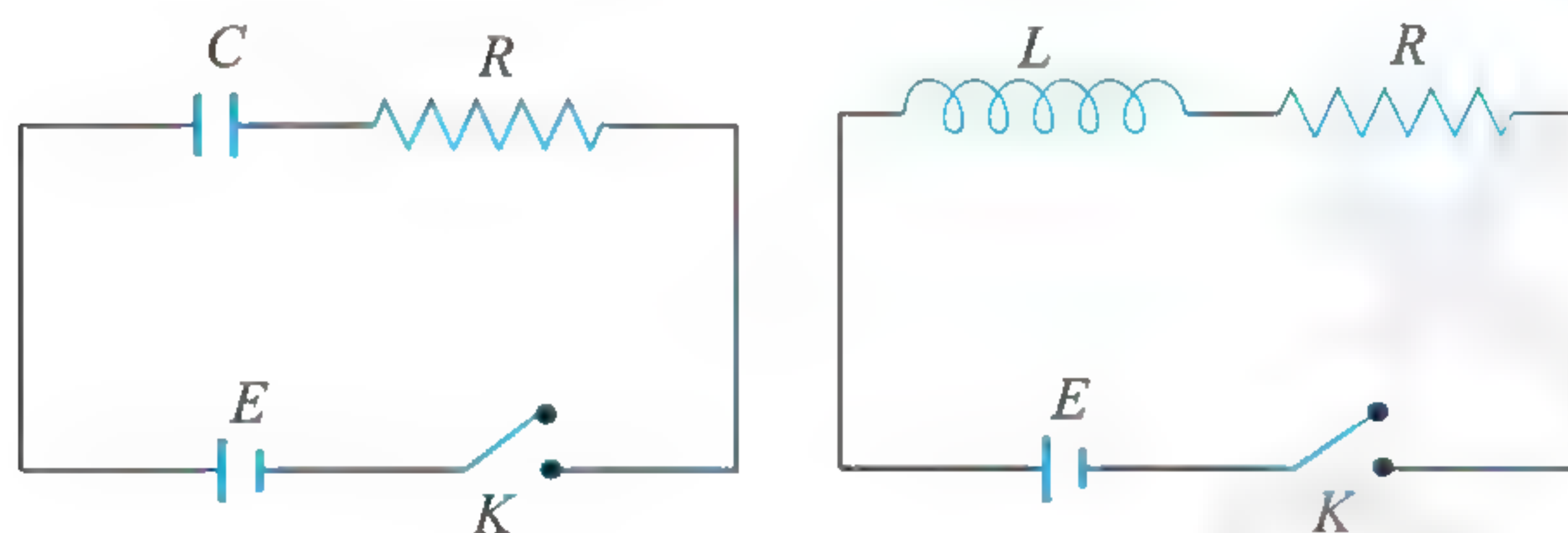
- (1) voltage across R is RI .

- (2) some part (not 100 %) of the energy supplied by the battery will be dissipated in R and remaining will continue to store in L .
- (3) voltage across L is equal to zero
- (4) magnetic energy stored is $\frac{1}{2}LI^2$

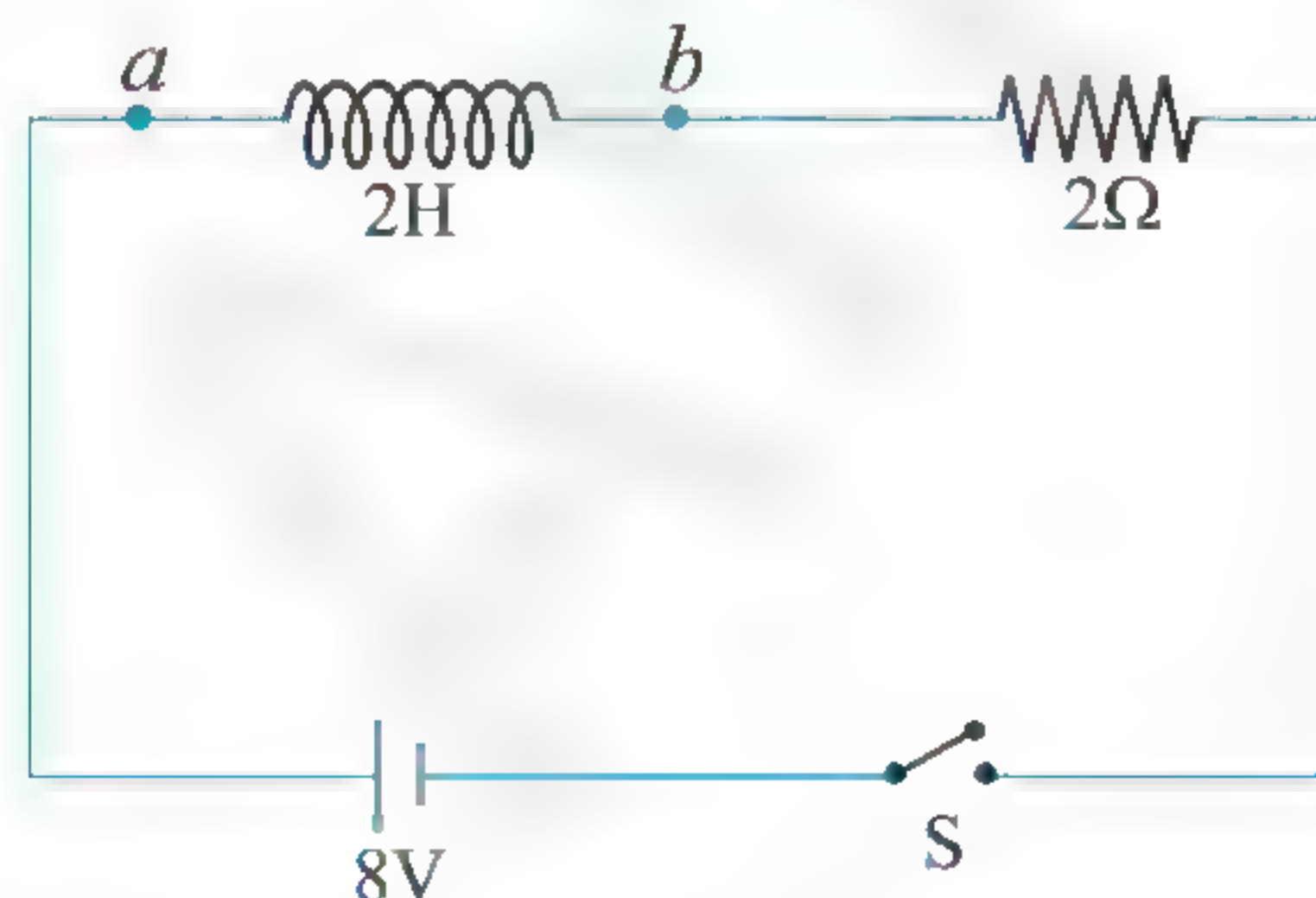
10. A time varying voltage $V = 2t$ (Volt) is applied across an ideal inductor of inductance $L = 2\text{H}$ as shown in figure. Then (assume current to be zero at $t = 0$)



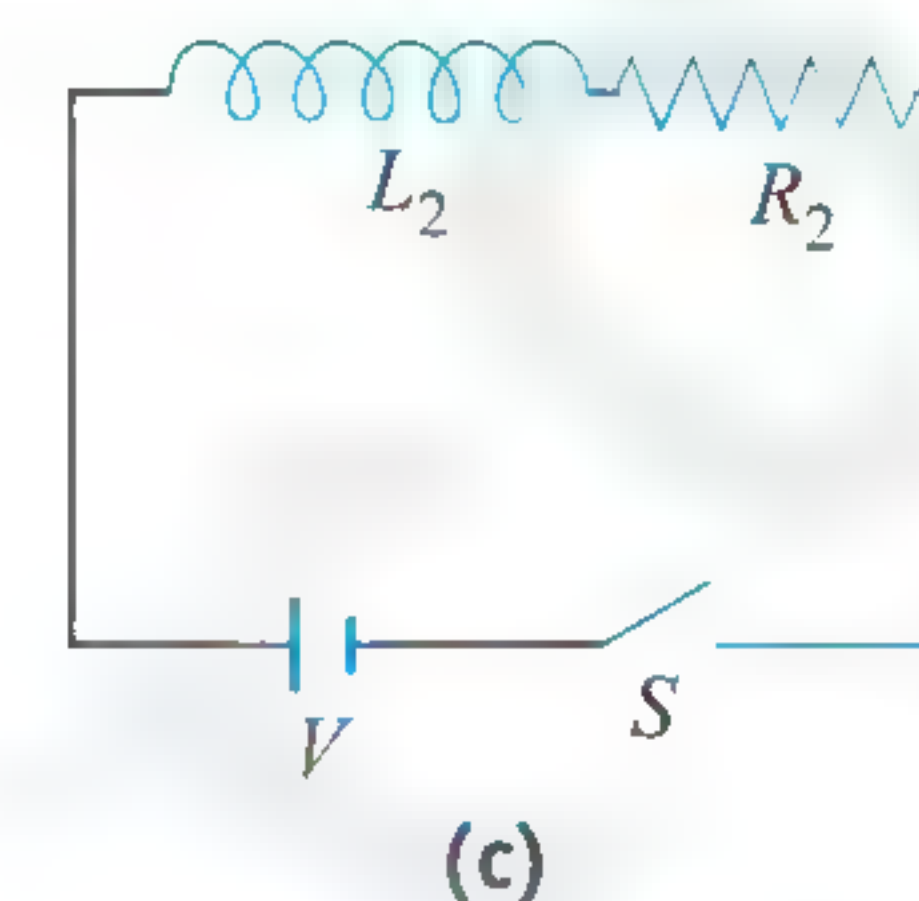
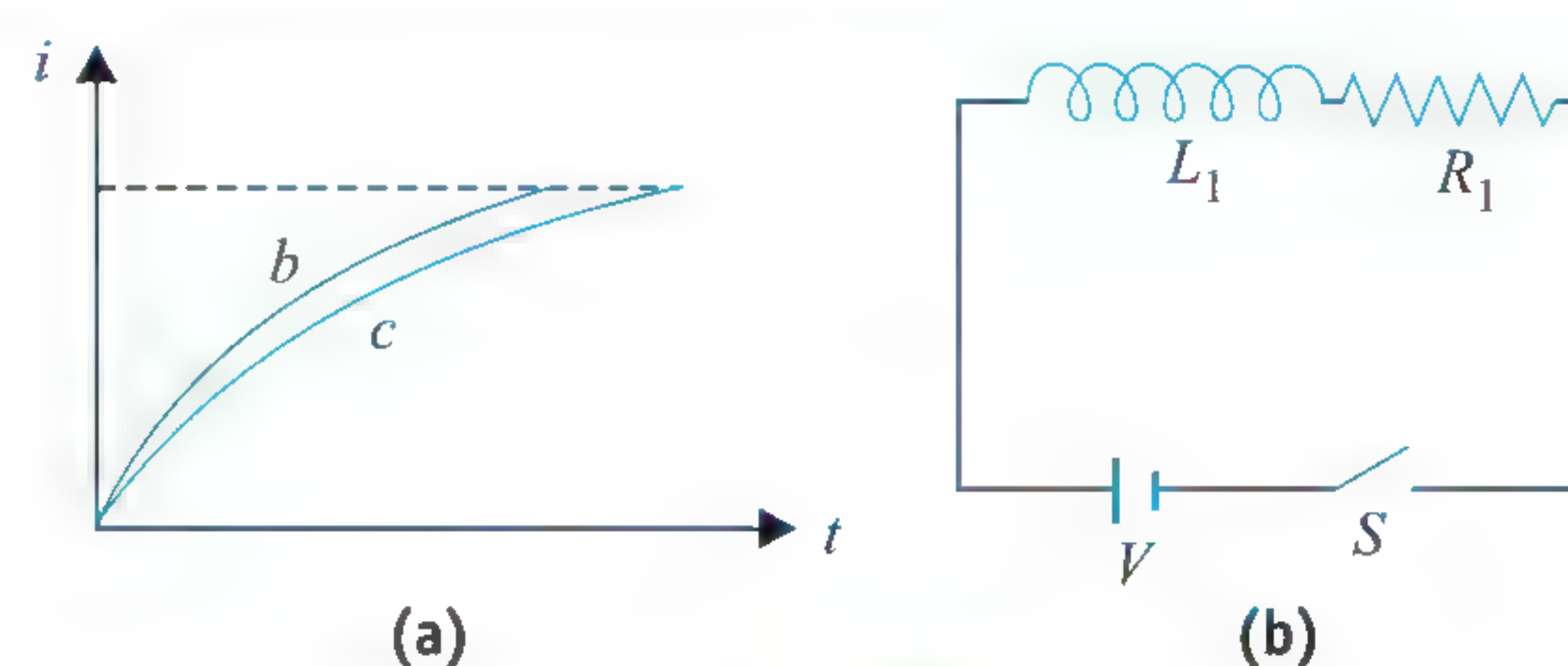
- (1) current versus time graph is a parabola
- (2) energy stored in magnetic field at $t = 2\text{ s}$ is 4 J
- (3) potential energy at time $t = 1\text{ s}$ in magnetic field is increasing at a rate of 1 J/s
- (4) energy stored in magnetic field is zero all the time
11. An LR circuit with a battery connected at $t = 0$. Which of the following quantities will be zero just after the connection?
- (1) Current in the circuit
- (2) Magnetic field energy in the inductor
- (3) Power delivered by battery
- (4) emf induced in the inductor
12. The switches in following figures are closed at $t = 0$ and re-open after a long time at $t = t_0$. Then



- (1) the charge on C just after $t = 0$ is EC
- (2) the charge on C long after $t = 0$ is EC
- (3) the current in L just before $t = t_0$ is E/R
- (4) the current in L long after $t = t_0$ is E/R
13. In the circuit shown in figure, circuit is closed at time $t = 0$. At time $t = \ln(2)$ second:



- (1) Rate of energy supplied by the battery is 16 J/s
- (2) Rate of heat dissipated across resistance is 16 J/s
- (3) Rate of heat dissipated across resistance is 8 J/s
- (4) $V_a - V_b = 2V$
14. The current growth in two L - R circuits (b) and (c) is as shown in Fig. (a). Let L_1, L_2, R_1 , and R_2 be the corresponding values in two circuits. Then

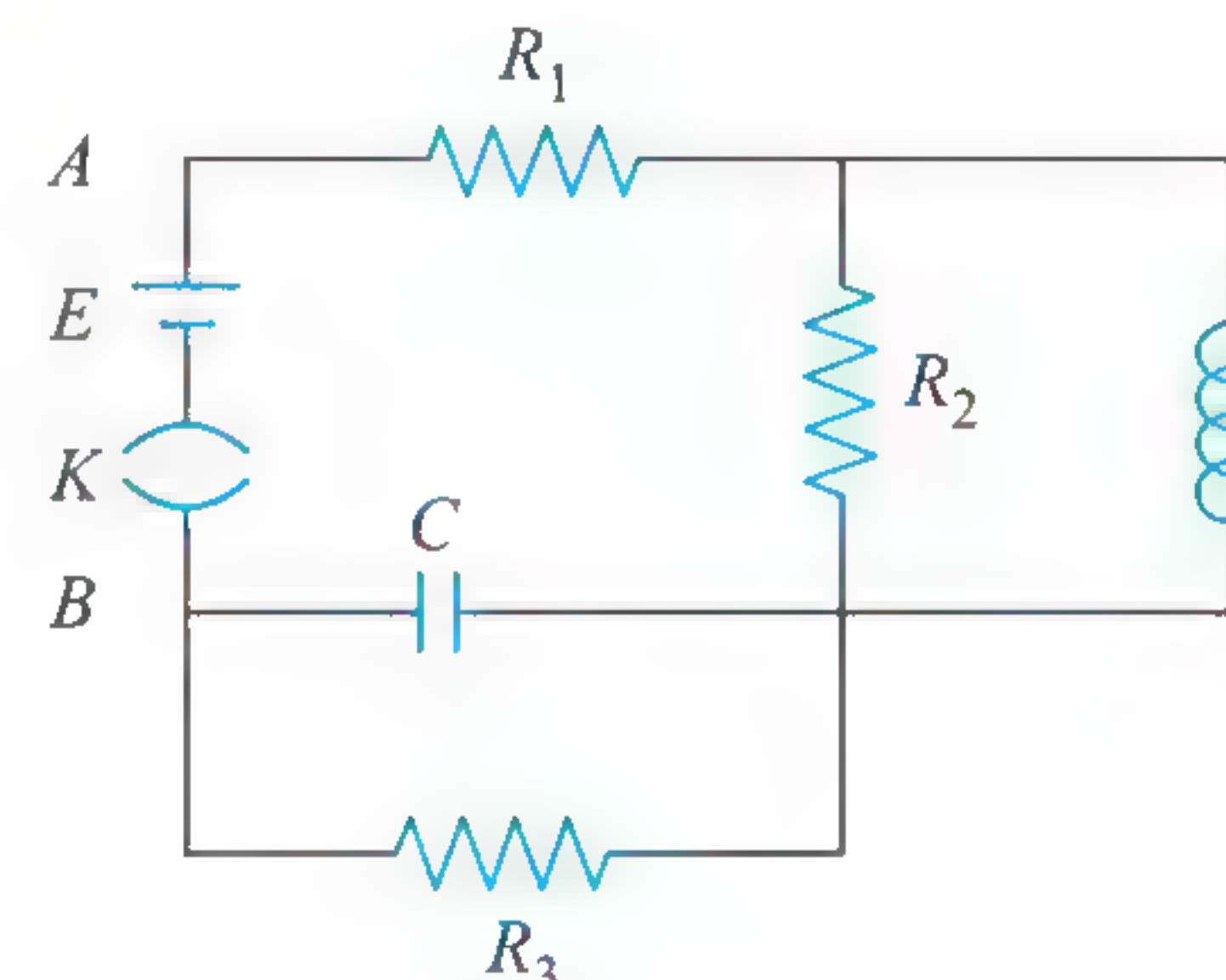


- (1) $R_1 > R_2$
- (2) $R_1 = R_2$
- (3) $L_1 > L_2$
- (4) $L_1 < L_2$

Linked Comprehension Type

For Problems 1–2

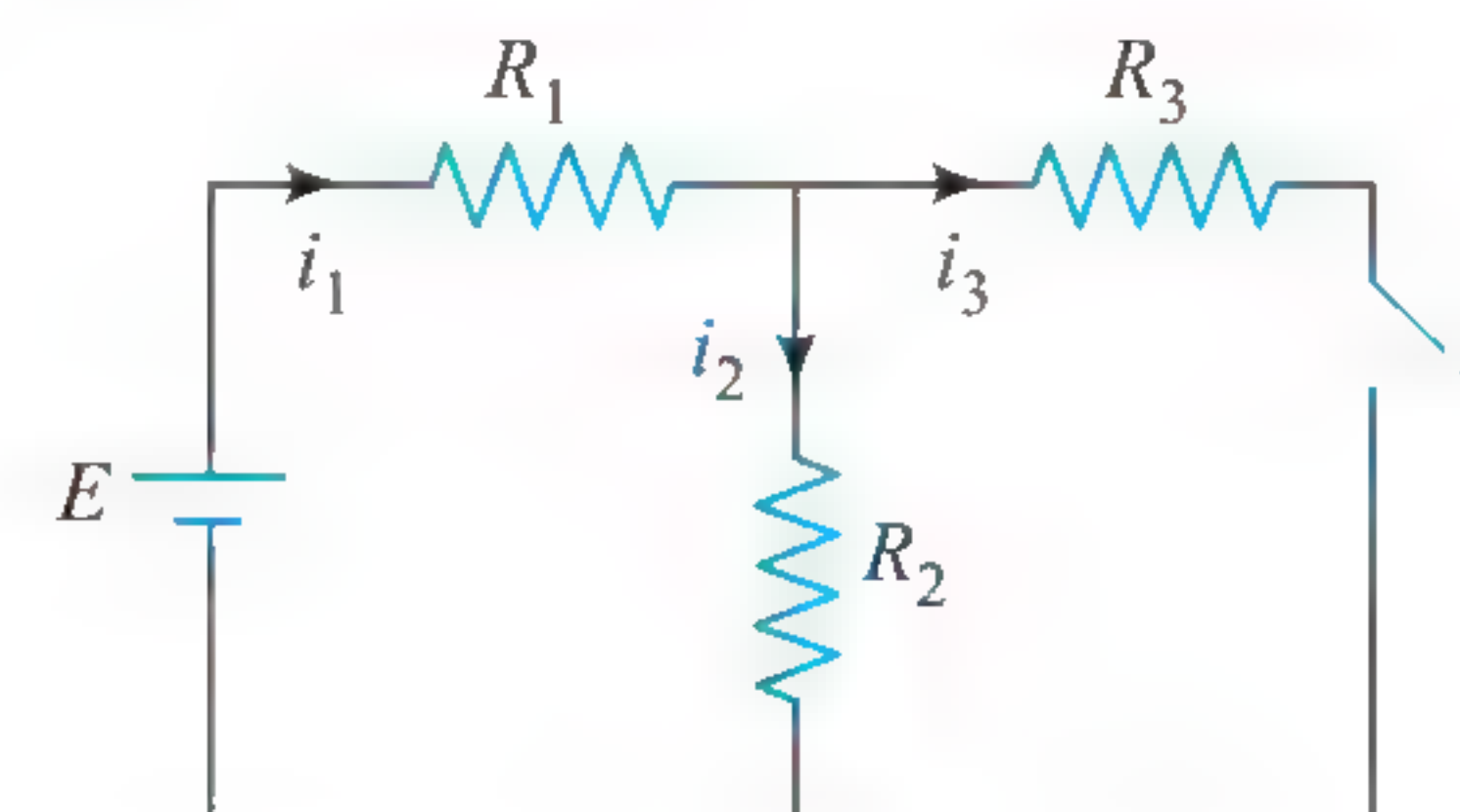
In the given circuit (figure), all the symbols have their usual meanings. At $t = 0$, key K is closed. Now answer the following questions.



1. At $t = 0$, the equivalent resistance between A and B is
- (1) $R_1 + R_2 + R_3$
- (2) $R_1 + R_2$
- (3) $R_1 + R_3$
- (4) indeterminate
2. At $t \rightarrow \infty$, the equivalent resistance between A and B is
- (1) $R_1 + R_2 + R_3$
- (2) $R_1 + R_2$
- (3) $R_1 + R_3$
- (4) none of these

For Problems 3–5

In the circuit shown in figure, $E = 15\text{ V}$, $R_1 = 1\Omega$, $R_2 = 1\Omega$, $R_3 = 2\Omega$, and $L = 1.5\text{ H}$. The currents flowing through R_1, R_2 , and R_3 are i_1, i_2 , and i_3 , respectively.



3. Immediately after turning switch S on,
- (1) $i_1 = i_2 = 7.5\text{ A}$, $i_3 = 0\text{ A}$
- (2) $i_1 = i_3 = 5\text{ A}$, $i_2 = 0\text{ A}$
- (3) $i_1 = i_2 = 9\text{ A}$, $i_3 = 0\text{ A}$
- (4) $i_1 = i_2 = i_3 = 0\text{ A}$
4. After the circuit reaches the steady state,
- (1) $i_1 = 9\text{ A}$, $i_2 = 6\text{ A}$, $i_3 = 3\text{ A}$
- (2) $i_1 = 9\text{ A}$, $i_2 = 3\text{ A}$, $i_3 = 6\text{ A}$

(3) $i_1 = 6 \text{ A}$, $i_2 = 6 \text{ A}$, $i_3 = 0 \text{ A}$

(4) $i_1 = 0 \text{ A}$, $i_2 = 0 \text{ A}$, $i_3 = 0 \text{ A}$

5. Immediately after connecting switch S ,

(1) $i_3 = 0 \text{ A}$ and $\frac{di_3}{dt} = 0 \text{ A s}^{-1}$

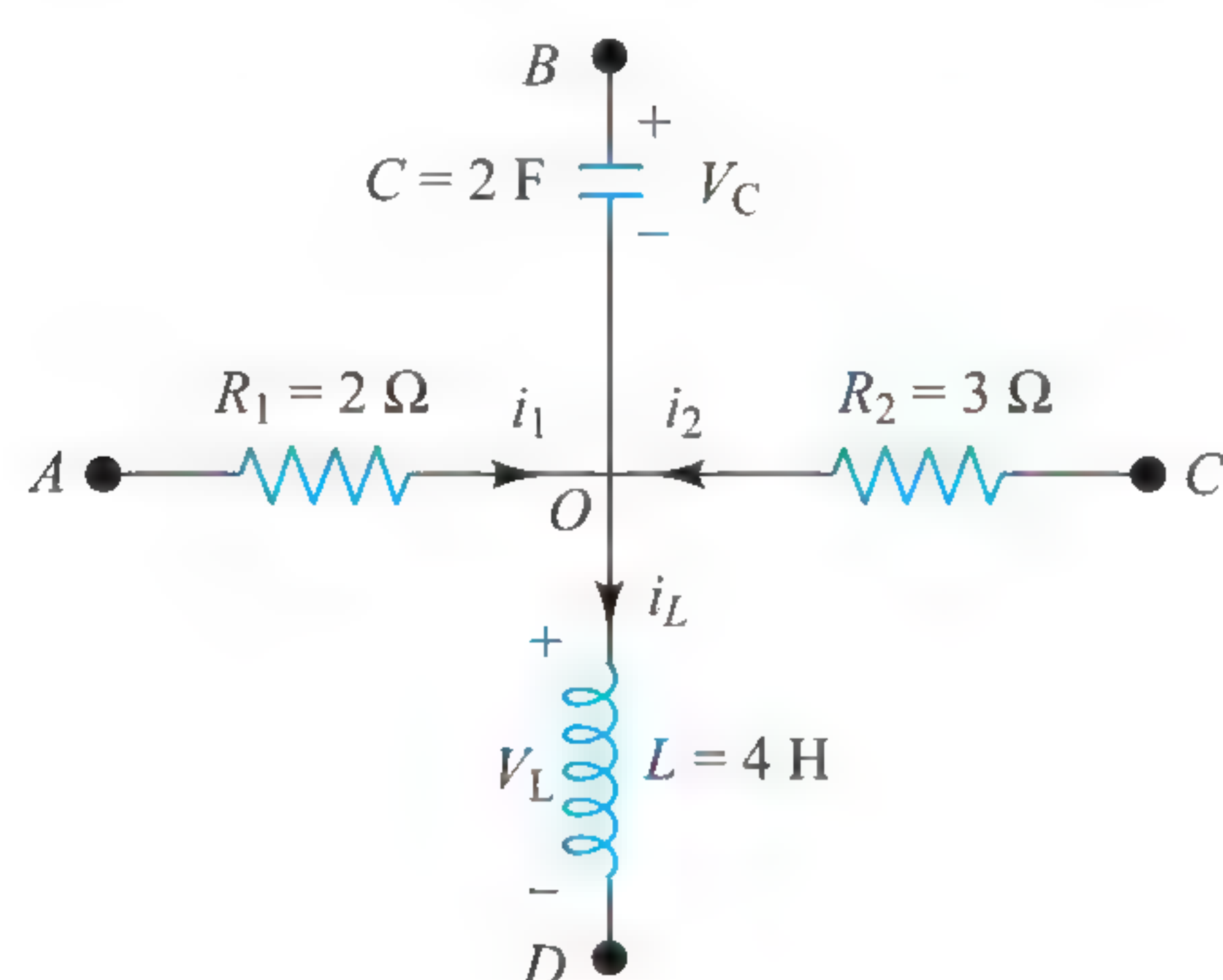
(2) $i_3 = 0 \text{ A}$ and $\frac{di_3}{dt} \neq 0 \text{ A s}^{-1}$

(3) $i_3 = 0 \text{ A}$ and the rate at which magnetic energy stored is not zero

(4) none of these

For Problems 6–11

In figure, $i_1 = 10 e^{-2t} \text{ A}$, $i_2 = 4 \text{ A}$, and $V_C = 3e^{-2t} \text{ V}$.

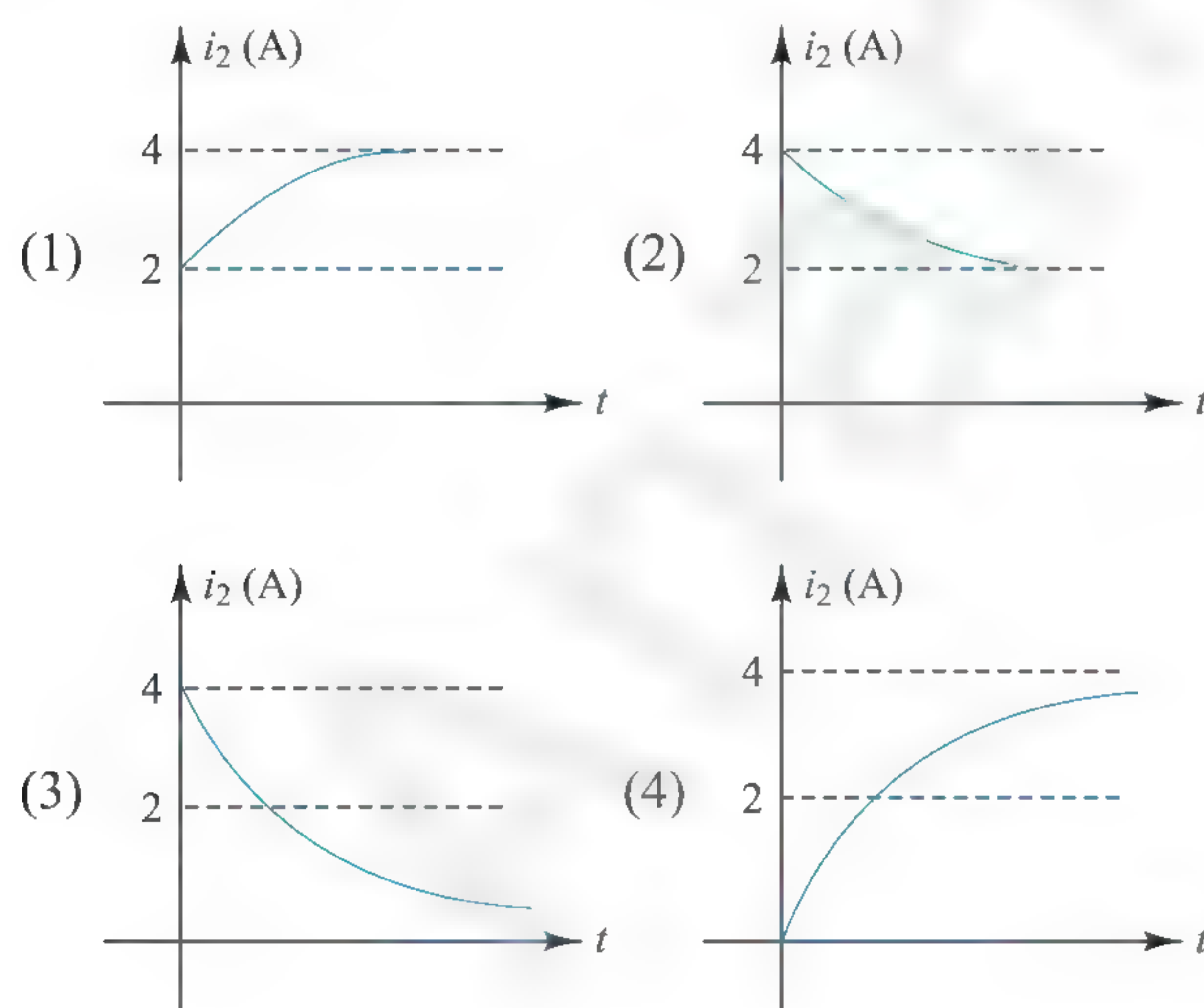


6. The current i_L is

(1) $[2 - 2(1 - e^{-2t})] \text{ A}$ (2) $[2 + 2(1 - e^{-2t})] \text{ A}$

(3) $[3 - 2(1 - e^{-2t})] \text{ A}$ (4) $[2 + 3(1 - e^{-2t})] \text{ A}$

7. The variation of current in the inductor with time can be represented as

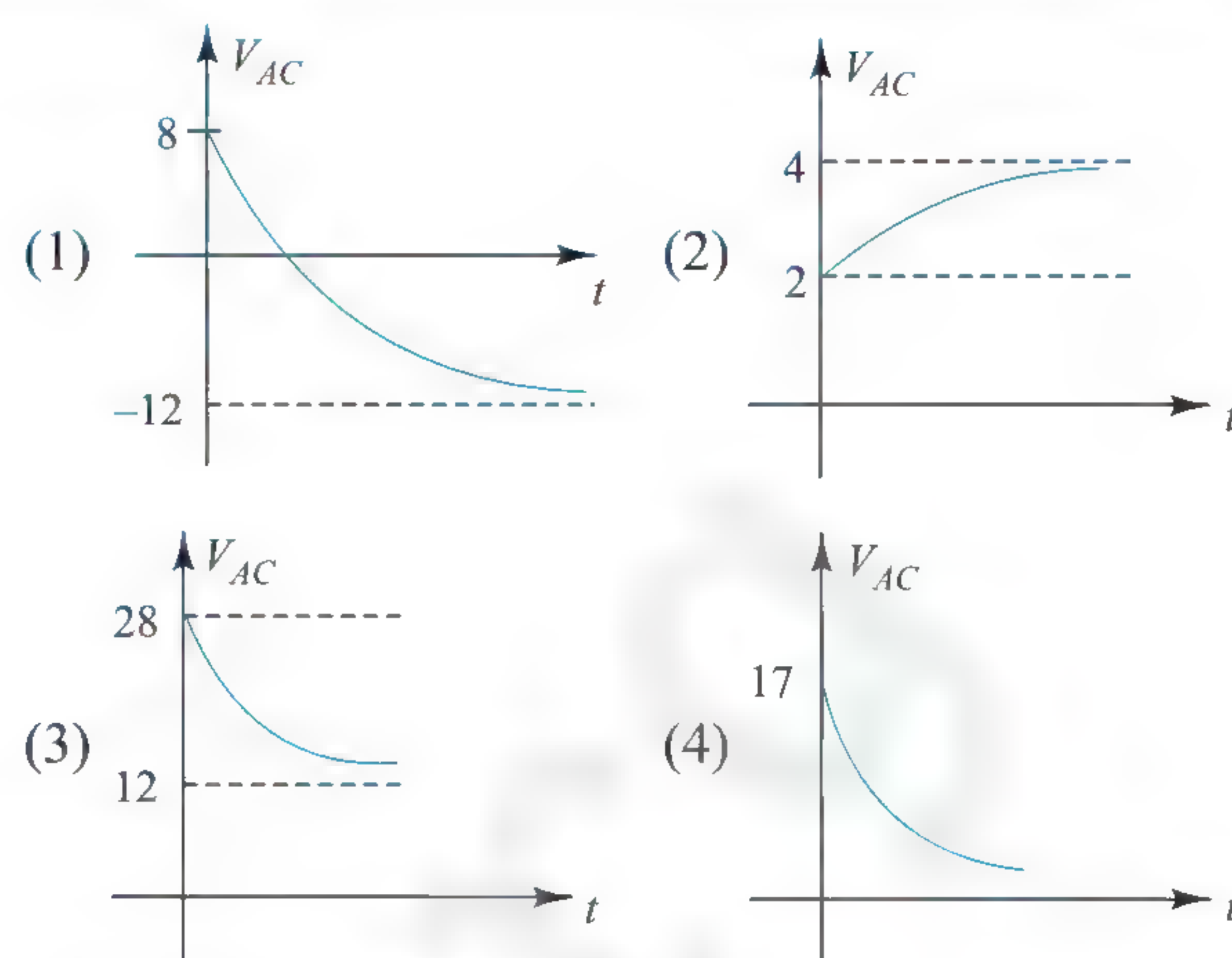


8. The potential difference across inductor V_L is

(1) $8e^{-2t} \text{ V}$ (2) $9e^{-2t} \text{ V}$

(3) $16e^{-2t} \text{ V}$ (4) $18e^{-2t} \text{ V}$

9. The variation of potential difference across A and C (V_{AC}) with time can be represented as

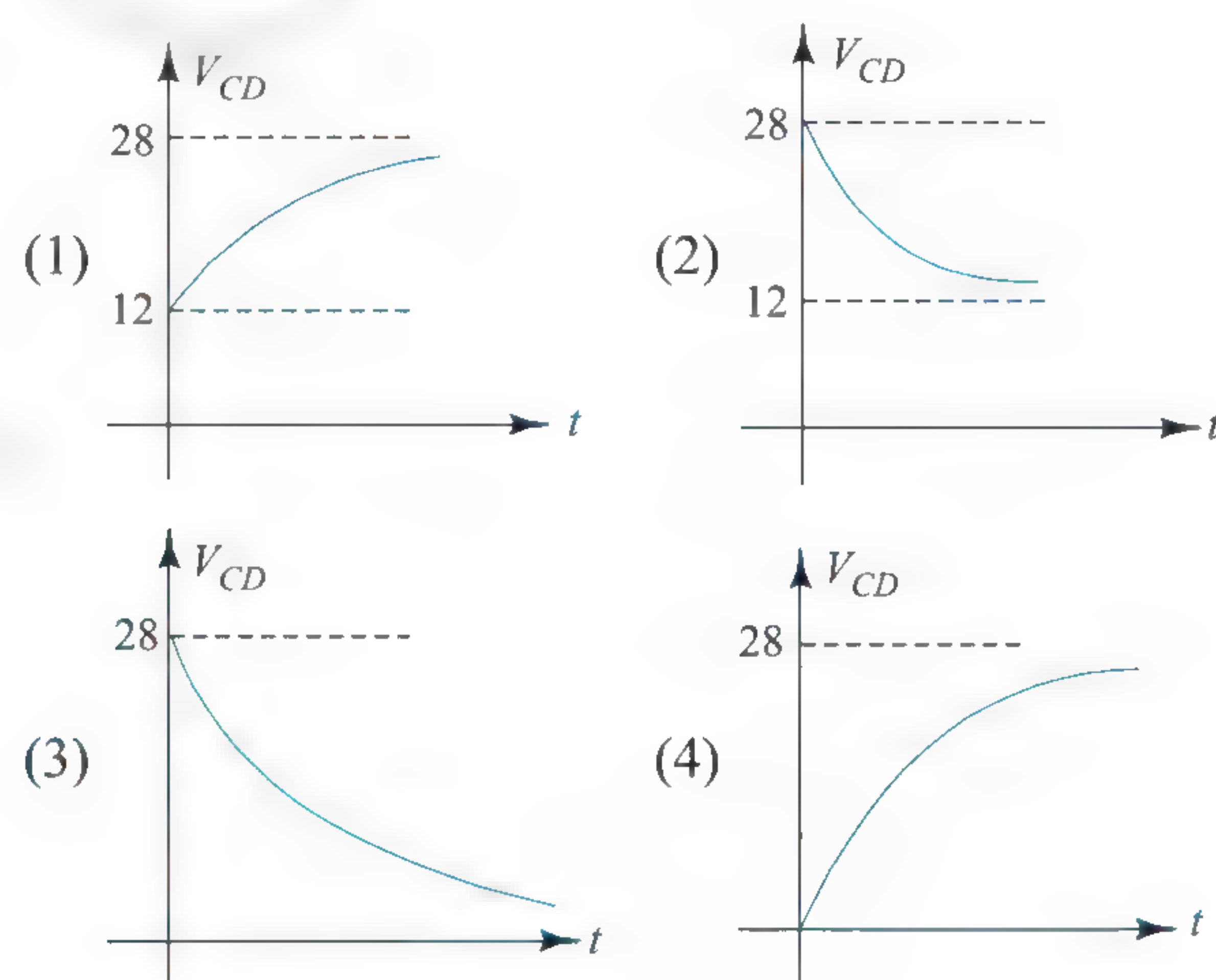


10. The potential difference across AB (V_{AB}) is

(1) $8e^{-2t} \text{ V}$ (2) $\frac{1}{2} e^{-3t} \text{ V}$

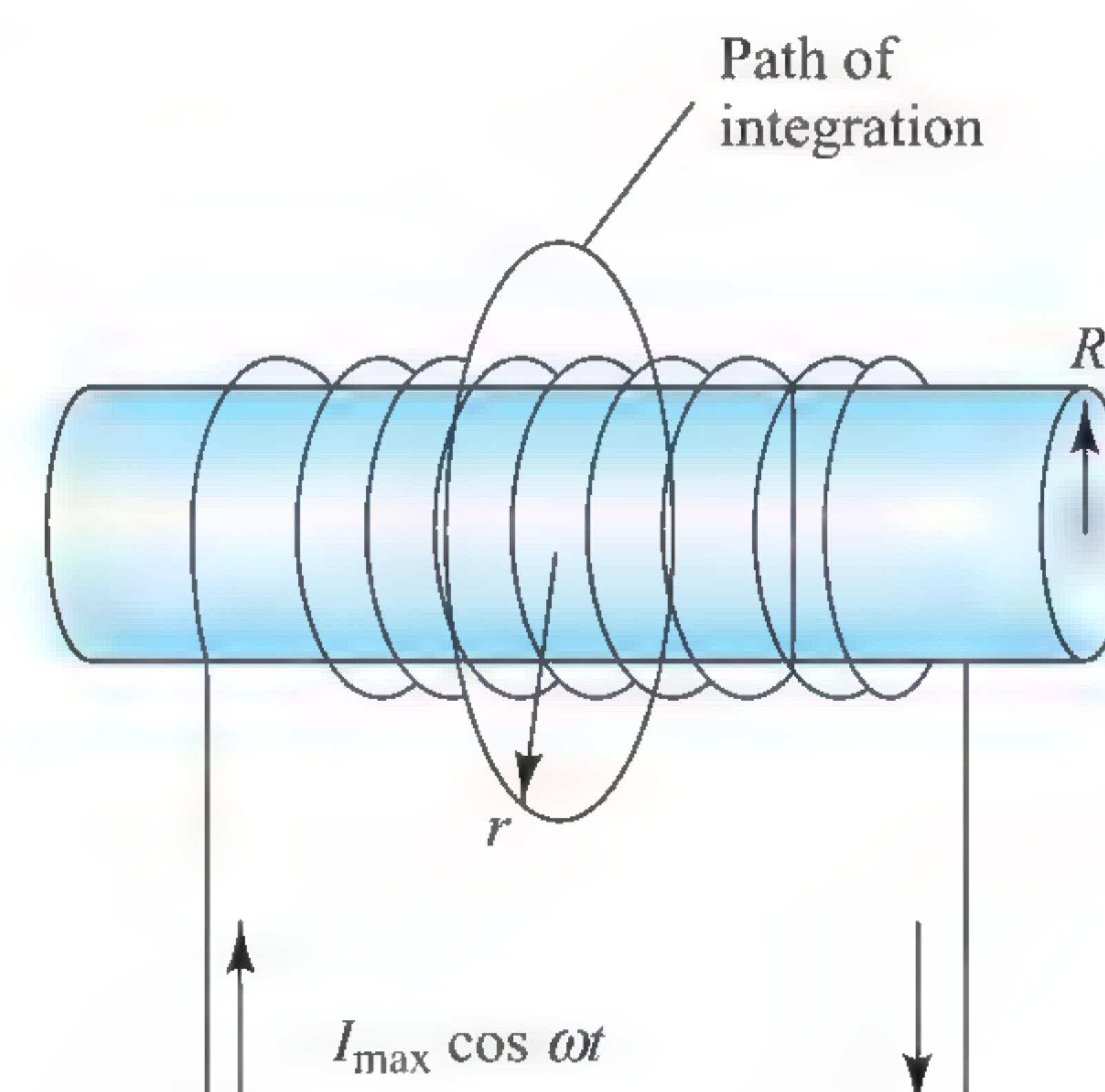
(3) $17e^{-2t} \text{ V}$ (4) $16e^{-2t} \text{ V}$

11. The variation of potential difference across C and D (V_{CD}) with time can be expressed as



For Problems 12–13

A long solenoid of radius R has n turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{\max} \cos \omega t$, where $I_{\max}$ is the maximum current and ω is the angular frequency of the alternating current source (shown in figure).



12. The magnitude of the induced electric field inside the solenoid, a distance $r < R$ from its long central axis is

$$(1) \frac{3\mu_0 n I_{\max} \omega}{2} r \sin \omega t \quad (2) \frac{\mu_0 n I_{\max} \omega}{2} r \cos \omega t$$

$$(3) \mu_0 n I_{\max} \omega r \sin \omega t \quad (4) \frac{\mu_0 n I_{\max} \omega}{2} r \sin \omega t$$

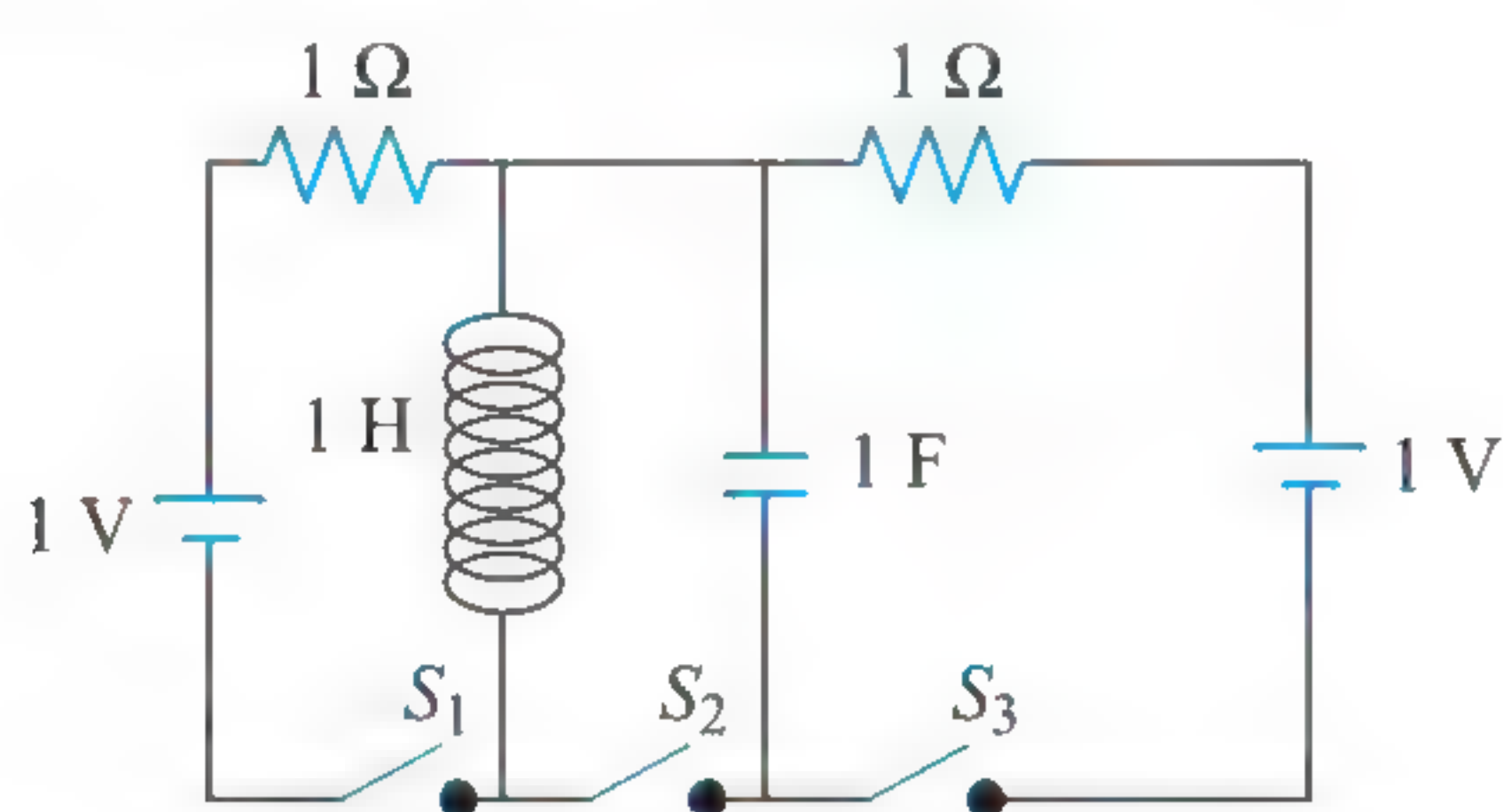
13. The magnitude of electric field outside the solenoid at a distance $r > R$ from its long central axis is

$$(1) \frac{\mu_0 n I_{\max} \omega R^2}{2r} \sin \omega t \quad (2) \frac{2\mu_0 n I_{\max} \omega R^2}{r} \sin \omega t$$

$$(3) \frac{\mu_0 n I_{\max} \omega R^2}{3r} \sin \omega t \quad (4) \frac{3\mu_0 n I_{\max} \omega R^2}{2r} \sin \omega t$$

For Problems 14–16

In the circuit shown (figure), switches S_1 and S_3 have been closed for 1 s and S_2 remained open. Just after 1 s, switch S_2 is closed and S_1 and S_3 are opened. Find after that instant ($t = 0$):



14. The maximum current in the circuit containing inductor and capacitor only (only S_2 is closed)

$$(1) \sqrt{3} \left(1 - \frac{1}{e}\right) \quad (2) \sqrt{2} \left(1 - \frac{1}{e}\right)$$

$$(3) \sqrt{3} \left(1 + \frac{1}{e}\right) \quad (4) \sqrt{2} \left(1 + \frac{1}{e}\right)$$

15. The maximum charge on the capacitor

$$(1) \sqrt{3} \left(1 + \frac{1}{e}\right) \quad (2) \sqrt{3} \left(1 - \frac{1}{e}\right)$$

$$(3) \sqrt{2} \left(1 + \frac{1}{e}\right) \quad (4) \sqrt{2} \left(1 - \frac{1}{e}\right)$$

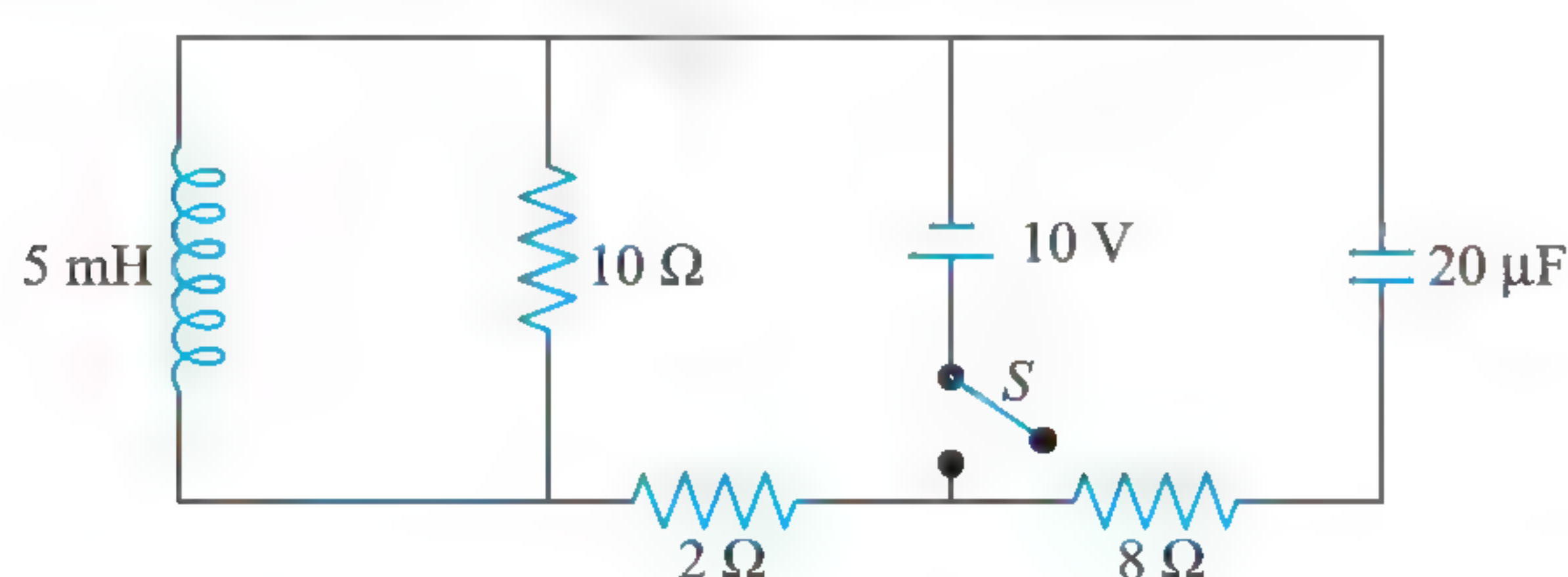
16. The charge on the upper plate of the capacitor as a function of time

$$(1) \sqrt{2} \left(1 - \frac{1}{e}\right) \sin \left(t + \frac{3\pi}{4}\right) \quad (2) \sqrt{2} \left(1 - \frac{1}{e}\right) \sin \left(t + \frac{\pi}{4}\right)$$

$$(3) \sqrt{3} \left(1 - \frac{1}{e}\right) \sin \left(t + \frac{\pi}{4}\right) \quad (4) \sqrt{3} \left(1 + \frac{1}{e}\right) \sin \left(t + \frac{\pi}{4}\right)$$

For Problems 17–19

In the given circuit at $t = 0$, switch S is closed.



17. The current through the 10Ω resistor at any instant t ($0 < t < \infty$) will be

$$(1) \frac{1}{6} e^{-(1000/3)t} \quad (2) \frac{5}{6} e^{-(1000/3)t}$$

$$(3) \frac{1}{6} e^{(1000/3)t} \quad (4) \frac{6}{5} e^{(1000/3)t}$$

18. The energy stored in the inductor at any instant t ($0 < t < \infty$) will be

$$(1) \frac{1}{2} [5 - 5e^{-(1000/3)t}]^2 \text{ mJ} \quad (2) \frac{125}{2} [1 - e^{-(1000/3)t}]^2 \text{ mJ}$$

$$(3) \frac{25}{2} [1 - e^{-(1000/3)t}]^2 \text{ mJ} \quad (4) \frac{5}{2} [1 - e^{-(1000/3)t}]^2 \text{ mJ}$$

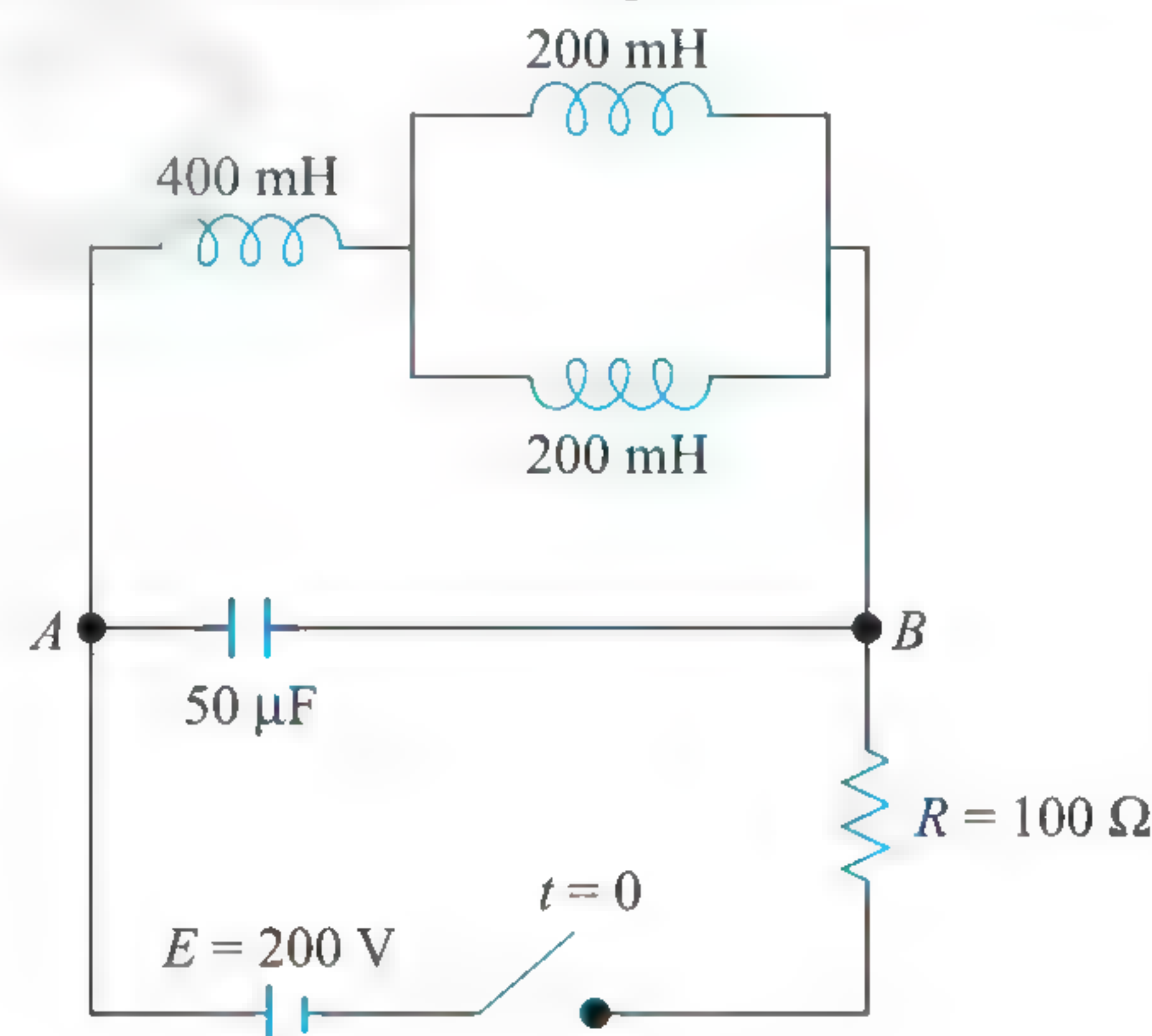
19. The energy stored in the capacitor and inductor, respectively, at $t \rightarrow \infty$ will be

$$(1) 1 \text{ mJ and } 62.5 \text{ mJ} \quad (2) 62.5 \text{ mJ and } 1 \text{ mJ}$$

$$(3) 2 \text{ mJ and } 62.5 \text{ mJ} \quad (4) 1 \text{ mJ and } 60 \text{ mJ}$$

For Problem 20–21

In the circuit shown in figure, switch S_1 was closed for a long time. At time $t = 0$ the switch is opened.



20. Find the maximum potential difference across the plates of the capacitor after the switch is opened.

$$(1) 100 \text{ V} \quad (2) 200 \text{ V}$$

$$(3) 300 \text{ V} \quad (4) 400 \text{ V}$$

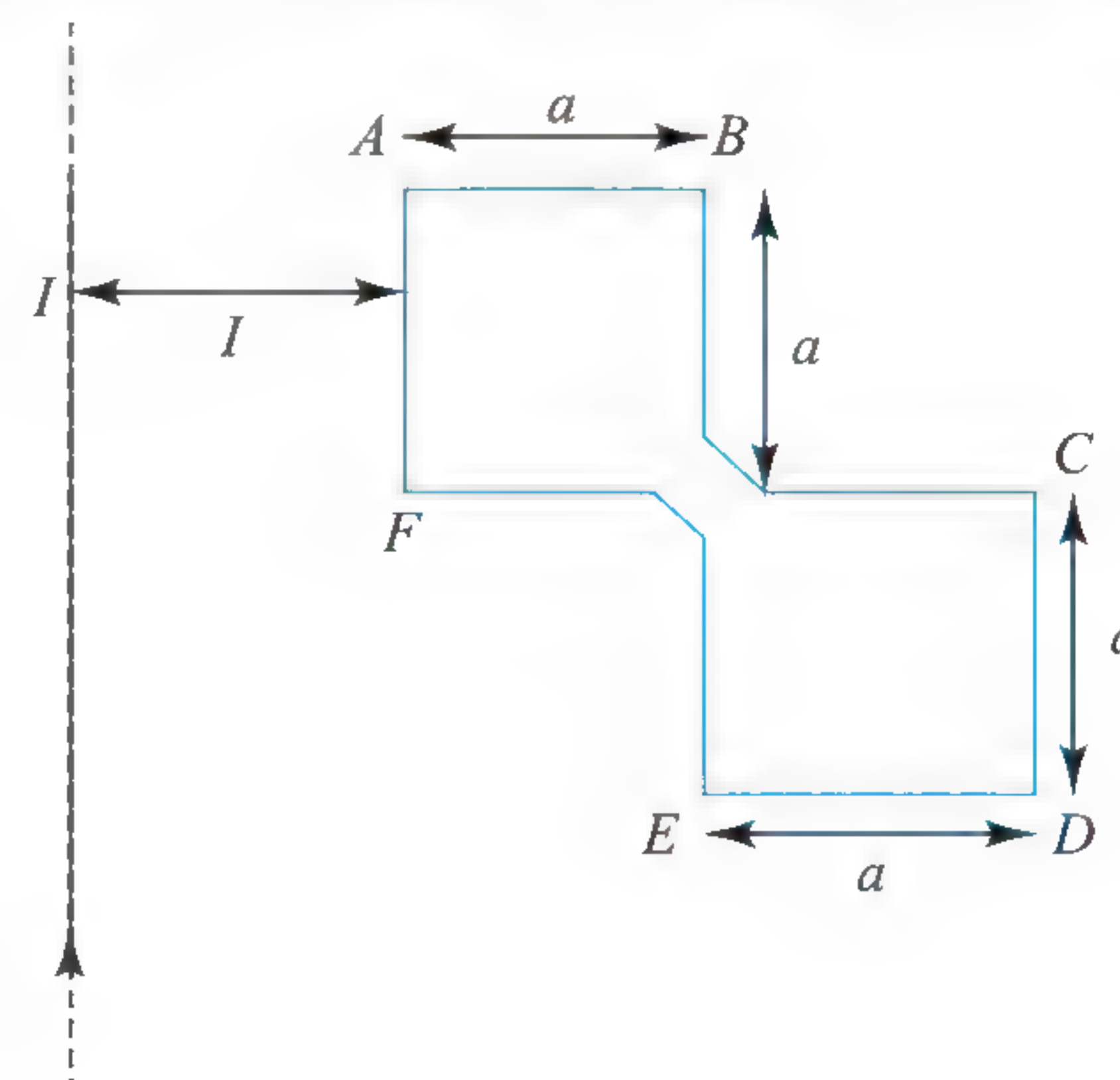
21. Find the angular frequency of oscillation of the charge on the capacitor.

$$(1) 100 \text{ rad s}^{-1} \quad (2) 200 \text{ rad s}^{-1}$$

$$(3) 300 \text{ rad s}^{-1} \quad (4) 400 \text{ rad s}^{-1}$$

For Problems 22–24

In figure, there is a conducting loop $ABCDEF$ of resistance λ per unit length placed near a long straight current-carrying wire. The dimensions are shown in the figure. The long wire lies in the plane of the loop. The current in the long wire varies as $I = I_0(t)$.



22. The mutual inductance of the pair is

- (1) $\frac{\mu_0 a}{2\pi} \ln\left(\frac{2a+l}{l}\right)$ (2) $\frac{\mu_0 a}{2\pi} \ln\left(\frac{2a-l}{l}\right)$
 (3) $\frac{2\mu_0 a}{\pi} \ln\left(\frac{a+l}{l}\right)$ (4) $\frac{\mu_0 a}{\pi} \ln\left(\frac{a+l}{l}\right)$

23. The emf induced in the closed loop is

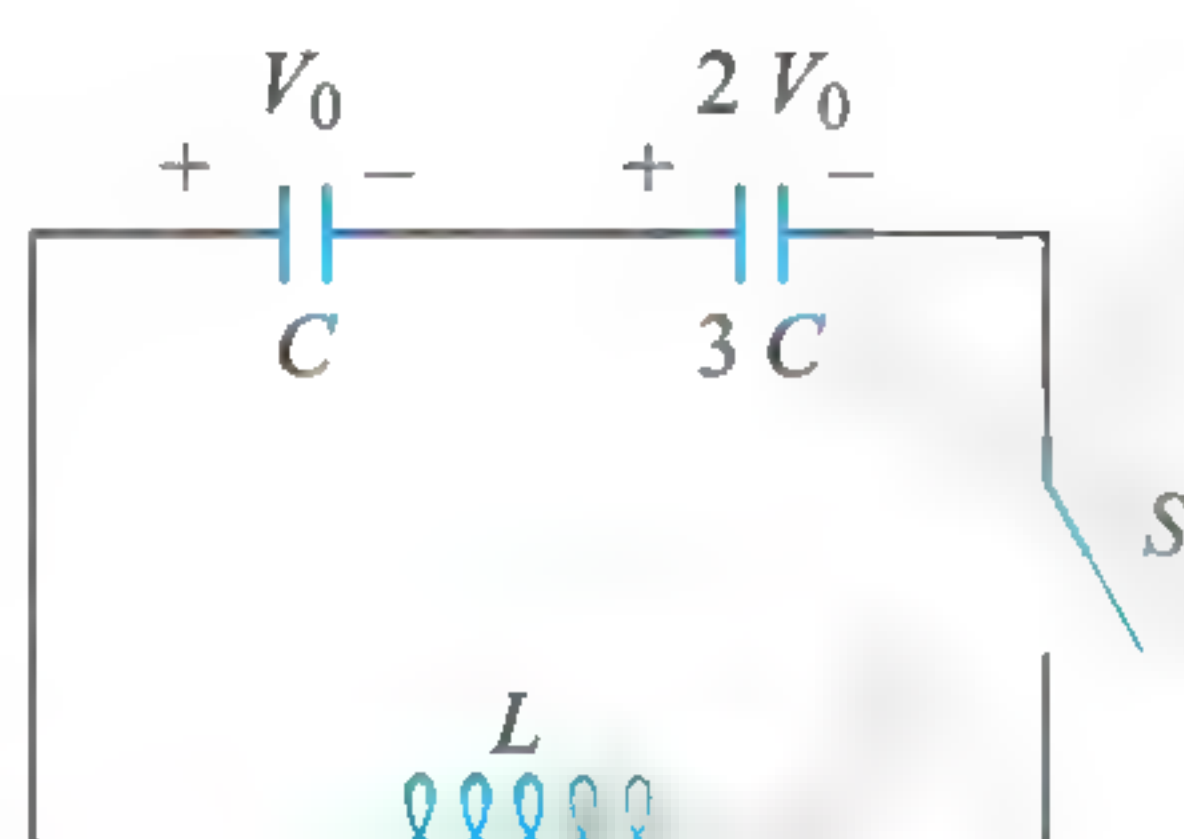
- (1) $\frac{\mu_0 I_0 a}{2\pi} \ln\left(\frac{2a+l}{l}\right)$ (2) $\frac{\mu_0 I_0 a}{2\pi} \ln\left(\frac{2a-l}{l}\right)$
 (3) $\frac{2\mu_0 I_0 a}{\pi} \ln\left(\frac{a+l}{l}\right)$ (4) $\frac{\mu_0 I_0 a}{\pi} \ln\left(\frac{a+l}{l}\right)$

24. The heat produced in the loop in time t is

- (1) $\frac{\left[\frac{\mu_0 I_0}{2\pi} \ln\left(\frac{a+l}{l}\right)\right]^2}{4\lambda} at$ (2) $\frac{\left[\frac{\mu_0 I_0}{2\pi} \ln\left(\frac{2a+l}{l}\right)\right]^2}{8\lambda} at$
 (3) $\frac{\left[\frac{2\mu_0 I_0}{\pi} \ln\left(\frac{a+l}{l}\right)\right]^2}{3\lambda} at$ (4) $\frac{\left[\frac{\mu_0 I_0}{2\pi} \ln\left(\frac{3a+l}{l}\right)\right]^2}{6\lambda} at$

For Problems 25–27

Two capacitors of capacitance C and $3C$ are charged to potential difference V_0 and $2V_0$, respectively, and connected to an inductor of inductance L as shown in figure. Initially, the current in the inductor is zero. Now, switch S is closed.



25. The maximum current in the inductor is

- (1) $\frac{3V_0}{2} \sqrt{\frac{3C}{L}}$ (2) $V_0 \sqrt{\frac{3C}{L}}$
 (3) $2V_0 \sqrt{\frac{3C}{L}}$ (4) $V_0 \sqrt{\frac{C}{L}}$

26. Potential difference across capacitor of capacitance C when the current in the circuit is maximum is

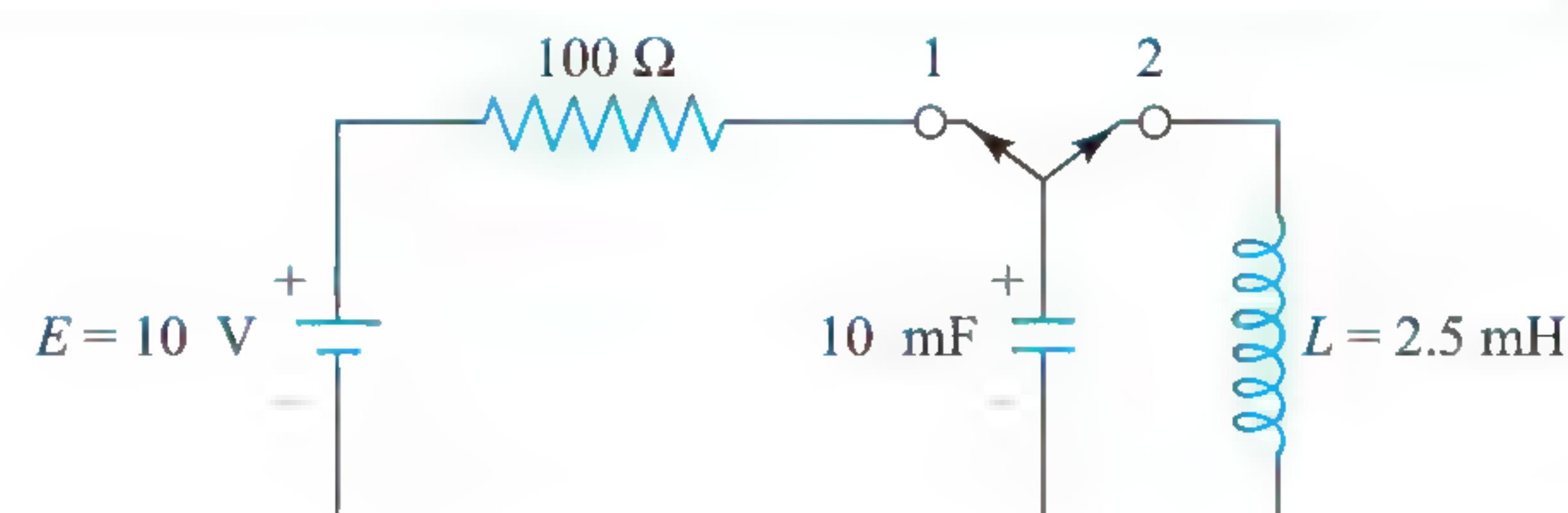
- (1) $\frac{V_0}{4}$ (2) $\frac{3V_0}{4}$
 (3) $\frac{5V_0}{4}$ (4) none of these

27. Potential difference across capacitor of capacitance $3C$ when the current in the circuit is maximum is

- (1) $\frac{V_0}{4}$ (2) $\frac{V_0}{4}$
 (3) $\frac{5V_0}{4}$ (4) none of these

For Problems 28–30

In figure, initially the capacitor is charged to a potential of 5 V and then connected to position 1 with the shown polarity for 1 s. After 1 s, it is connected across the inductor at position 2.



28. The potential across the capacitor after 1 s of its connection to position 1 is

- (1) $5 \times 10^3 \left(2 + \frac{1}{e}\right) \text{ V}$ (2) $5 \times 10^3 \left(2 - \frac{1}{e}\right) \text{ V}$
 (3) $5 \times 10^3 \left(1 + \frac{2}{e}\right) \text{ V}$ (4) none of these

29. The maximum current flowing in the LC circuit when the capacitor is connected across the inductor is

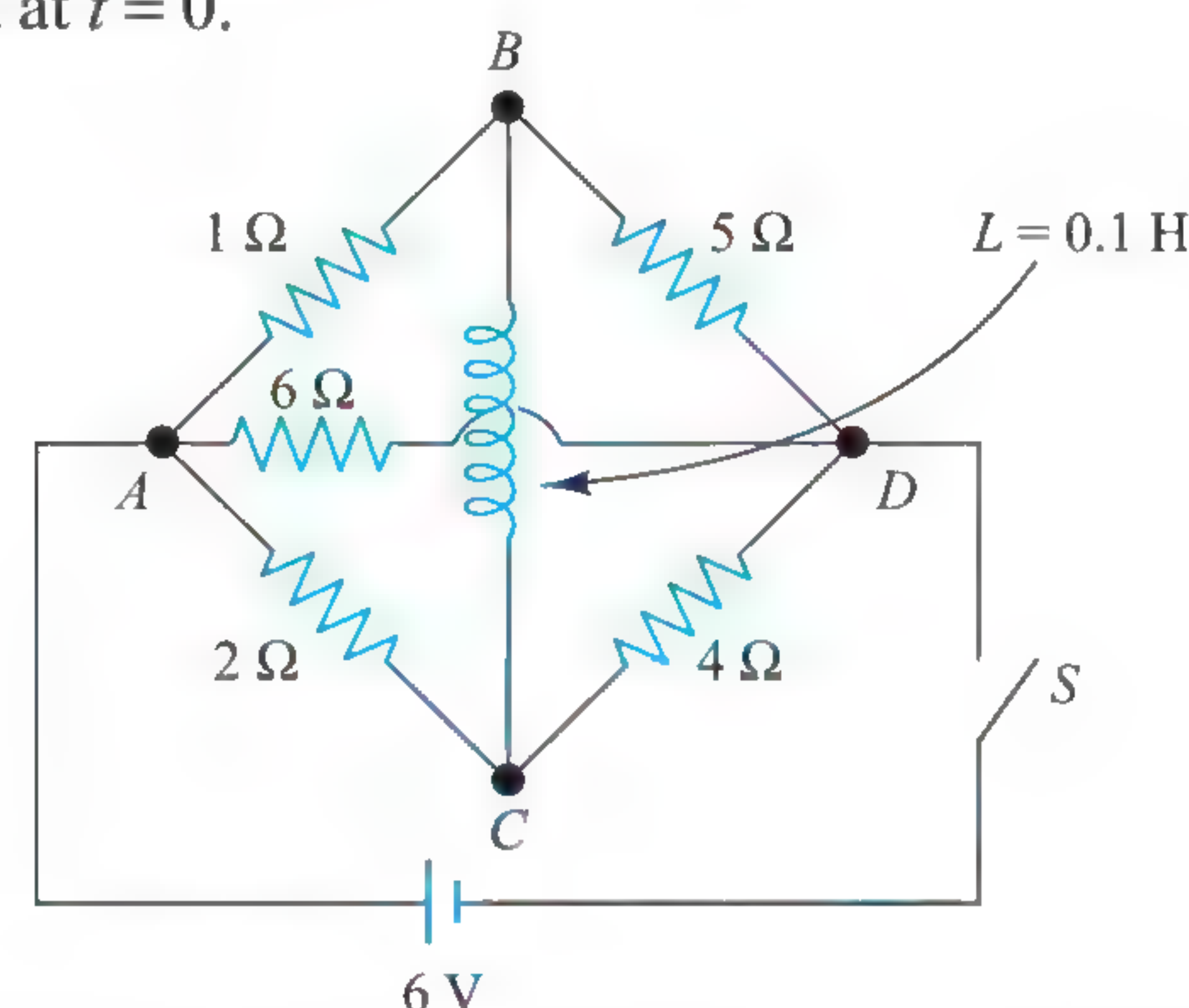
- (1) $\left(2 - \frac{1}{e}\right) \times 10^4 \text{ A}$ (2) $\left(1 + \frac{2}{e}\right) \times 10^4 \text{ A}$
 (3) $\left(1 - \frac{2}{e}\right) \times 10^4 \text{ A}$ (4) none of these

30. The frequency of LC oscillations is

- (1) $(20/\pi) \text{ Hz}$ (2) $(2/\pi) \text{ Hz}$
 (3) $(40/\pi) \text{ Hz}$ (4) $100/\pi \text{ Hz}$

For Problems 31–33

There is no current in any part of this circuit for time $t < 0$. Switch S is closed at $t = 0$.



31. The rate at which the current through the inductor increases initially is

- (1) zero (2) 10 A s^{-1}
 (3) 1 A s^{-1} (4) 5 A s^{-1}

32. Current through the 6Ω resistor

- (1) increases linearly with time
 (2) increases non-linearly with time
 (3) decreases non-linearly with time
 (4) remains constant

33. The current through the inductor after a long time will be

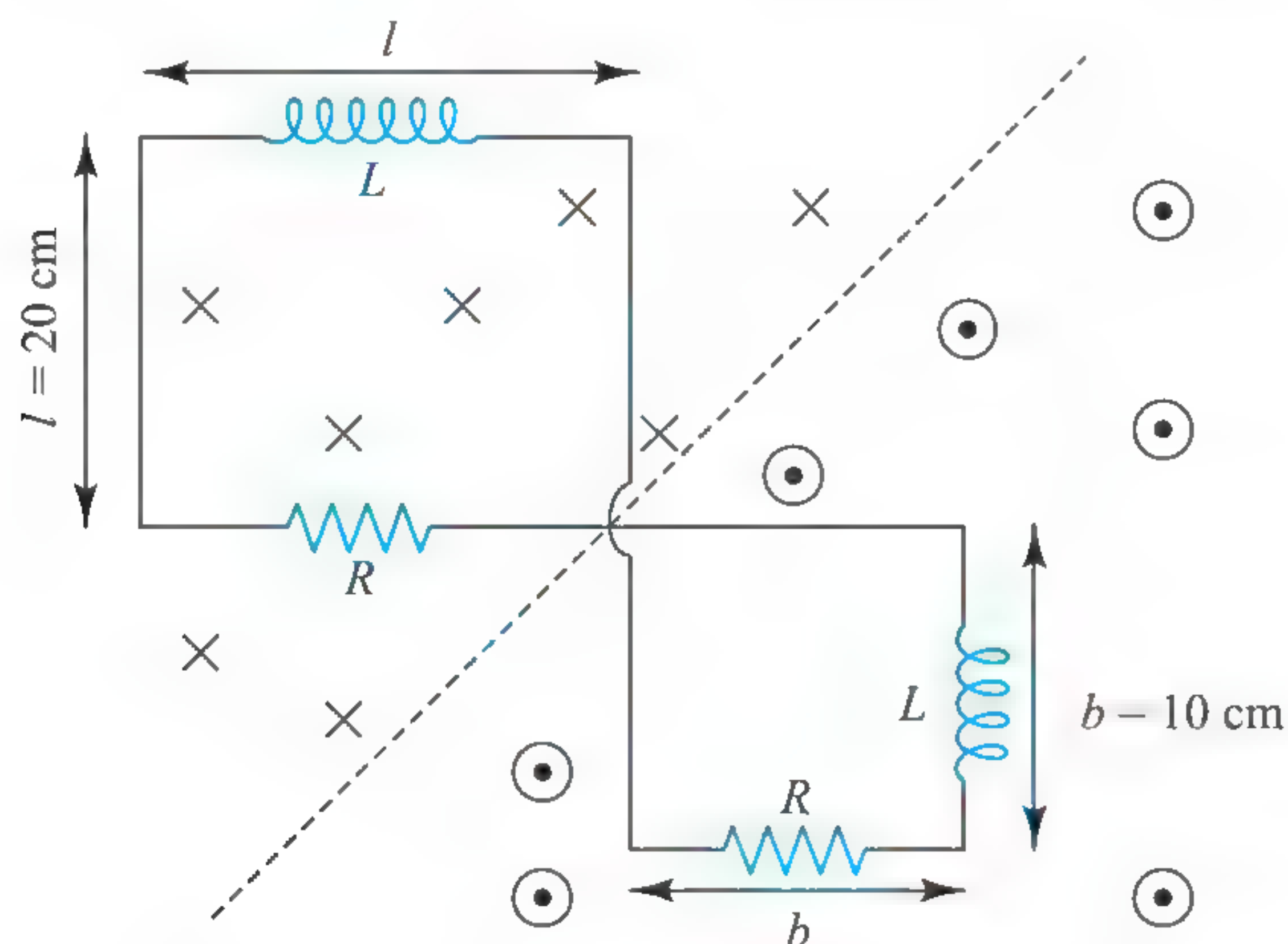
- (1) zero (2) infinite
 (3) $\frac{6}{13} \text{ A}$ (4) none of these

For Problems 34–36

In figure, there is a frame consisting of two square loops having resistors and inductors as shown. This frame is placed in a

uniform but time-varying magnetic field in such a way that one of the loops is placed in crossed magnetic field and the other is placed in dot magnetic field. Both magnetic fields are perpendicular to the planes of the loops.

If the magnetic field is given by $B = (20 + 10t) \text{ Wb m}^{-2}$ in both regions [$l = 20 \text{ cm}$, $b = 10 \text{ cm}$, and $R = 10 \Omega$, $L = 10 \text{ H}$],



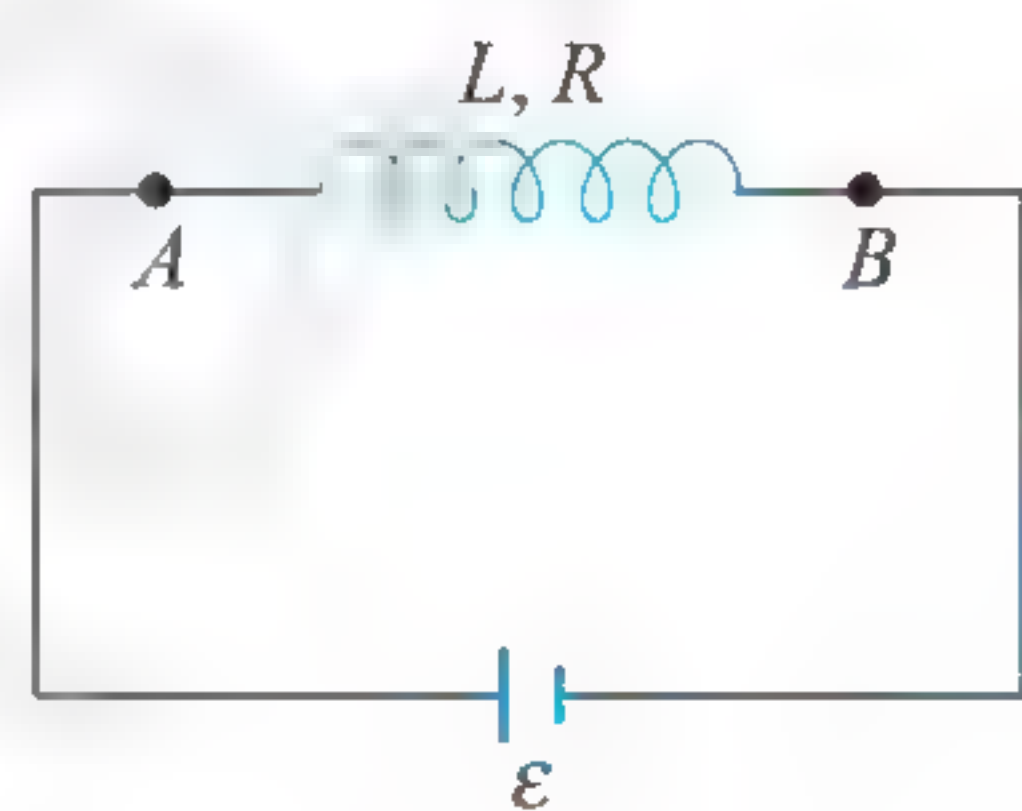
34. The direction of induced current in the bigger loop will be
 (1) clockwise
 (2) anticlockwise
 (3) first clockwise for some time, then anticlockwise, and so on
 (4) first anticlockwise for some time, then clockwise, and so on
35. The induced emf in the frame only due to the variation of magnetic field will be
 (1) 0.3 V
 (2) 0.1 V
 (3) 0.5 V
 (4) 0.4 V

36. The current in the frame as a function of time will be

- (1) $\frac{1}{20} (1 - e^{-t})$
 (2) $\frac{1}{40} (1 - e^{-t})$
 (3) $\frac{1}{20} e^{-t}$
 (4) $\frac{1}{10} e^{-t}$

For Problems 37–39

An inductor having self inductance L with its coil resistance R is connected across a battery of emf $\mathcal{E}$. When the circuit is in steady state at $t = 0$, an iron rod is inserted into the inductor due to which its inductance becomes nL ($n > 1$).



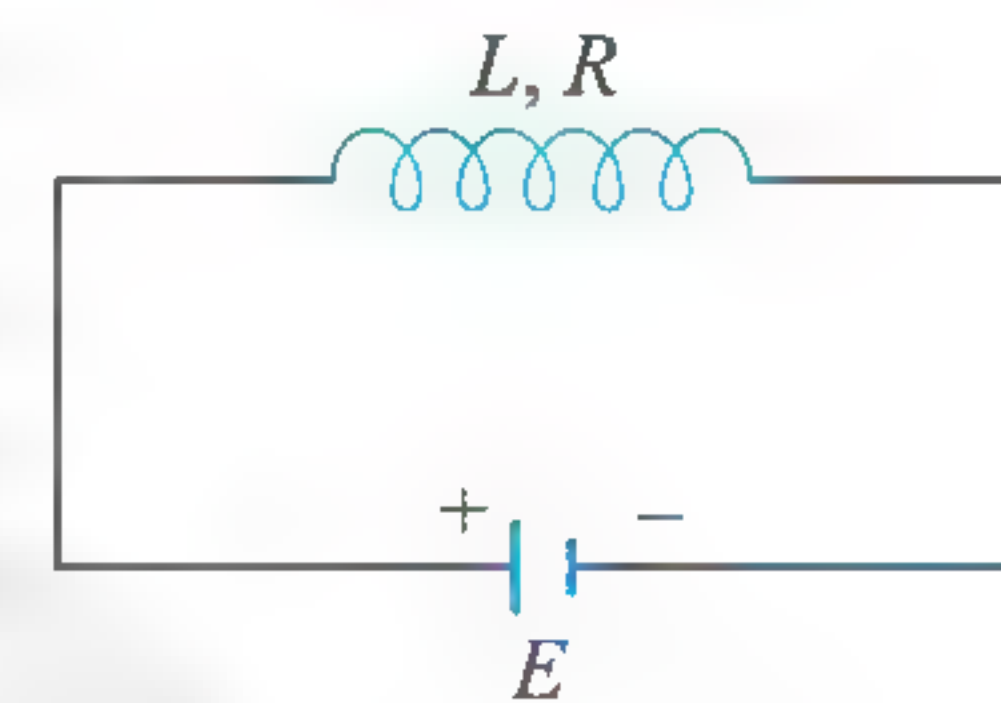
37. After insertion of rod which of the following quantities will change with time?
 (a) Potential difference across terminals A and B
 (b) Inductance
 (c) Rate of heat produced in coil
 (1) only (a)
 (2) (a) and (c)
 (3) only (c)
 (4) (a), (b) and (c)
38. After insertion of rod, current in the circuit:
 (1) Increases with time
 (2) Decreases with time
 (3) Remains constant with time
 (4) First decreases with time then becomes constant

39. When again circuit is in steady state, the current in it is

- (1) $I < \mathcal{E}/R$
 (2) $I > \mathcal{E}/R$
 (3) $I = \mathcal{E}/R$
 (4) None of these

For Problems 40–42

A solenoid of resistance R and inductance L has a piece of soft iron inside it. A battery of emf E and of negligible internal resistance is connected across the solenoid as shown in figure. At any instant, the piece of soft iron is pulled out suddenly so that inductance of the solenoid decreases to ηL ($\eta < 1$) with battery remaining connected.



40. The work done to pull out the soft iron piece is

- (1) $\frac{\eta L E^2}{2R^2}$
 (2) $\frac{(1-\eta) L E^2}{2R^2}$
 (3) $\frac{(1-\eta) L E^2}{\eta R^2}$
 (4) $\frac{(1-\eta) L E^2}{2\eta R^2}$

41. Assume $t = 0$ is the instant when iron piece has been pulled out, the current as a function of time after this is

- (1) $i = \frac{E}{R} \left[1 - \left(1 - \frac{1}{\eta} \right) e^{-t/\lambda} \right]$
 (2) $i = \frac{E}{R} \left[1 + \left(1 + \frac{1}{\eta} \right) e^{-t/\lambda} \right]$
 (3) $i = \frac{E}{R} \left[1 - \left(1 + \frac{1}{\eta} \right) e^{-t/\lambda} \right]$
 (4) $i = \frac{E}{R} \left[1 + \left(1 - \frac{1}{\eta} \right) e^{-t/\lambda} \right]$

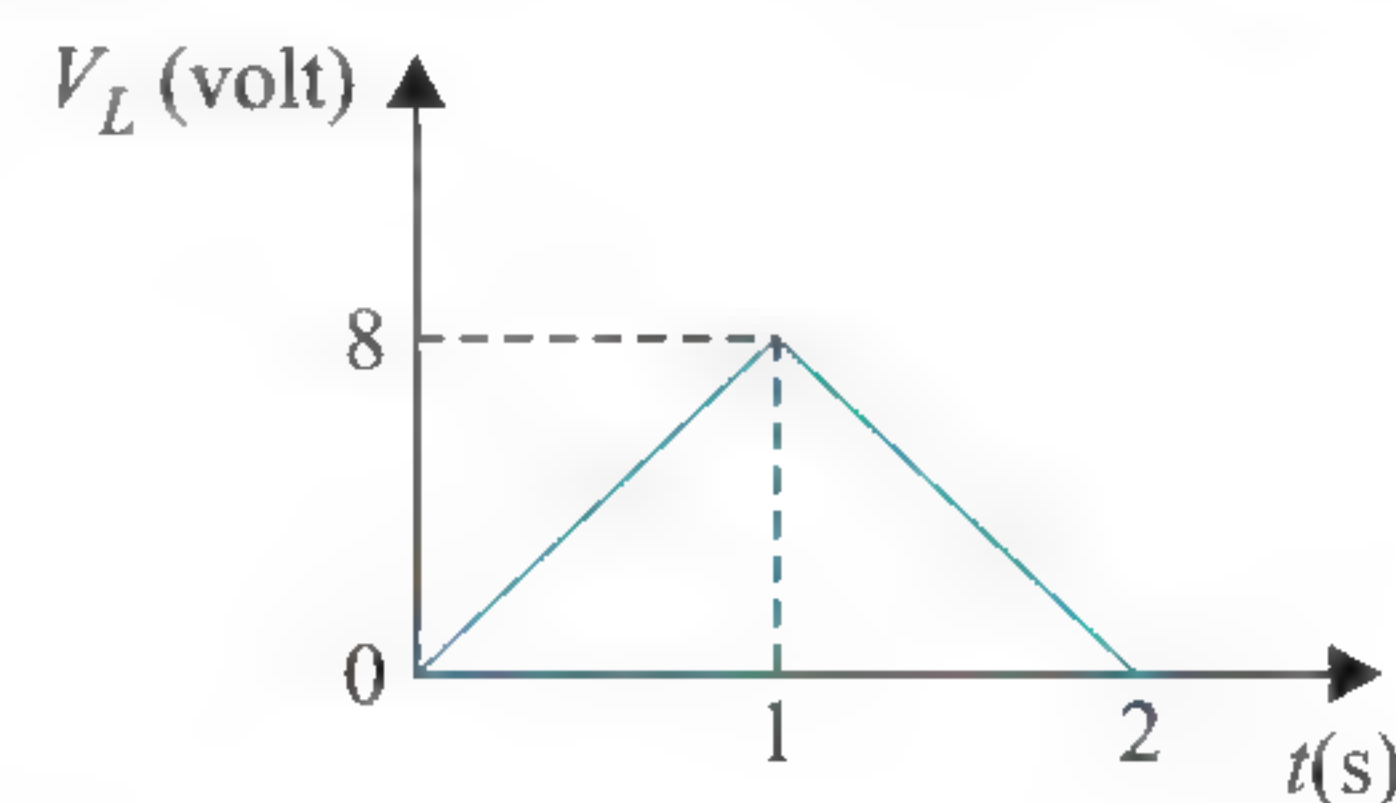
(where $\lambda = \frac{\eta L}{R}$)

42. Power supplied by the battery as a function of time

- (1) $P = \frac{E^2}{R} \left[1 + \left(1 - \frac{1}{\eta} \right) e^{-t/\lambda} \right]$
 (2) $P = \frac{E^2}{R} \left[1 + \left(1 + \frac{1}{\eta} \right) e^{-t/\lambda} \right]$
 (3) $P = \frac{E^2}{R} \left[1 - \left(1 + \frac{1}{\eta} \right) e^{-t/\lambda} \right]$
 (4) $P = \frac{E^2}{R} \left[1 - \left(1 - \frac{1}{\eta} \right) e^{-t/\lambda} \right]$

For Problems 43–45

The potential difference across a 2-H inductor as a function of time is shown in figure. At time $t = 0$, current is zero.



43. Area under V_L - t graph gives

- (1) current
 (2) change in current

- (3) product of inductance and current
 (4) product of inductance and change in current

44. Current at $t = 1$ s is

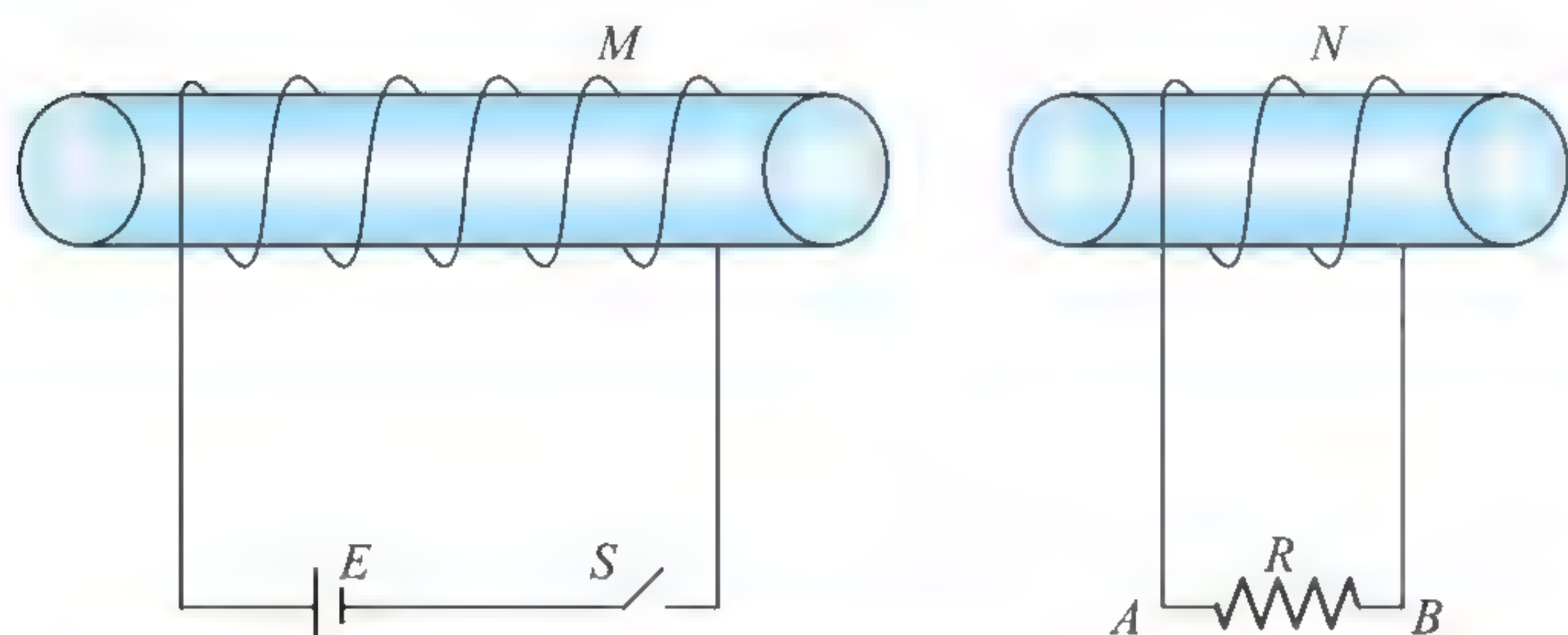
- (1) 1 A (2) 2 A
 (3) 4 A (4) 8 A

45. The variation of current versus time is

- (1) a straight line (2) a parabola
 (3) a hyperbola (4) exponential

Matrix Match Type

1. Figure shows two coaxial coils M and N . Column I is regarding some operations done with coil M and column II about induced current in coil N .



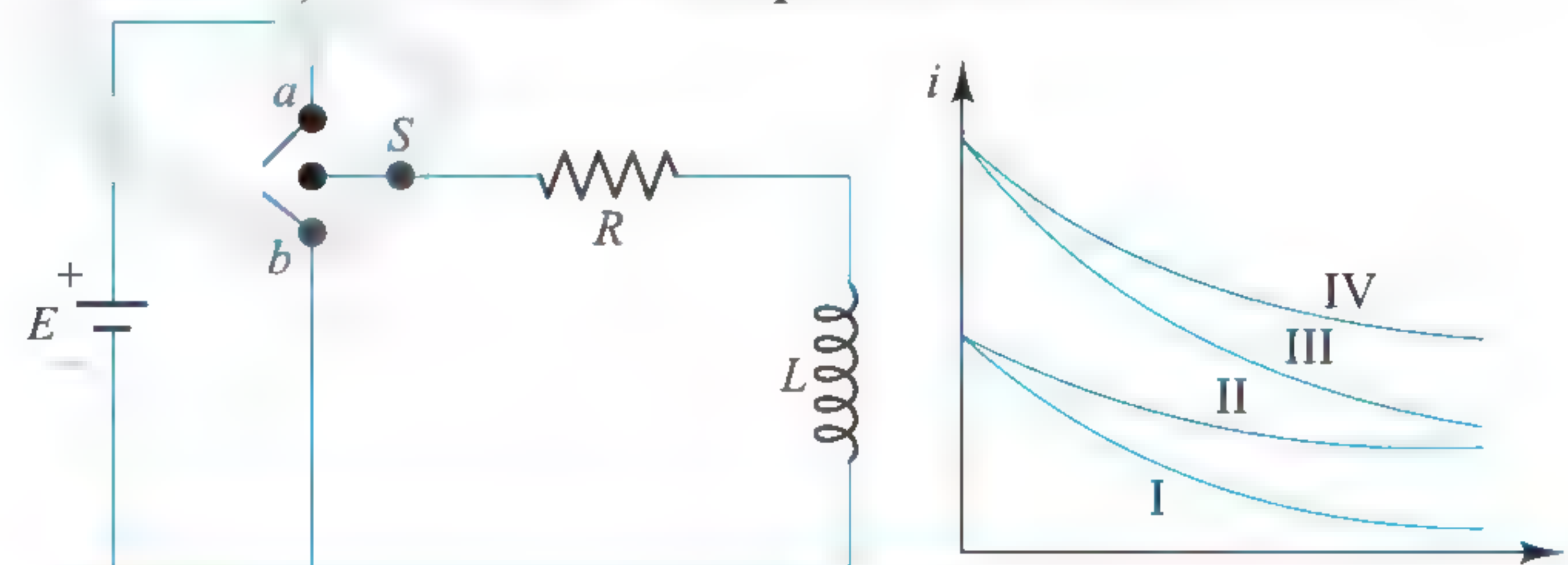
Column I	Column II
i. Just after switch S is closed	a. Current is induced from A to B
ii. Switch S is opened after keeping it closed for a long time.	b. Current is induced from B to A
iii. After switch S is closed for a long time	c. No current is induced
iv. Just after switch S is closed while moving M away from N	d. the wire will attract or repel the loop.

2. In column I, some circuits are given. In all the circuits except in (i), switch S remains closed for a long time and then it is opened at $t = 0$; while for (i), the situation is reversed. Column II tells something about the circuit quantities. Match the entries of column I with the entries of column II.

Column I	Column II
i.	a. Induced emf can be greater than E .
ii.	b. Induced emf would be less than E .

iii.	c. Finally, energy stored in inductor is zero.
iv.	d. Finally, energy stored in inductor is non-zero.

3. The switch S in the circuit is connected with point a for a very long time, then it is shifted to position b . The resulting current through the inductor is shown by curves in the graph for four sets of values for the resistance R and inductance L (given in column 1). Which set corresponds with which curve?



Column I	Column II
i. R_0 and L_0	a. I
ii. $2R_0$ and L_0	b. II
iii. R_0 and $2L_0$	c. III
iv. $2R_0$ and $2L_0$	d. IV

4. Column I shows some ac circuits. Column II gives some possibilities in the circuit. Match appropriately.

Column I	Column II
i.	a. $ \cos \phi \neq 1$ (ϕ is the phase difference between current and emf)
ii.	b. Current leads the emf
iii.	c. Current lags the emf
iv.	d. Average power dissipated is zero

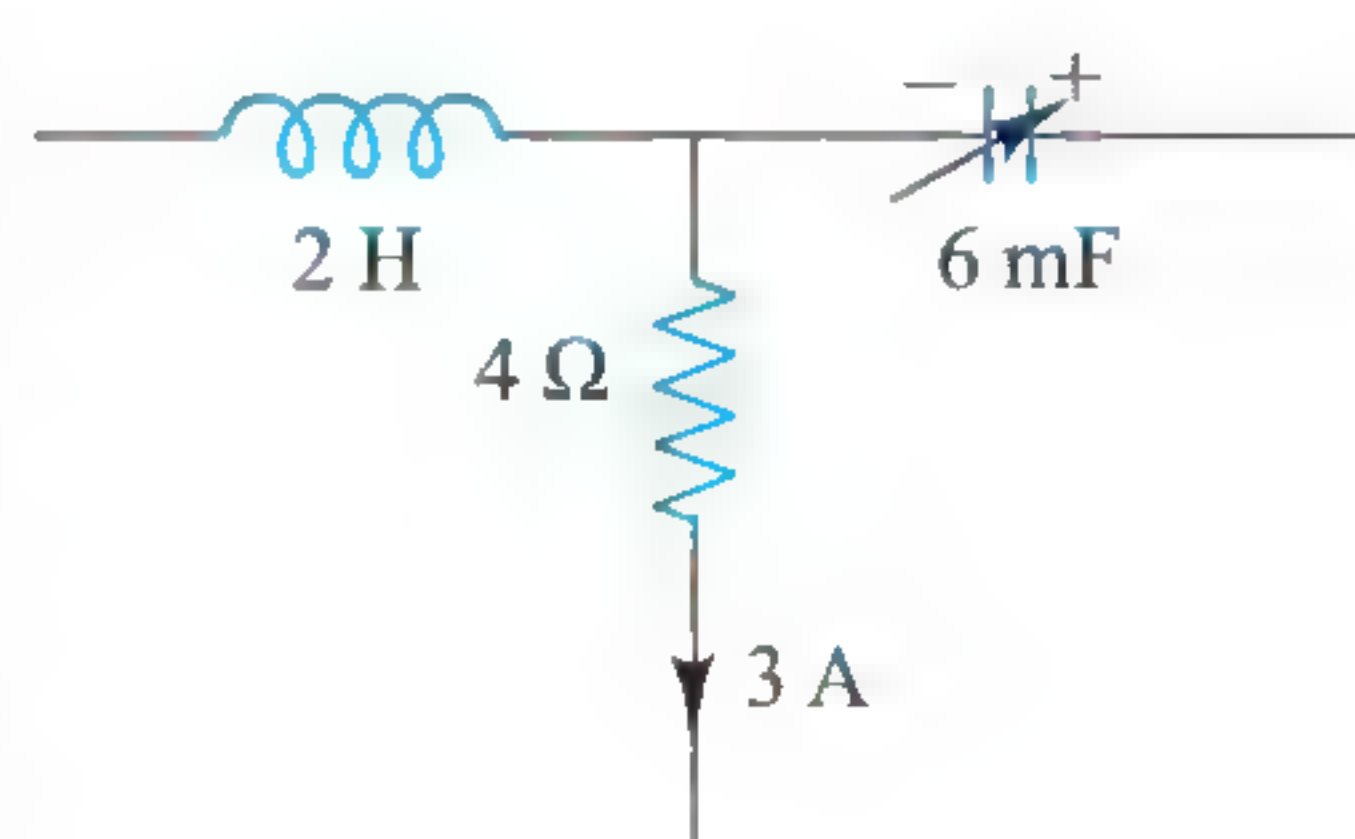
Numerical Value Type

1. In the circuit (figure) what is potential difference $V_B - V_A$ (in V) when current I is 5 A and is decreasing at the rate of 10^3 A s^{-1} ?



2. A current of 2 A is increasing at a rate of 4 A s^{-1} through a coil of inductance 1 H. Find the energy stored in the inductor per unit time in the units of J s^{-1} .
3. Two coils, 1 and 2 have a mutual inductance $M = 5 \text{ H}$ and resistance $R = 10 \Omega$ each. A current flows in coil 1, which varies with time as: $I_1 = 2t^2$, where t is time. Find the total charge (in C) that has flown through coil 2 between $t = 0$ and $t = 2 \text{ s}$.

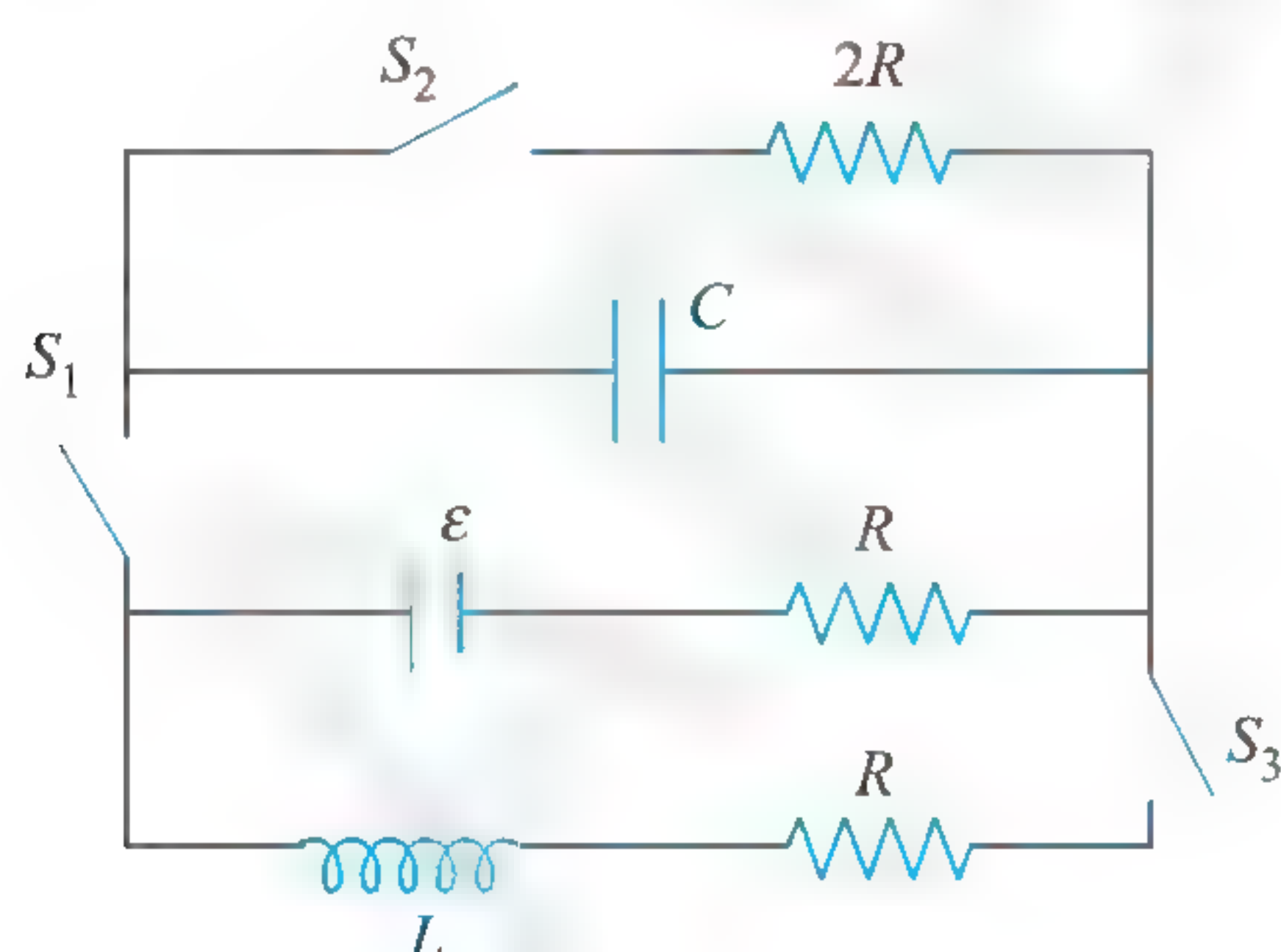
4. Figure shows a part of a bigger circuit. The capacity of the capacitor is 6 mF and is decreasing at the constant rate 0.5 mF s^{-1} . The potential difference across the capacitor at the shown moment is changing as follows:



$$\frac{dV}{dt} = 2 \text{ V s}^{-1}, \frac{d^2V}{dt^2} = \frac{1}{2} \text{ V s}^{-2}$$

The current in the 4Ω resistor is decreasing at the rate of 1 mA s^{-1} . What is the potential difference (in mV) across the inductor at this moment?

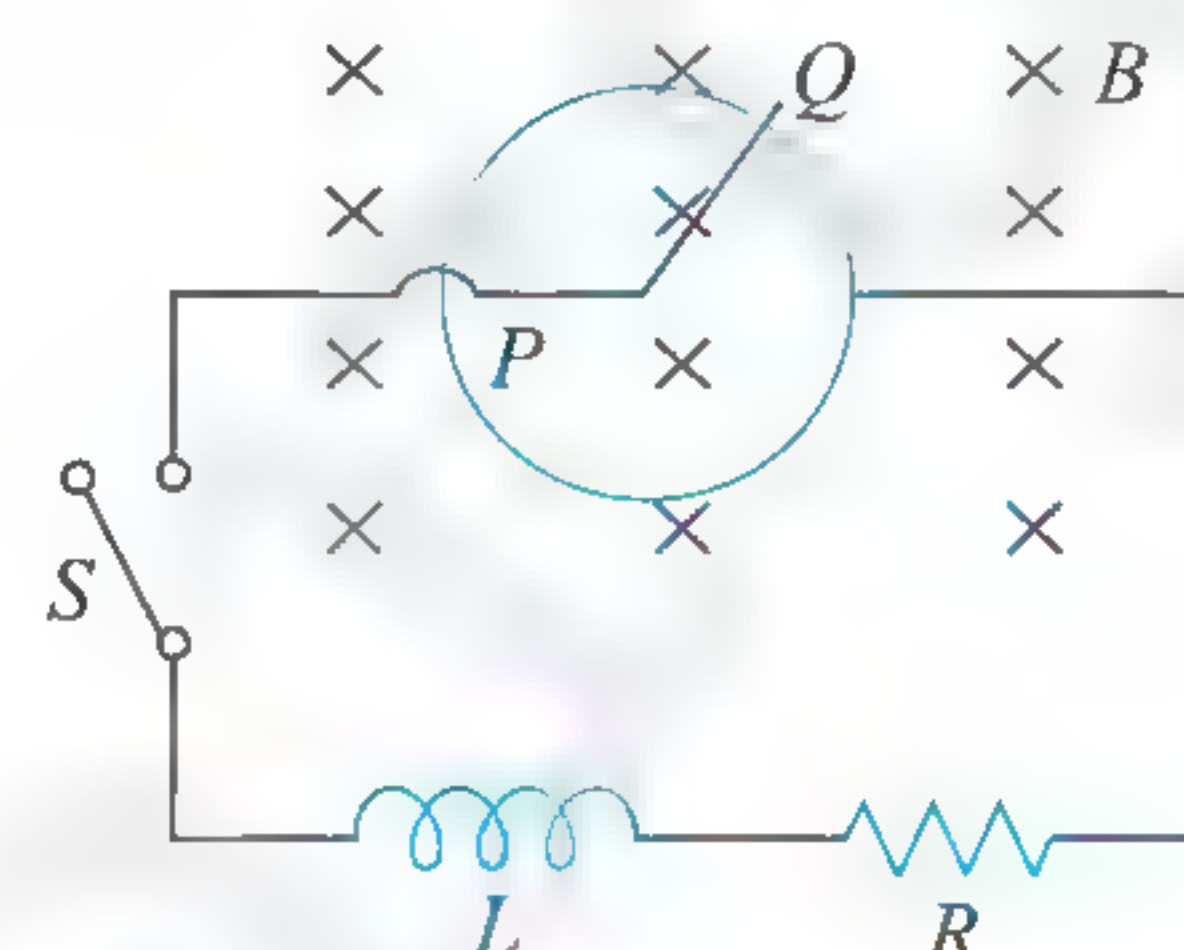
5. In the given circuit, initially switch S_1 is closed, and S_2 and S_3 are open. After charging of capacitor, at $t = 0$, S_1 is opened and S_2 and S_3 are closed. If the relation between inductance, capacitance and resistance is $L = 4CR^2$, then find the time (in s) after which current passing through capacitor and inductor will be same (given $R = \ln 2 \text{ m}\Omega$, $L = 2 \text{ mH}$)



6. A long solenoid of diameter 0.1 m has 2×10^4 turns per metre. At the center of the solenoid, a 100-turn coil of radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid is decreased at a constant rate from +2 A to -2 A in 0.05 s. Find the total charge (in μC) flowing through the coil during this time when the resistance of the coil is $40 \pi^2 \Omega$.

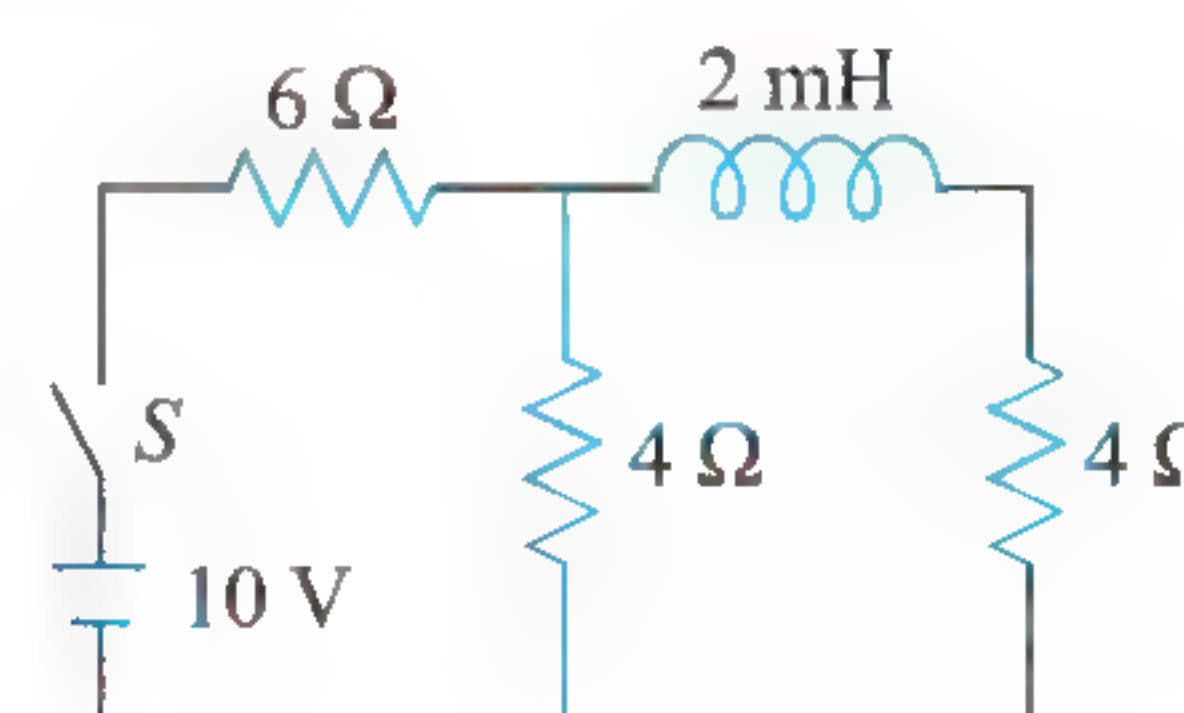
7. A capacitor of capacity $2 \mu\text{F}$ is charged to a potential difference of 12 V. It is then connected across an inductor of inductance $6 \mu\text{H}$. What is the current (in A) in the circuit at a time when the potential difference across the capacitor is 6.0 V?

8. Figure shows a circuit having a coil of resistance $R = 2.5 \Omega$ and inductance L connected to a conducting rod PQ which can slide on a perfectly conducting circular ring of radius 10 cm with its center at P .



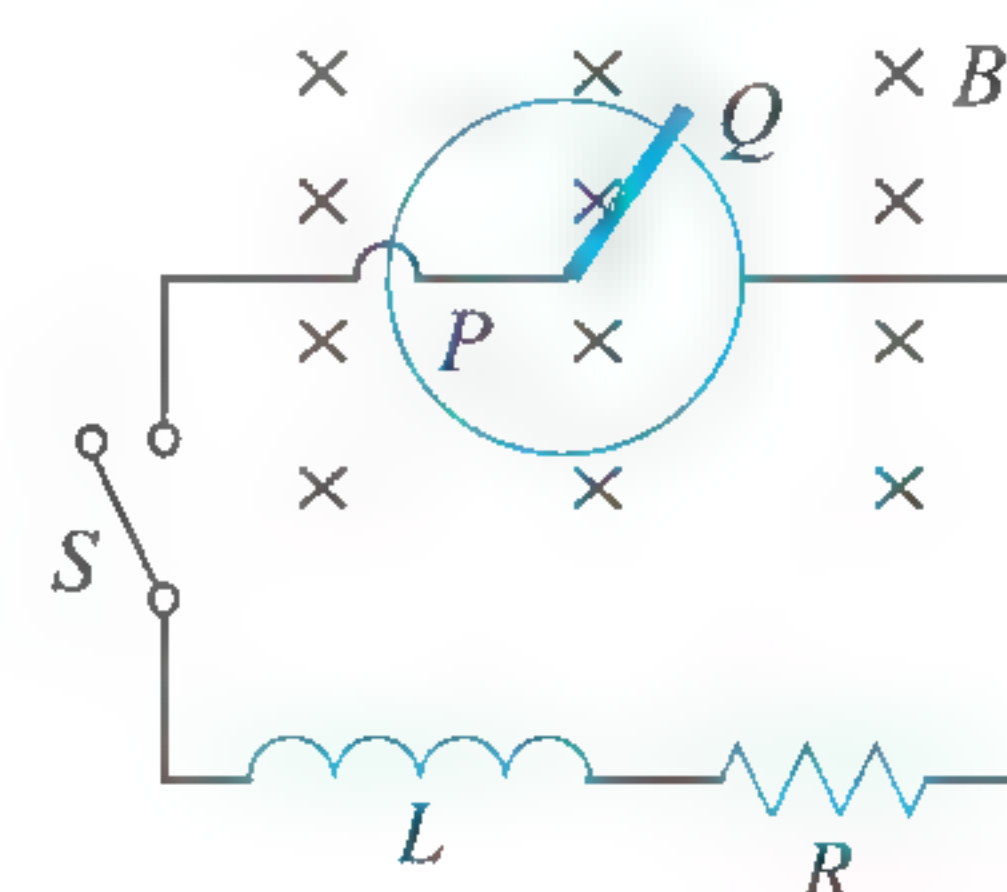
Assume that friction and gravity are absent and a constant uniform magnetic field of 5 T exists as shown in figure. At $t = 0$, the circuit is switched on and simultaneously a time-varying external torque is applied on the rod so that it rotates about P with a constant angular velocity 40 rad s^{-1} . Find the magnitude of this torque (in mNm) when current reaches half of its maximum value. Neglect the self inductance of the loop formed by the circuit.

9. In the given circuit, what is the current I (in A) drawn from battery at time $t = 0$.



10. At any instant a current of 2 A is increasing at a rate of 1 A/s through a coil of inductance 2 H. Find the energy (in SI units) being stored in the inductor per unit time at that instant.

11. The diagram shows a circuit having a coil of resistance $R = 2.5 \Omega$ and inductance L connected to a conducting rod PQ which can slide on a perfectly conducting circular ring of radius 10 cm with its centre at ' P '. Assume that friction and gravity are absent and a constant uniform magnetic field of 5 T exists as shown in figure.



At $t = 0$, the circuit is switched on and simultaneously a time varying external torque is applied on the rod so that it rotates about P with a constant angular velocity 40 rad/s . Find magnitude of this torque (in milli Nm) when current reaches half of its maximum value. Neglect the self-inductance of the loop formed by the circuit.

12. A certain volume of copper is drawn into a wire of radius a and is wrapped in shape of a helix having radius r ($\gg a$). The windings are as close as possible without overlapping. Self-inductance of the inductor so obtained is L_1 . Another

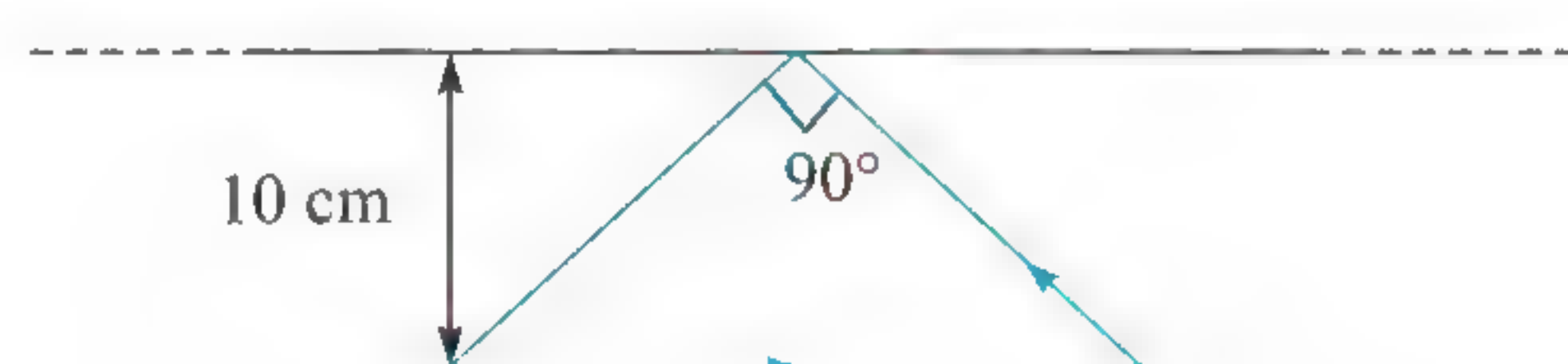
wire of radius $2a$ is drawn using same volume of copper and wound in the fashion as described above. This time the inductance is L_2 . Find $\frac{L_1}{L_2}$.

Archives

JEE ADVANCED

Multiple Correct Answers Type

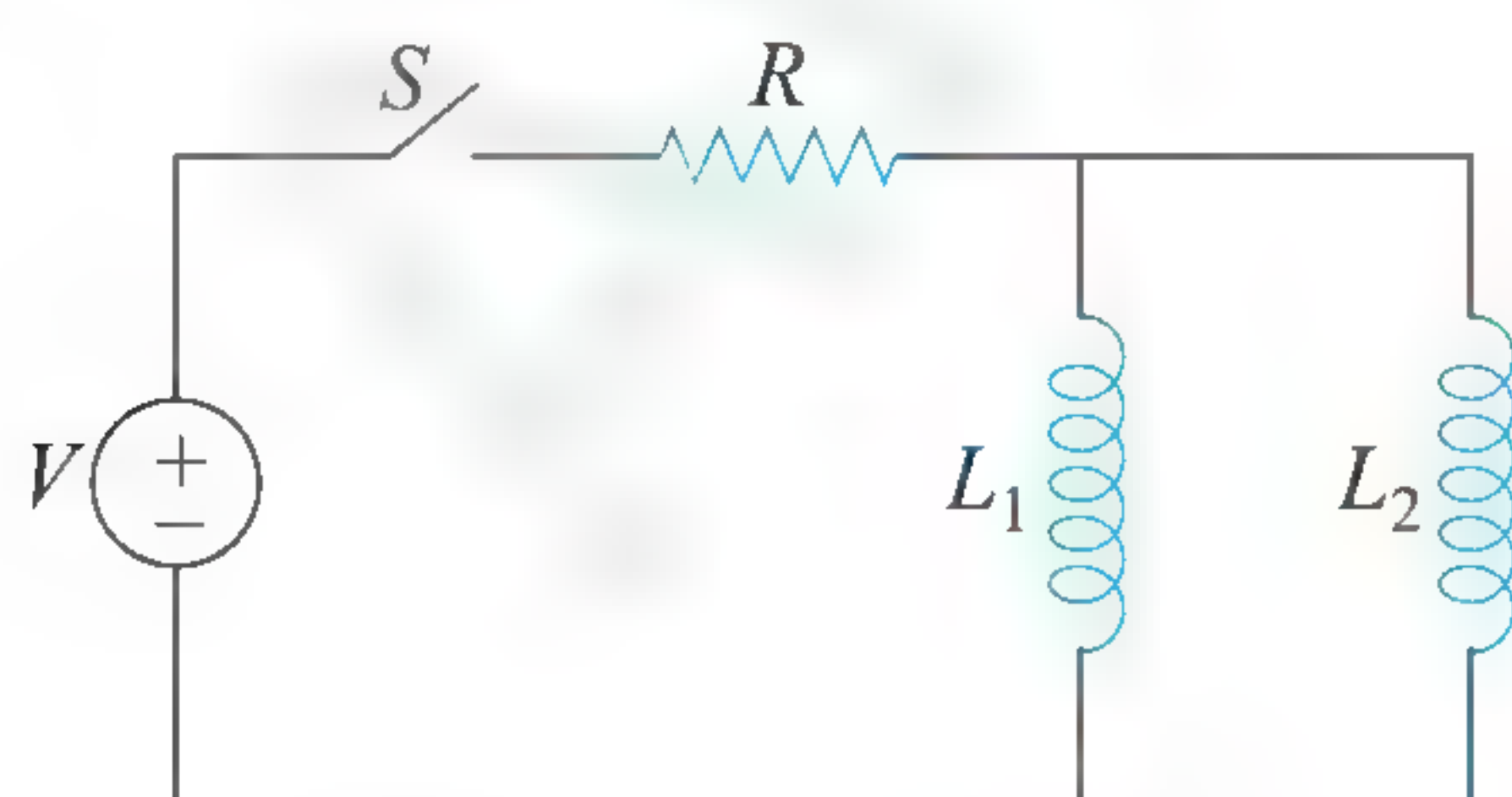
1. A conducting loop in the shape of right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at constant rate of 10 A s^{-1} . Which of the following statement(s) is (are) true?



- (1) The induced current in the wire is in opposite direction to the current along the hypotenuse.
- (2) There is a repulsive force between the wire and the loop.
- (3) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire.
- (4) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.

(JEE Advanced 2016)

2. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?



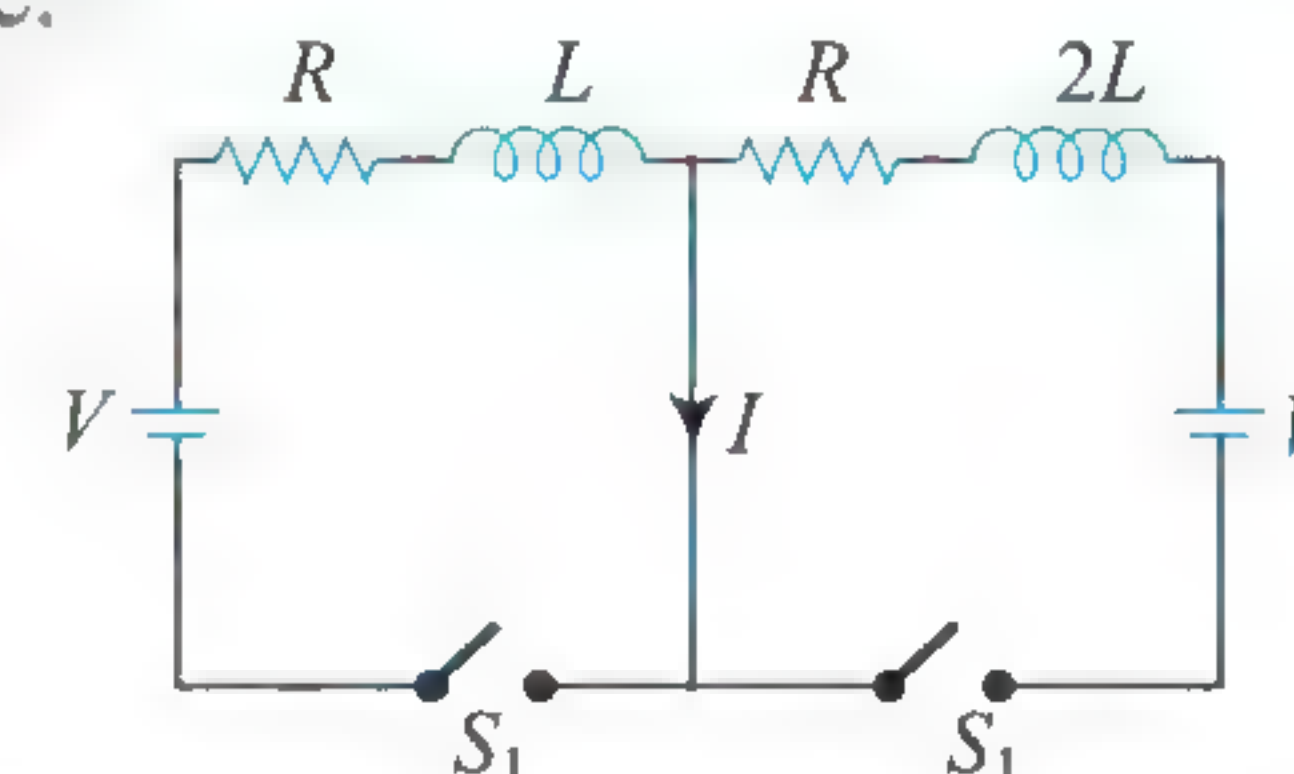
- (1) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$)
- (2) After a long time, the current through L_1 will be $\frac{V}{R} \frac{L_2}{L_1 + L_2}$

- (3) After a long time, the current through L_2 will be $\frac{V}{R} \frac{L_1}{L_1 + L_2}$

- (4) At $t = 0$, the current through the resistance R is $\frac{V}{R}$

(JEE Advanced 2017)

3. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure.



Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude $I_{\max}$ at time $t = \tau$. Which of the following statement(s) is (are) true?

(1) $I_{\max} = \frac{V}{2R}$

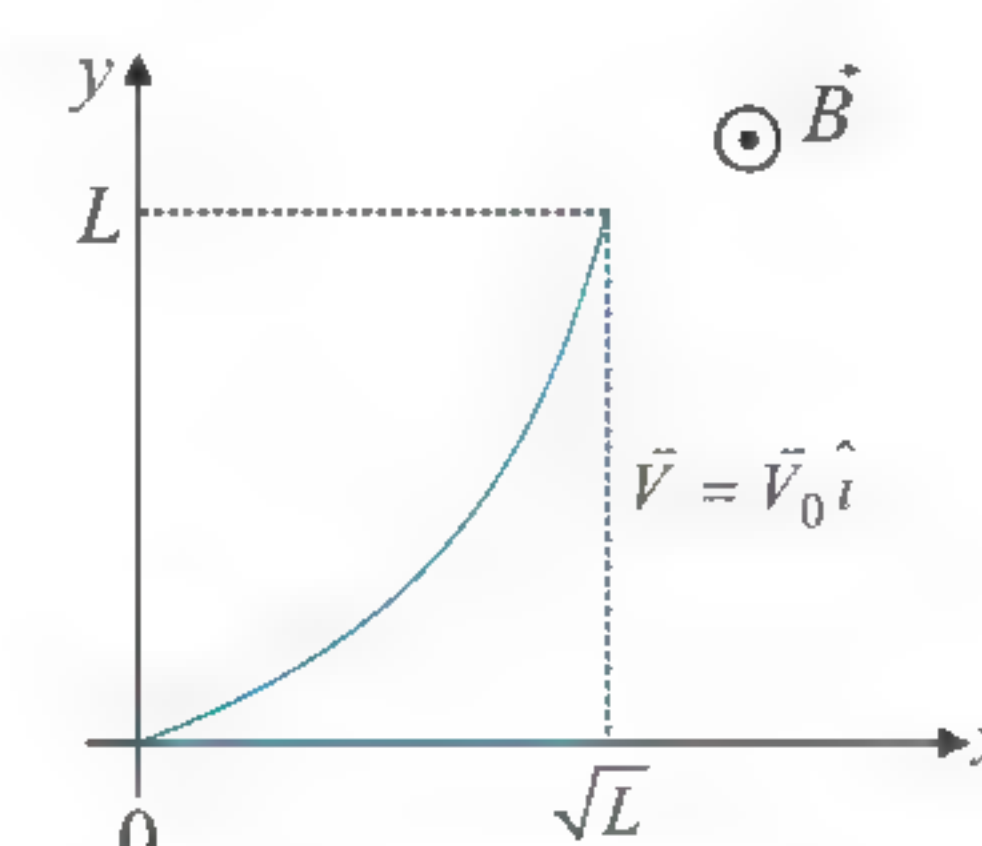
(2) $I_{\max} = \frac{V}{4R}$

(3) $\tau = \frac{L}{R} \ln 2$

(4) $\tau = \frac{2L}{R} \ln 2$

(JEE Advanced 2018)

4. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0 \hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0 \left(1 + \left(\frac{y}{L}\right)^\beta\right) \hat{k}$, as shown in figure. If V_0 , B_0 , L and β are positive constants and $\Delta\phi$ is potential difference developed between the ends of the wire; then the correct statement(s) is/are



- (1) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$
- (2) $|\Delta\phi|$ is proportional to the length of the wire projected on y-axis

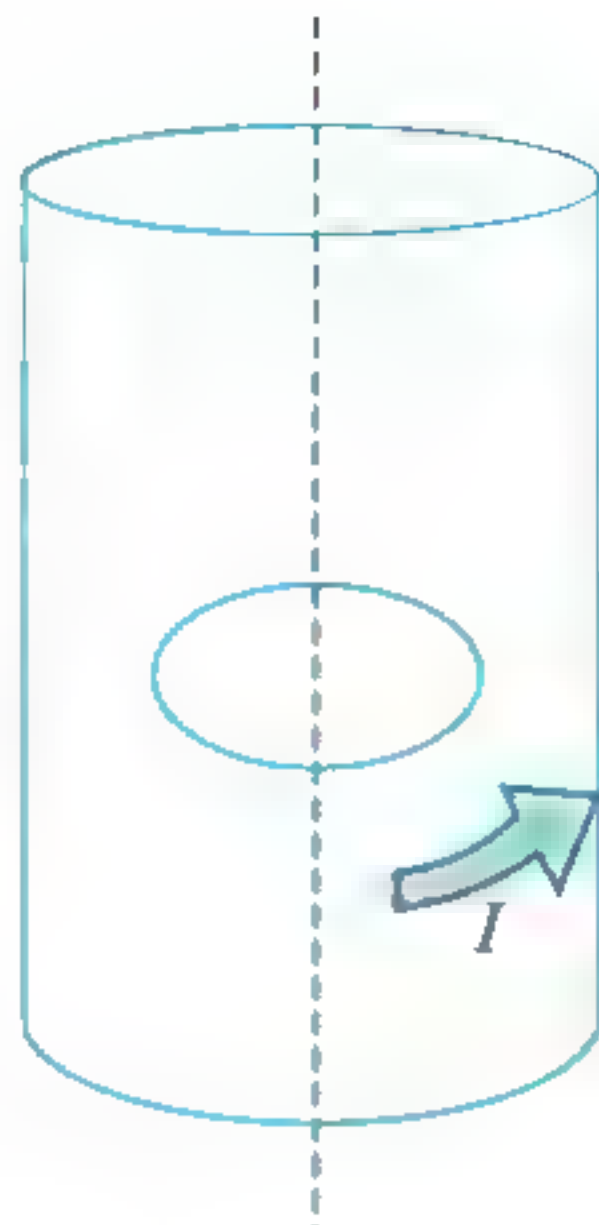
$$(3) |\Delta\phi| = \frac{1}{2} B_0 V_0 L \text{ for } \beta = 0$$

$$(4) |\Delta\phi| = \frac{4}{3} B_0 V_0 L \text{ for } \beta = 2$$

(JEE Advanced 2019)

Numerical Value Type

1. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown in figure. A wire-loop of resistance 0.005Ω and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300t)$, then N is _____.

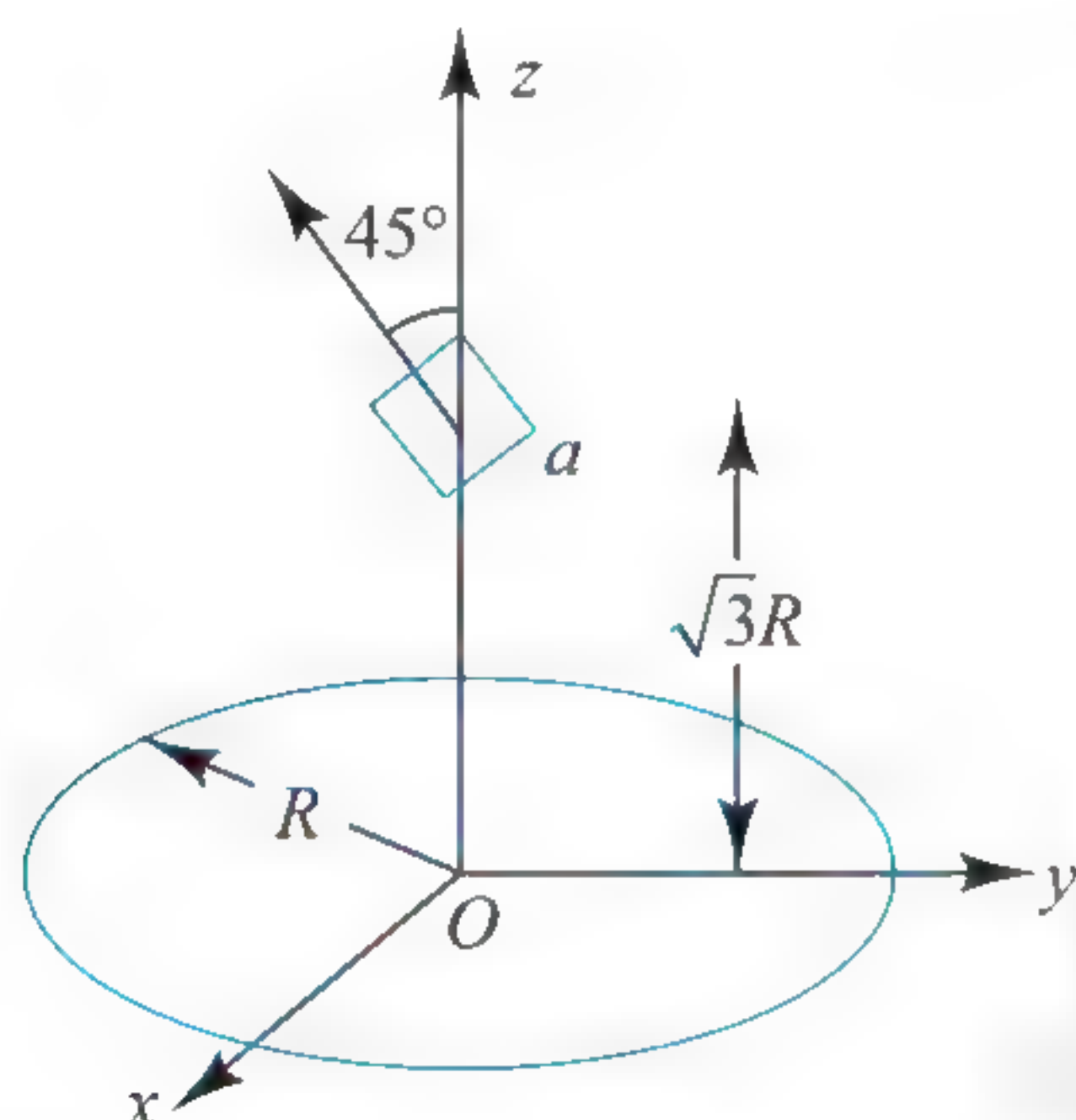


(IIT JEE, 2011)

2. A circular wire loop of radius R is placed in the x - y plane centered at the origin O . A square loop of side a ($a \ll R$) having two turns is placed with its center at along the axis of the circular wire loop as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z -axis. If the mutual inductance between the loops is given

by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is _____.

(IIT JEE 2012)



3. Two inductors L_1 (inductance 1 mH, internal resistance 3Ω) and L_2 (inductance 2 mH, internal resistance 4Ω), and a resistor R (resistance 12Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time

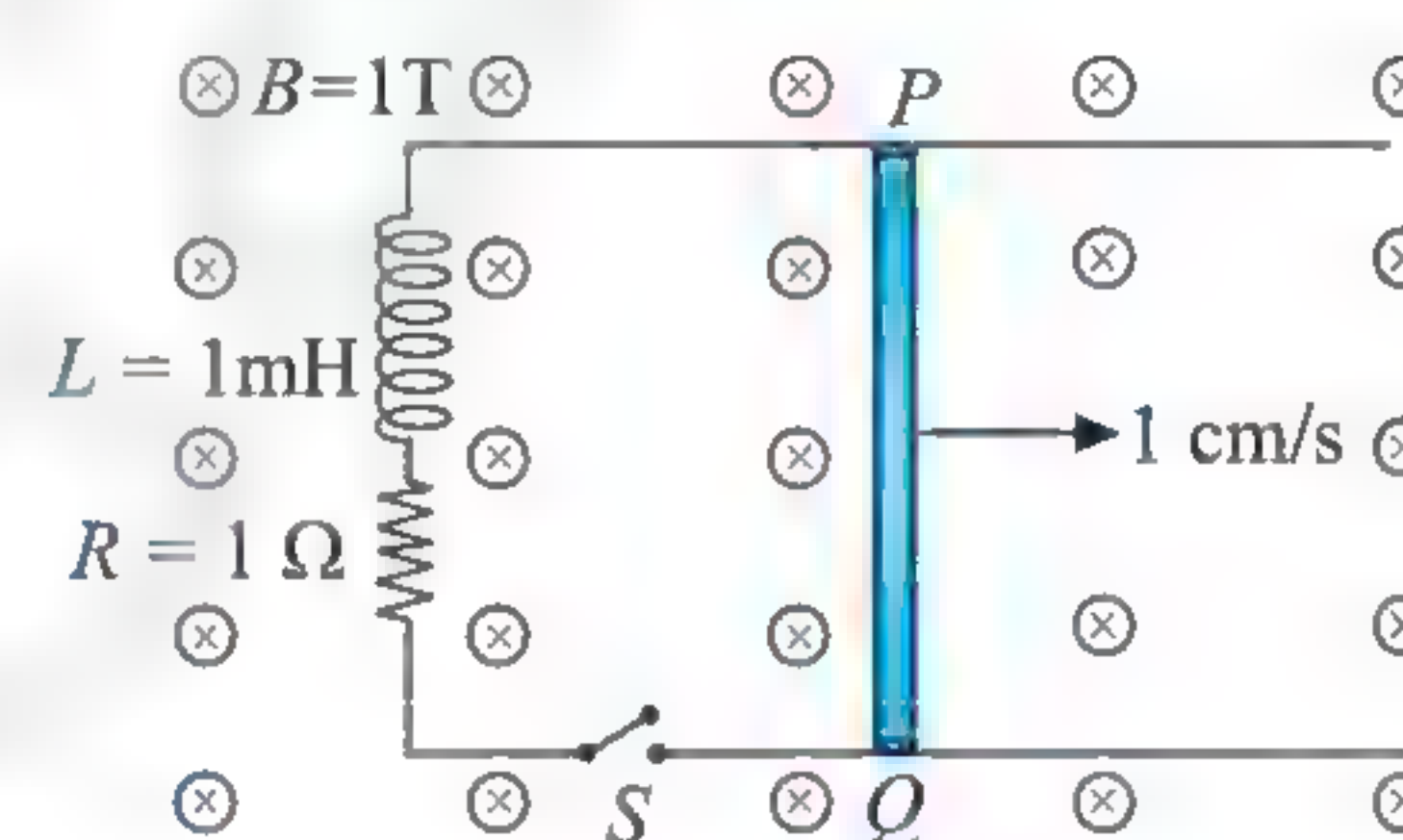
$t = 0$. The ratio of the maximum to the minimum current ($I_{\max}/I_{\min}$) drawn from the battery is _____.

(JEE Advanced 2016)

4. A 10 cm long perfectly conducting wire PQ is moving, with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor $L = 1 \text{ mH}$ and a resistance $R = 1 \Omega$ as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field $B = 1 \text{ T}$ perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3} \text{ A}$, where the value of x is _____.

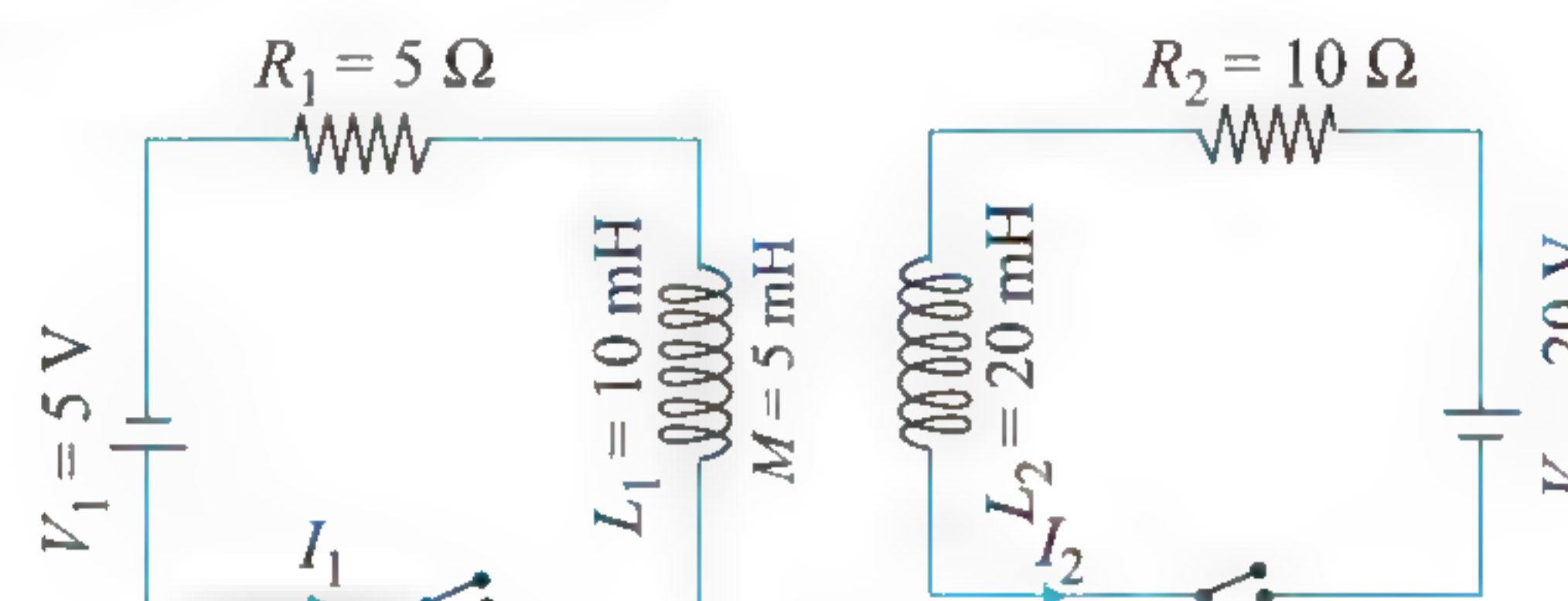
[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed.]

Given: $e^{-1} = 0.37$, where e is base of the natural logarithm]



(JEE Advanced 2019)

5. The inductors of two LR circuits are placed next to each other, as shown in the figure. The values of the self-inductance of the inductors, resistances, mutual-inductance and applied voltages are specified in the given circuit. After both the switches are closed simultaneously, the total work done by the batteries against the induced EMF in the inductors by the time the currents reach their steady state values is _____ mJ.



(JEE Advanced 2020)

Answers Key

EXERCISES

Single Correct Answer Type

1. (1)

6. (3)

11. (3)

16. (4)

21. (2)

26. (3)

31. (2)

36. (1)

41. (2)

46. (2)

51. (1)

56. (3)

61. (2)

66. (3)

71. (1)
2. (2)

7. (2)

12. (1)

17. (1)

22. (1)

27. (1)

32. (2)

37. (3)

42. (3)

47. (3)

52. (1)

57. (4)

62. (4)

67. (2)

72. (1)
3. (3)

8. (4)

13. (2)

18. (2)

23. (3)

28. (2)

33. (1)

38. (1)

43. (3)

48. (1)

53. (1)

58. (1)

63. (2)

68. (1)
4. (2)

9. (2)

14. (4)

19. (3)

24. (4)

29. (1)

34. (4)

39. (3)

44. (4)

49. (3)

54. (1)

59. (2)

64. (1)

69. (4)
5. (2)

10. (4)

15. (2)

20. (2)

25. (4)

30. (2)

35. (2)

40. (2)

45. (1)

50. (1)

55. (3)

60. (1)

65. (4)

70. (1)

Multiple Correct Answers Type

1. (3),(4)

4. (1),(3)

7. (1),(2)

10. (1),(2),(3)

13. (1),(3)
2. (1),(3)

5. (1),(2)

8. (1),(2),(3),(4)

11. (1),(2),(3)

14. (2),(4)
3. (1),(2)

6. (1),(3)

9. (1),(3),(4)

12. (2),(3)

Linked Comprehension Type

1. (2)

6. (2)
2. (3)

7. (1)
3. (1)

8. (3)
4. (1)

9. (1)
5. (2)

10. (3)

11. (2)

16. (1)

21. (2)

26. (3)

31. (2)

36. (2)

41. (1)
12. (4)

17. (2)

22. (1)

27. (3)

32. (4)

37. (3)

42. (4)
13. (1)

18. (2)

23. (1)

28. (2)

33. (3)

38. (1)

43. (4)
14. (2)

19. (1)

24. (2)

29. (1)

34. (2)

39. (3)

44. (2)
15. (4)

20. (2)

25. (1)

30. (4)

35. (3)

40. (4)

45. (2)

Matrix Match Type

1. i. $\rightarrow$ a.; ii. $\rightarrow$ b.; iii. $\rightarrow$ c.; iv. $\rightarrow$ a.
2. i. $\rightarrow$ c.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a., c.; iv. $\rightarrow$ a., c.
3. i. $\rightarrow$ c.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ b.
4. i. $\rightarrow$ a., c.; ii. $\rightarrow$ a., c., d.; iii. $\rightarrow$ a., b.; iv. $\rightarrow$ a., c., d.

Numerical Value Type

1. (8)

6. (8)

11. (5)
2. (8)

7. (6)

12. (8)
3. (4)

8. (5)
4. (4)

9. (1)
5. (1)

10. (4)

ARCHIVES

JEE Advanced

Multiple Correct Answers Type

1. (2),(4)

4. (1),(2),(4)
2. (1),(2),(3)
3. (2),(4)

Numerical Value Type

1. (6)

2. (7)

3. (8)

4. (0.63)

5. (55.00)

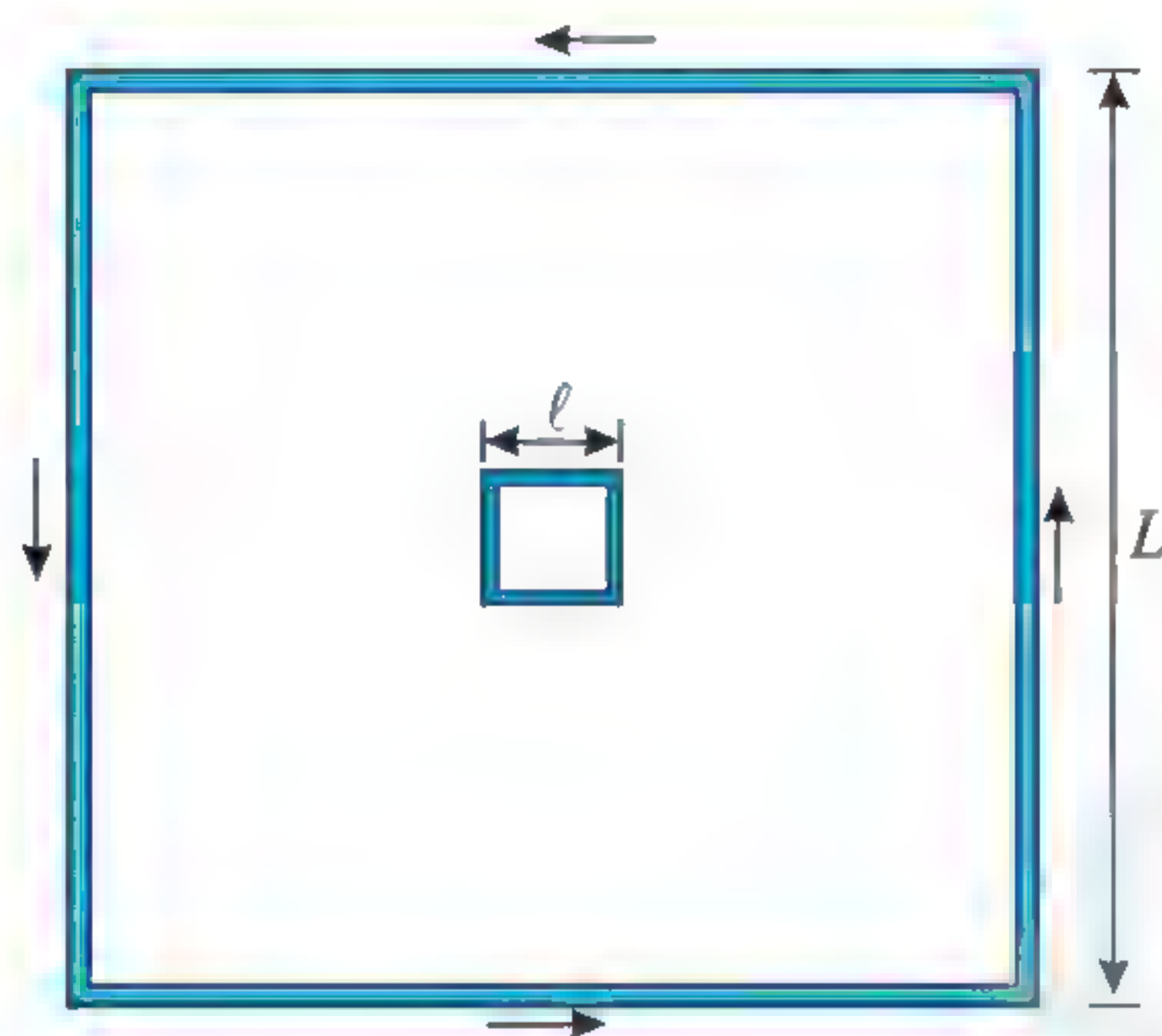
Chapter 5

Concept Application Exercises

Exercise 5.1

1. $L = \frac{\epsilon}{\frac{di}{dt}} = \frac{6}{600} = 10^{-2} \text{ H}$
2. $\langle V_L \rangle = L \frac{\Delta i}{\Delta t} = 4 \times 10^{-3} \left(\frac{2 - 0.25}{0.5} \right) = 14 \text{ mV}$
3. $M = \frac{\phi_2}{i_1} = \frac{\mu_0 i_1 l^2}{2R i_1} = \frac{\mu_0 l^2}{2R} = \frac{(4\pi \times 10^{-7})(10^{-4})}{(2)\left(\frac{1}{4}\right)} = 8\pi \times 10^{-11} \text{ H}$
4. $M = \frac{\phi_2}{i_1} = \frac{\mu_0 i_1 A}{2\pi x} / i_1$
 $= \frac{\mu_0 A}{2\pi x} = \frac{4\pi \times 10^{-7} \times 10^{-8}}{2\pi \times 0.5} = 4 \times 10^{-15} \text{ H}$
5. Considering the larger loop to be made up of four rods, each of length L , the field at the center, i.e., at a distance $L/2$ from each rod, will be

$$B = \frac{\mu_0 I}{4\pi d} [\sin \alpha + \sin \beta] = \frac{\mu_0 I}{4\pi (L/2)} 2 \sin 45^\circ = \frac{\mu_0 I}{\sqrt{2}\pi L}$$



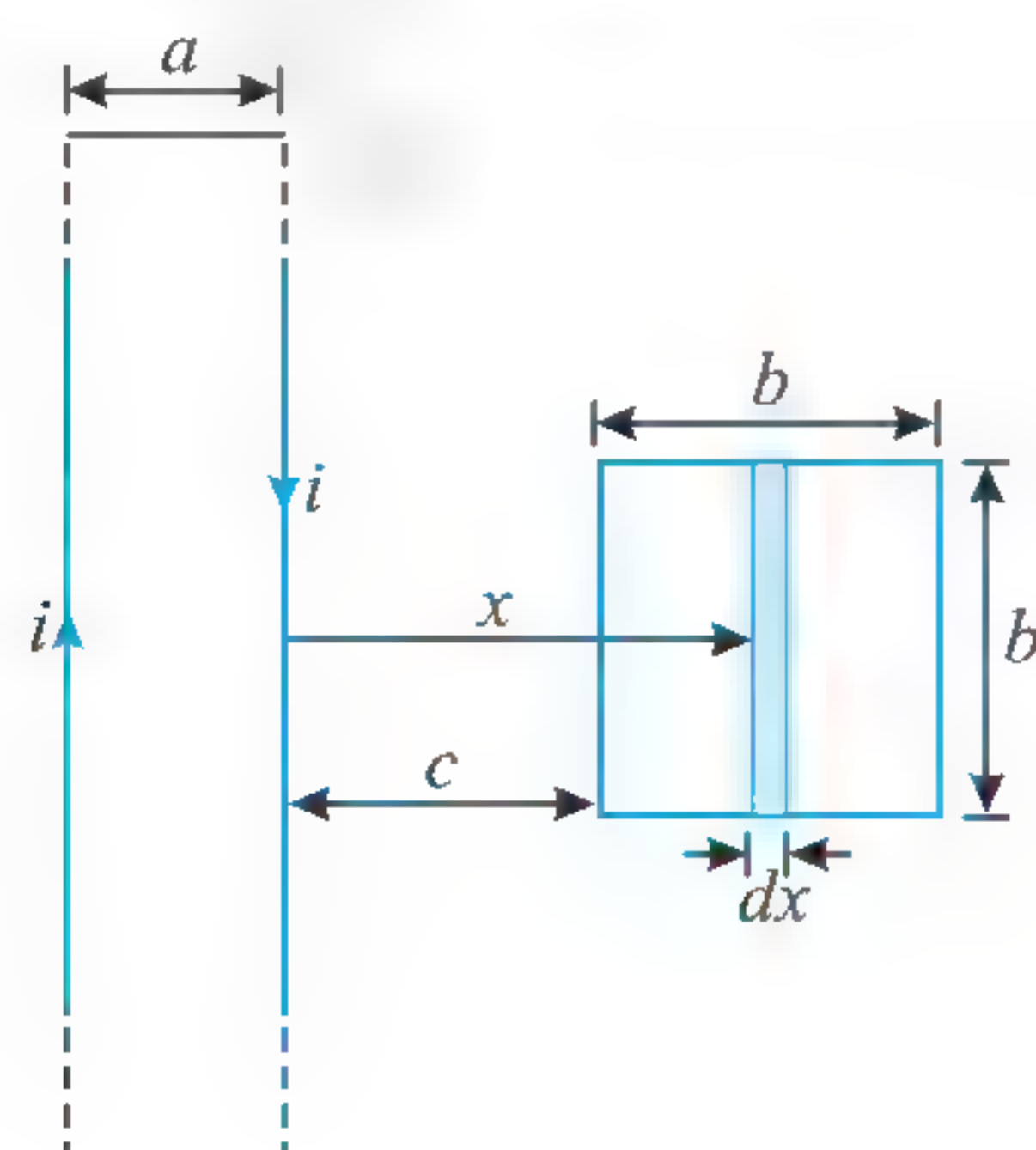
Net field: $B_1 = 4B_0 = 2\sqrt{2} \frac{\mu_0 I}{\pi L}$

So the flux linked with the smaller loop is

$$\phi = B_1 S_2 = \frac{2\sqrt{2}\mu_0 I}{\pi L} l^2$$

Hence, $M = \frac{\phi}{I} = 2\sqrt{2} \frac{\mu_0 l^2}{\pi L}$

6. Let us assume the rectangular loop of infinite size have current i . Consider a segment of width dx at a distance x as shown.



The flux through the segment, $d\phi = \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)} \right] b dx$.

$$\phi = \int_c^{c+b} \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)} \right] b dx$$

$$= \frac{\mu_0 i b}{2\pi} \left[\ln \frac{c+b}{c} - \ln \frac{a+b+c}{a+c} \right]$$

The mutual inductance between two rectangular loops is

$$M = \frac{\phi}{i} = \frac{\mu_0 b}{2\pi} \ln \frac{(c+b)(a+c)}{c(a+b+c)}$$

7. If a current I_1 flows around the first wire then a uniform axial magnetic field of strength $B_1 = \frac{\mu_0 N_1 I_1}{\ell}$ is generated in the core.

The magnetic field in the region outside the core is of negligible magnitude. The flux linked to a single turn of the second wire is $B_1 A$. Thus, the flux linked to all N_2 turns of the second wire is

Now, $\phi = N_2 B_1 A = \frac{\mu_0 N_1 N_2 A I_1}{\ell} = M I_1$

$\therefore M = \frac{\mu_0 N_1 N_2 A}{\ell}$

As described previously, M is a geometric quantity depending on the dimensions of the core and the manner in which the two wires are wound around the core, but not on the actual currents flowing through the wires.

8. Let a current i flows in coil of radius a_2 .

Magnetic field at the centre of the coil = $\frac{\mu_0 i}{2a_2}$

Flux in smaller coil $\phi = \frac{\mu_0 i}{2a_2} \pi a_1^2$

or $M i = \frac{\mu_0 i}{2a_2} \pi a_1^2$ or $M = \frac{\mu_0 \pi a_1^2}{2a_2}$

9. Let a current i flows in the coil of radius a_1 . The magnetic field at the centre of this coil will now be parallel to the plane of the smaller coil and hence no flux will pass through it, so $M = 0$.
10. If current i flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of the larger coil. Now, the area vector of the smaller coil which is perpendicular to the plane of the smaller coil will make an angle θ with the magnetic field.

Thus, flux $\vec{B} \cdot \vec{A} = \frac{\mu_0 i}{2a_2} \pi a_1^2 \cos \theta$

or $M = \frac{\mu_0 \pi a_1^2 \cos \theta_1}{2a_2}$

11. (a) To find mutual inductance, it does not matter in which coil we consider current and in which coil flux is calculated (reciprocity theorem). Let current i be flowing in the larger coil. Magnetic field at the centre = $\frac{\mu_0 i}{2b}$.

Flux through the smaller coil = $\frac{\mu_0 i}{2b} \pi a^2$

$\therefore M = \frac{\mu_0}{2b} \pi a^2$

$$(b) \text{ | emf induced in larger coil |} = \left[\left(\frac{di}{dt} \right) \text{ in smaller coil} \right]$$

$$= \frac{\mu_0}{2b} \pi a^2 (2) = \frac{\mu_0 \pi a^2}{b}$$

$$(c) \text{ current in the larger coil} = \frac{\mu_0 \pi a^2}{bR}.$$

Exercise 5.2

1. (i) (a) The bulb glows brightly right away, and then more and more faintly as the capacitor charges up. (b) The bulb gradually gets brighter and brighter, changing rapidly at first and then more and more slowly. (c) The bulb immediately becomes bright. (d) The bulb glows brightly right away, and then more and more faintly as the inductor starts carrying more and more current (the inductor eventually acts as a short).

(ii) (a) The bulb goes out immediately because current stops immediately (charge ceases to flow). (b) The bulb glows for a moment as a spark jumps across the switch. (c) The bulb stays lit for a while, gradually getting fainter and fainter as the capacitor discharges through the bulb. (d) The bulb suddenly glows brightly. Then its brightness decreases to zero, changing rapidly at first and then more and more slowly.

2. The equivalent resistance is

$$L_{eq} = L_1 + L_4 + L_{23} = L_1 + L_4 + \frac{L_2 L_3}{L_2 + L_3}$$

$$= 30.0 + 12.0 + \frac{(40.0)(10.0)}{(40.0 + 10.0)} = 50.0 \text{ mH}.$$

3. (a) In given situation the coil oppose the changes, so if it pushes current rightward (when the current is already going rightward), then i must be in the process of decreasing.

(b) From Eq. (in absolute value) we get,

$$L = \left| \frac{\varepsilon}{di/dt} \right| = \frac{6.0}{1.5 \times 10^3} = 4.0 \times 10^{-3} \text{ H}.$$

4. The voltages are related as

$$\varepsilon - iR - L \frac{di}{dt} = 0 \Rightarrow \varepsilon = iR + L \frac{di}{dt} = 0$$

When the current is increasing:

$$9.0 = (2.0)R + L(0.50) \quad \dots(i)$$

When the current is decreasing:

$$5.0 = (2.0)R + L(-0.50) \quad \dots(ii)$$

(a) Subtracting [ii] from [i] gives

$$9.0 - 5.0 - L(1.0) \Rightarrow L = 4.0 \text{ H}$$

(b) Substituting the value for L in either equation gives

$$7.0 = (2.0)R \Rightarrow R = 3.5 \Omega$$

5. Applying the loop theorem, $\varepsilon - L \left(\frac{di}{dt} \right) = iR$,

we solve for the (time-dependent) emf, with SI units understood:

$$\varepsilon = L \frac{di}{dt} + iR = L \frac{d}{dt}(3.0 + 5.0t) + (3.0 + 5.0t)R$$

$$= (6.0)(5.0) + (3.0 + 5.0t)(4.0)$$

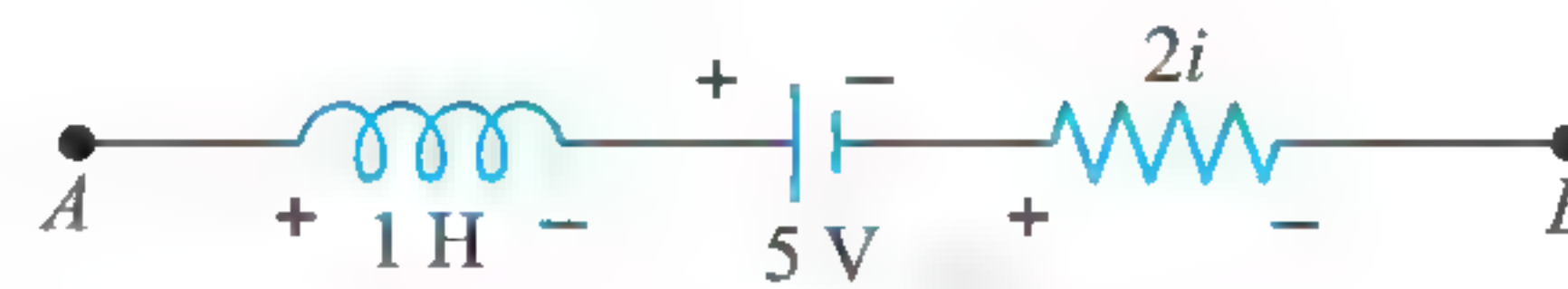
$$= (42 + 20t) \text{ V}$$

6. (a) Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop: $\varepsilon - L di/dt = 0$.

So $i = \varepsilon t/L$. As the fuse blows at $t = t_0$, $i = i_0 = 3.0 \text{ A}$. Thus,

$$t_0 = \frac{i_0 L}{\varepsilon} = \frac{(3.0 \text{ A})(5.0 \text{ H})}{10 \text{ V}} = 1.5 \text{ s}$$

7. Applying KVL from A to B ,



$$V_A - 1 \frac{di}{dt} - 5 - 2i = V_B$$

- (a) Put $i = 2$, $\frac{di}{dt} = 0$; $V_A - 5 - 4 = V_B$

$$\therefore V_A - V_B = 9 \text{ V}$$

- (b) Put $i = 2$, $\frac{di}{dt} = 1$

$$V_A - 1 - 5 - 4 = V_B$$

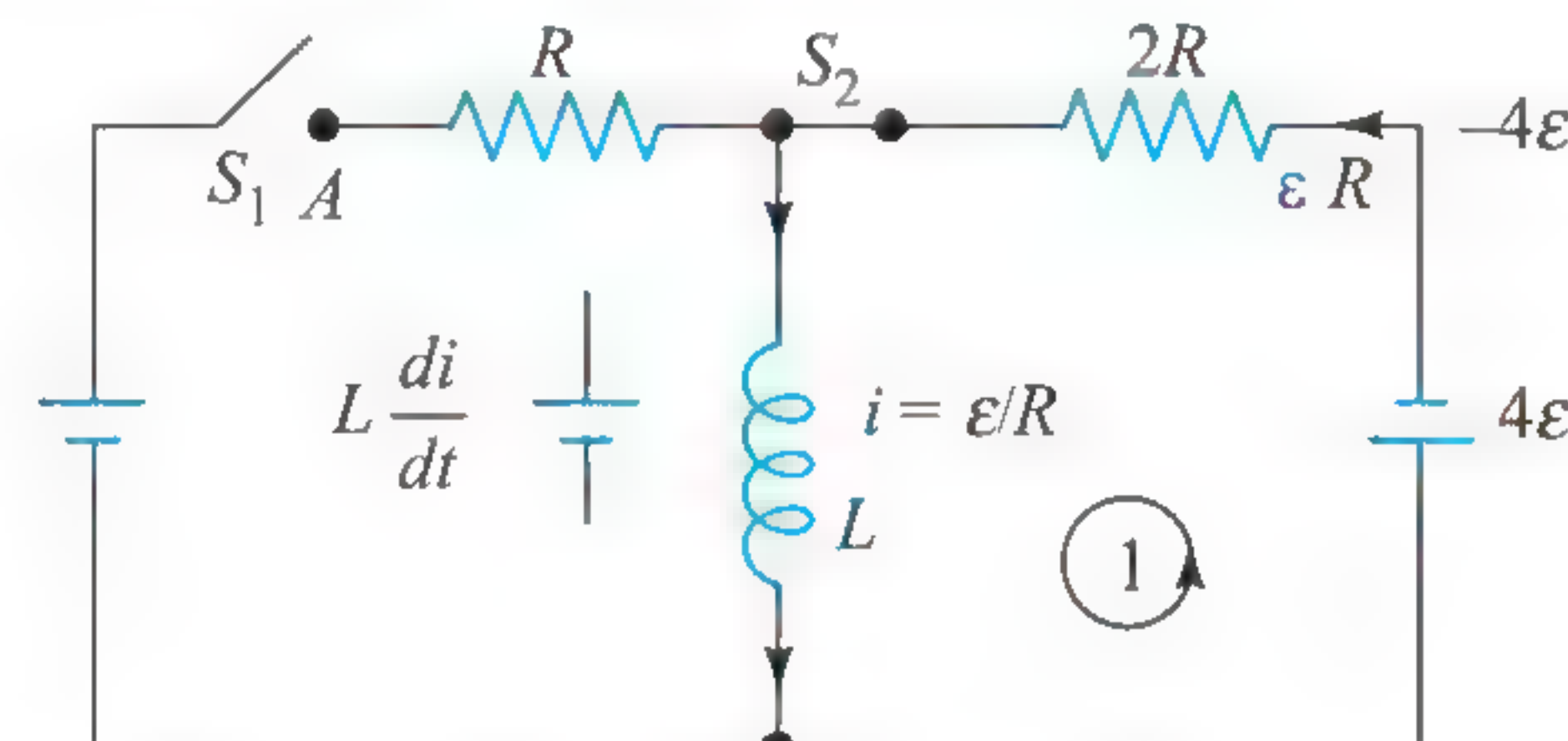
$$\text{or } V_A - V_B = 10 \text{ V}$$

- (c) Put $i = 2$, $\frac{di}{dt} = -1$

$$V_A + 1 - 5 - 2 \times 2 = V_B$$

$$V_A = 8 \text{ V}$$

8. Before S_2 is closed and S_1 is opened, current in the left part of the circuit $= \varepsilon/R$. Now when S_2 is closed and S_1 is opened, current through the inductor cannot change suddenly, current ε/R will continue to flow through the inductor.

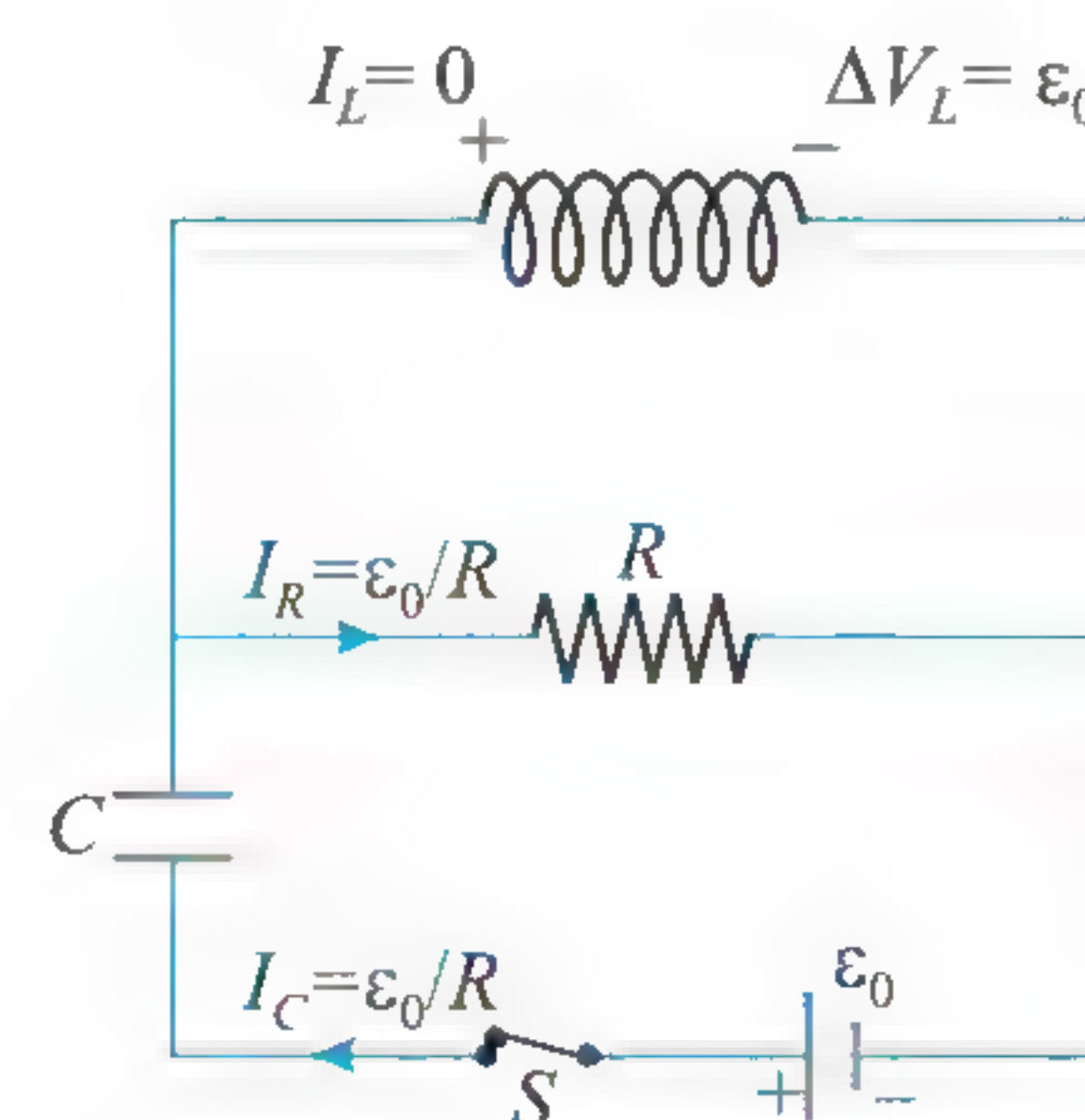


Applying KVL in loop 1,

$$L \frac{di}{dt} + \frac{\varepsilon}{R} (2R) + 4\varepsilon = 0 \Rightarrow \frac{di}{dt} = -\frac{6\varepsilon}{L}$$

Exercise 5.3

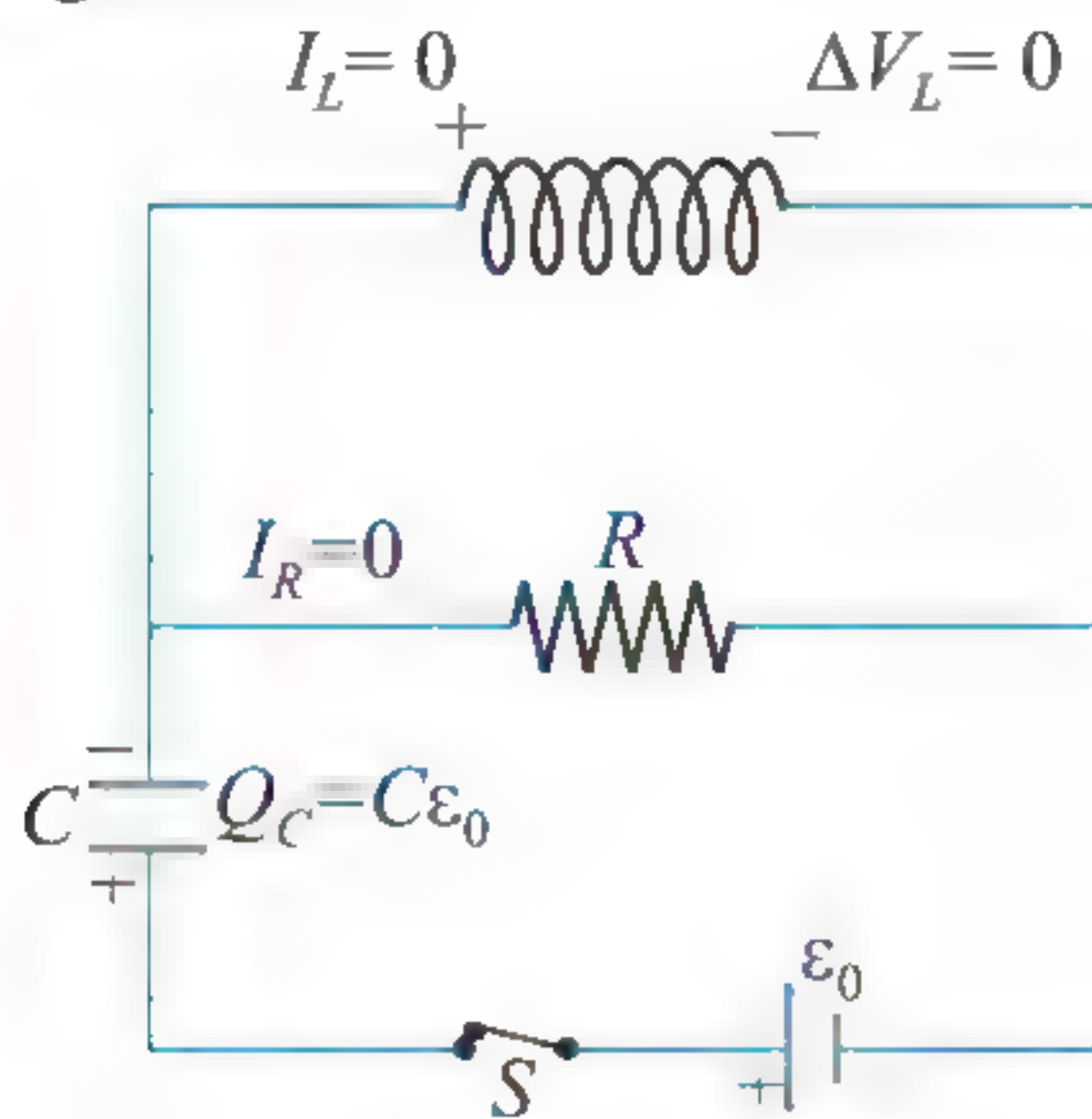
1. (a) The instant after the switch is closed, the capacitor acts as a conducting wire, and the inductor acts as infinite resistance to maintain zero current in itself. The situation is as shown in the circuit diagram



$$I_L = 0, I_C = \frac{\varepsilon_0}{R}, I_R = \frac{\varepsilon_0}{R}$$

$$\Delta V_L = \varepsilon_0, \Delta V_C = 0, \Delta V_R = \varepsilon_0$$

(b) After the switch has been closed a long time, the capacitor acts as an open switch. The steady state conditions will exist. The currents and voltages are:



$$I_L = 0, I_C = 0, I_R = 0$$

$$\Delta V_L = 0, \Delta V_C = \varepsilon_0, \Delta V_R = 0$$

2. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \frac{\varepsilon}{(R_1 + R_2)} = \frac{110}{(10 + 20)} = \frac{11}{3} \text{ A.}$$

(b) $i_2 = i_1 = \frac{11}{3} \text{ A.}$

(c) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0 \quad \dots(i)$$

$$\varepsilon - i_1 R_1 - (i_1 - i_2) R_3 = 0 \quad \dots(ii)$$

We solve these simultaneously for i_1 and i_2 , and find

$$i_1 = \frac{\varepsilon(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$i_1 = \frac{(110)(20 + 30)}{(10)(20) + (10)(30) + (20)(30)} = 5.0 \text{ A,}$$

(d) and $i_2 = \frac{\varepsilon R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$

$$i_2 = \frac{(110)(30)}{(10)(20) + (10)(30) + (20)(30)} = 3.0 \text{ A,}$$

(e) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$).

(f) The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is $5.0 \text{ A} - 3.0 \text{ A} = 2.0 \text{ A}$. The current in R_2 is the same but in the opposite direction as that in R_3 , that is, $i_2 = -2.0 \text{ A}$.

A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus,

(g) $i_1 = 0$, and

(h) $i_2 = 0$.

3. For the increasing current $i = \frac{\varepsilon}{R}(1 - e^{-Lt/R})$. The final value is $\frac{\varepsilon}{R}$, so the condition on Δt is

$$0.800 \frac{\varepsilon}{R} = \frac{\varepsilon}{R}(1 - e^{-L\Delta t/R})$$

$$e^{-L\Delta t/R} = 0.200 \Rightarrow e^{L\Delta t/R} = 5.00$$

$$\frac{L\Delta t}{R} = \ln 5.00 \Rightarrow \Delta t = \frac{R \ln 5.00}{L}$$

At the moment when the battery is removed, the current in the coil is quite precisely $\frac{\varepsilon}{R}$. During the decrease, $i = I_i e^{-Lt/R} = \frac{\varepsilon}{R} e^{-Lt/R}$.

(a) at $t = \Delta t = \frac{R \ln 5.00}{L}$,

$$\frac{i}{I_i} = e^{-L\Delta t/R} = 0.200 = 20.0\%$$

(b) at $t = 2\Delta t$,

$$\frac{i}{I_i} = e^{-L(2\Delta t)/R} = (e^{-L\Delta t/R})^2$$

$$= (0.200)^2 = 0.0400 = 4.00\%$$

4. (a) When $i = 2.00 \text{ A}$, the voltage across the resistor is

$$= \Delta V_R = iR = (2.0)(8.0) = 16.0 \text{ V}$$

Kirchhoff's loop rule tells us that the sum of the changes in potential around the loop must be zero:

$$\varepsilon - \Delta V_R - \varepsilon_L = 36.0 - 16.0 - \varepsilon_L = 0,$$

so $\varepsilon_L = 20.0 \text{ V}$ and $\frac{\Delta V_R}{\varepsilon_L} = \frac{16.0}{20.0} = 0.80$

(b) Similarly, for $i = 4.50 \text{ A}$, $\Delta V_R = iR = (4.50)(8.0) = 36.0 \text{ V}$ and $\varepsilon - \Delta V_R - \varepsilon_L = 36.0 \text{ V} - 36.0 \text{ V} - \varepsilon_L = 0$, so $\varepsilon_L = 0$.

5. (a) Applying the loop rule to the left loop gives $\varepsilon - i_1 R_1 = 0$, so $i_1 = \varepsilon/R_1 = 10 \text{ V}/5.0 \Omega = 2.0 \text{ A}$.

(b) Since now $\varepsilon_L = \varepsilon$, we have $i_2 = 0$.

(c) The junction rule gives $i_s = i_1 + i_2 = 2.0 \text{ A} + 0 = 2.0 \text{ A}$.

(d) Since $V_L = \varepsilon$, the potential difference across resistor 2 is $V_2 = \varepsilon - \varepsilon_L = 0$.

(e) The potential difference across the inductor is $V_L = \varepsilon = 10 \text{ V}$.

(f) The rate of change of current is $\frac{di_2}{dt} = \frac{V_L}{L} = \frac{\varepsilon}{L} = \frac{10 \text{ V}}{5.0 \text{ H}} = 2.0 \text{ A/s}$.

(g) After a long time, we still have $V_1 = \varepsilon$, so $i_1 = 2.0 \text{ A}$.

(h) Since now $V_L = 0$, $i_2 = \varepsilon/R_2 = 10 \text{ V}/10 \Omega = 1.0 \text{ A}$.

(i) The current through the switch is now

$$i_s = i_1 + i_2 = 2.0 \text{ A} + 1.0 \text{ A} = 3.0 \text{ A}.$$

(j) Since $V_L = 0$, $V_2 = \varepsilon - V_L = \varepsilon = 10 \text{ V}$.

(k) With the inductor acting as an ordinary connecting wire, we have $V_L = 0$.

(l) The rate of change of current in resistor 2 is $\frac{di_2}{dt} \cdot \frac{V_L}{L} = 0$.

6. Because of the decay of current that occurs after the switches are closed on B, the flux will decay according to,

$$\Phi_1 = L_1 i_1 = L_1 i_{01} e^{-t/\tau_1} = \phi_{01} e^{-t/\tau_1}, \text{ similarly } \Phi_2 = \phi_{02} e^{-t/\tau_2}$$

Setting the fluxes equal to each other and solving for time leads to

$$t = \frac{\ln(\Phi_{02}/\Phi_{01})}{(R_2/L_2) - (R_1/L_1)}$$

$$\Rightarrow t = \frac{\ln(2.0)}{(30/0.006) - (20/0.005)} = \frac{\ln(2.0)}{1000} \text{ s}$$

7. Using $i = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$, where $\tau_L = 2.0 \text{ ns}$, we find

$$t = \tau_L \ln\left(\frac{1}{1 - iR/\varepsilon}\right) \approx 1.0 \text{ ns}.$$

8. (a) As the switch closes at $t = 0$, the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf ($\mathcal{E}_{L1}$) of the $L_1 = 0.30$ H inductor to cancel that of the battery.

$$\frac{di}{dt} = \frac{|\mathcal{E}_{L1}|}{L_1} = \frac{6.0}{0.30} = 20 \text{ A/s.}$$

(b) What is being asked for is essentially the current in the battery when the emfs of the inductors vanish (as $t \rightarrow \infty$). Applying the loop rule to the outer loop, with $R_1 = 8.0 \Omega$, we have

$$\mathcal{E} - iR_1 - |\mathcal{E}_{L1}| - |\mathcal{E}_{L2}| = 0 \Rightarrow i = \frac{6.0 \text{ V}}{R_1} = 0.75 \text{ A.}$$

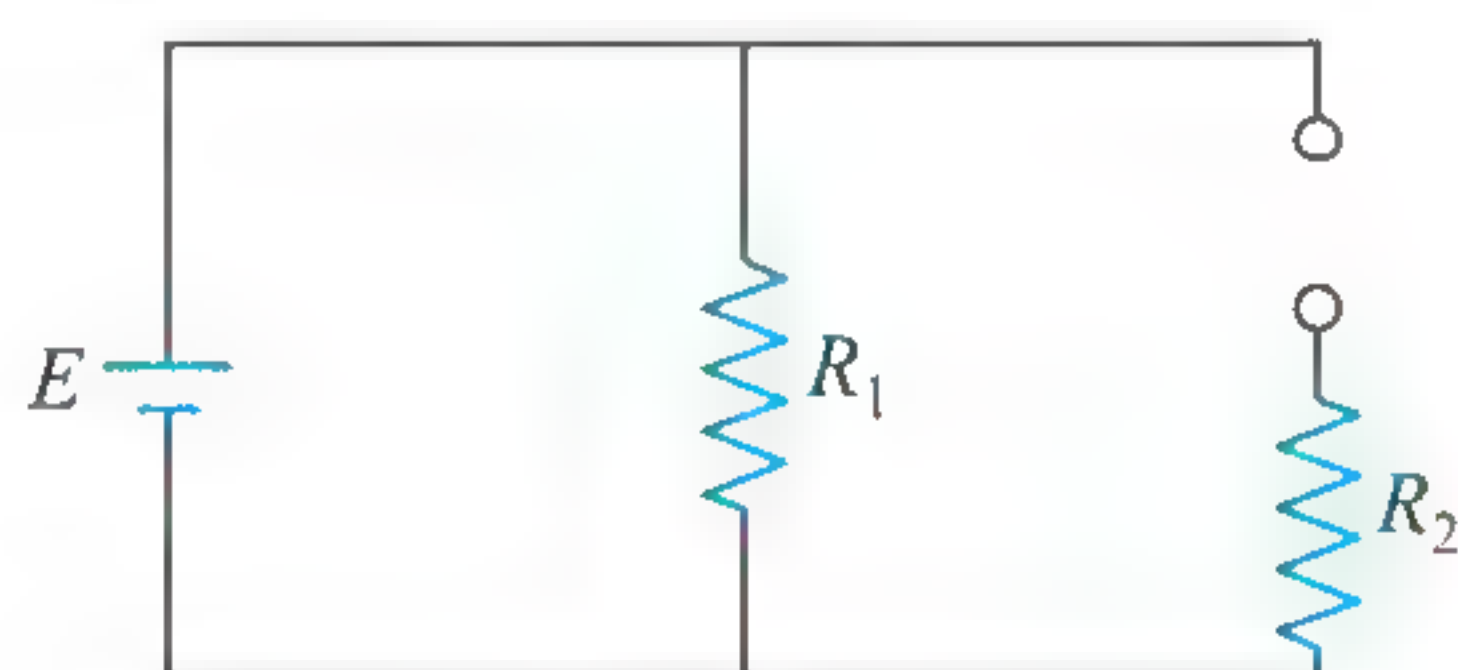
9. At $t = 0$, C will act as a simple wire, so R and L both will be short-circuited. So that there is no current in R and L .

$$I_1 = 0, I_2 = 0, I_3 = \frac{\mathcal{E}}{r}$$

At $t = \infty$, L will act as a simple wire so, R and C will be short-circuited.

$$I_1 = 0, I_2 = \frac{\mathcal{E}}{r}, I_3 = 0$$

10. (I) At $t = 0$, inductor acts as open circuit:



(a) $i_1 = \frac{E}{R_1} = 2 \text{ A}$

(b) $i_2 = 0 \text{ A}$

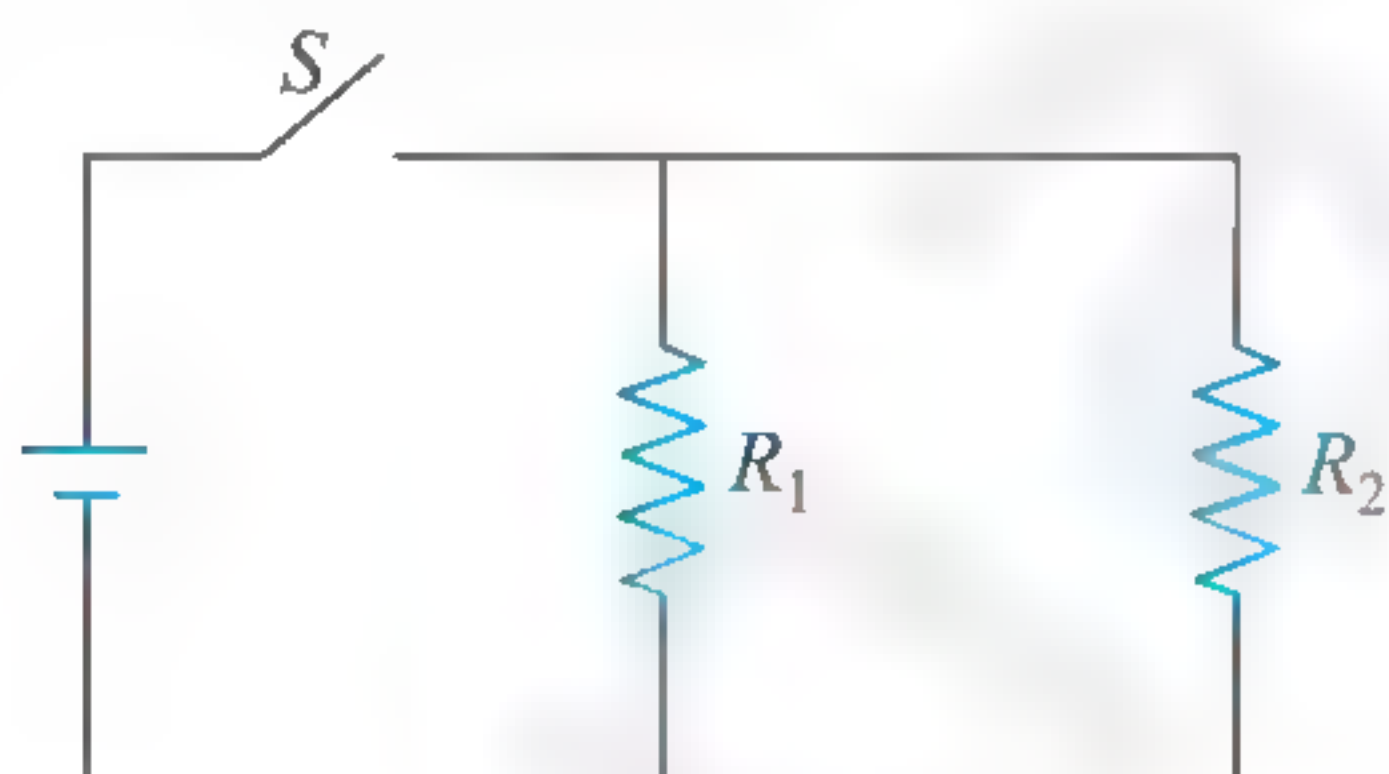
(c) $i = i_1 + i_2 = 2 + 0 = 2 \text{ A}$

(d) V across resistor $R_2 = 0$

(e) V across $L = 10 \text{ V}$

(f) $L \frac{di_2}{dt} = 10 \Rightarrow \frac{di_2}{dt} = \frac{10}{5} = 2 \text{ A/s}$

(II) At $t = \infty$, inductor acts as a conducting wire:



(a) $i_1 = \frac{E}{R_1} = 2 \text{ A}$

(b) $i_2 = \frac{E}{R_2} = \frac{10}{10} = 1 \text{ A}$

(c) $i_{\text{switch}} = i_1 + i_2 = 3 \text{ A}$

(d) $V_{R2} = 10 \text{ V}$

(e) $V_L = 0 \text{ V}$

(f) Current in R_2 is not changing with time, so

$$\frac{di_2}{dt} = 0$$

11. At $t = 0$, i_3 is zero since current cannot suddenly change due to the inductor.

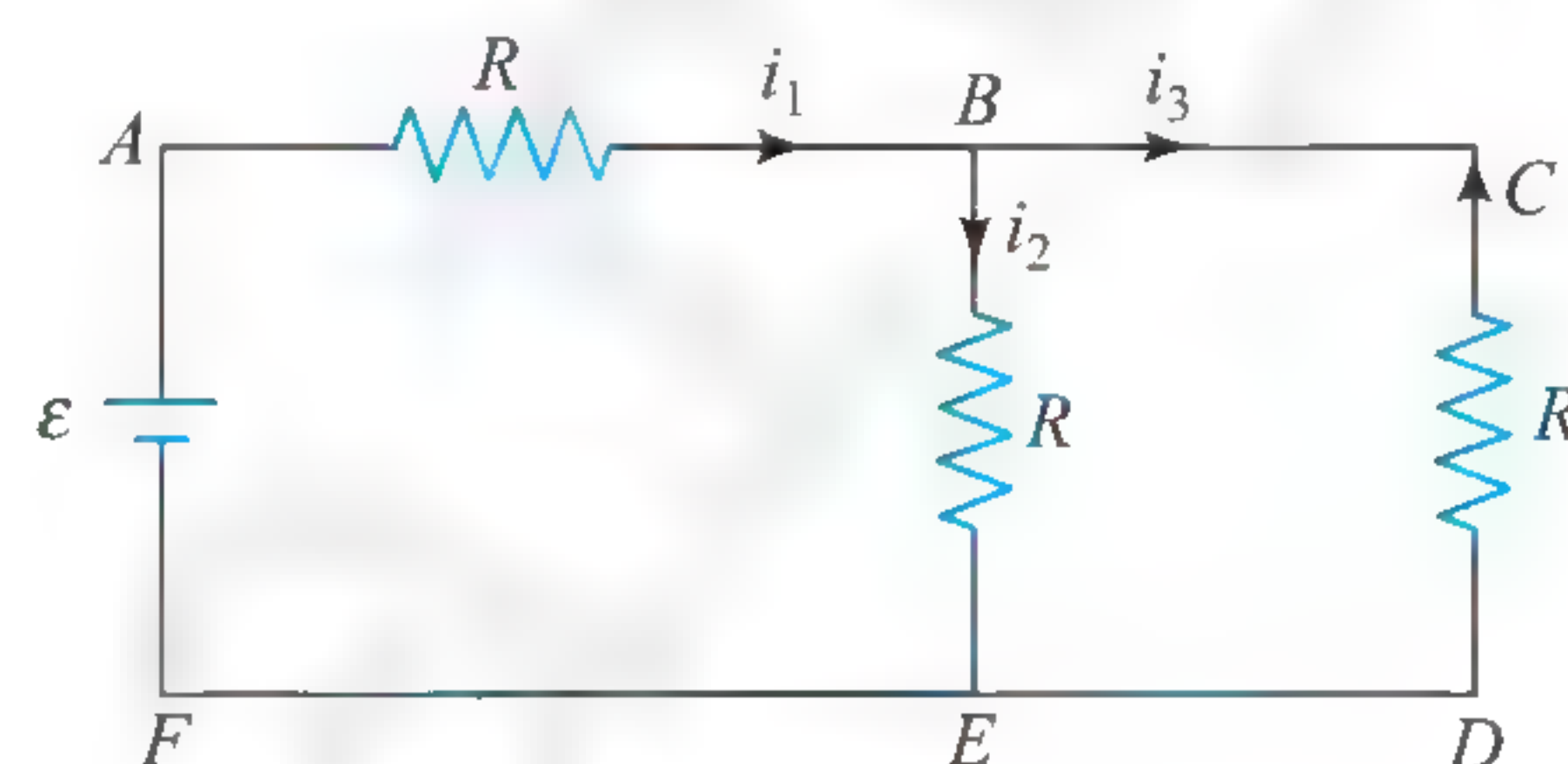
$$\therefore i_1 = i_2 \text{ (from KCL)}$$

Applying KVL in the part $ABEF$, we get $i_1 = i_2 = \frac{\mathcal{E}}{2R}$.

$$\mathcal{E} - i_1 R - L \frac{di_3}{dt} = 0$$

$$\Rightarrow \frac{di_3}{dt} = \frac{\mathcal{E} - i_1 R}{L} = \frac{\mathcal{E} - (\mathcal{E}/2R)R}{L} = \frac{\mathcal{E}}{2L}$$

At $t = \infty$, i_3 will become constant and hence potential difference across the inductor will be zero. It is just like a simple wire and the circuit can be solved assuming it to be equivalent to the circuit diagram shown below.



$$i_2 = i_3 = \frac{\mathcal{E}}{3R}, i_1 = \frac{2\mathcal{E}}{3R}, \frac{di_3}{dt} = 0$$

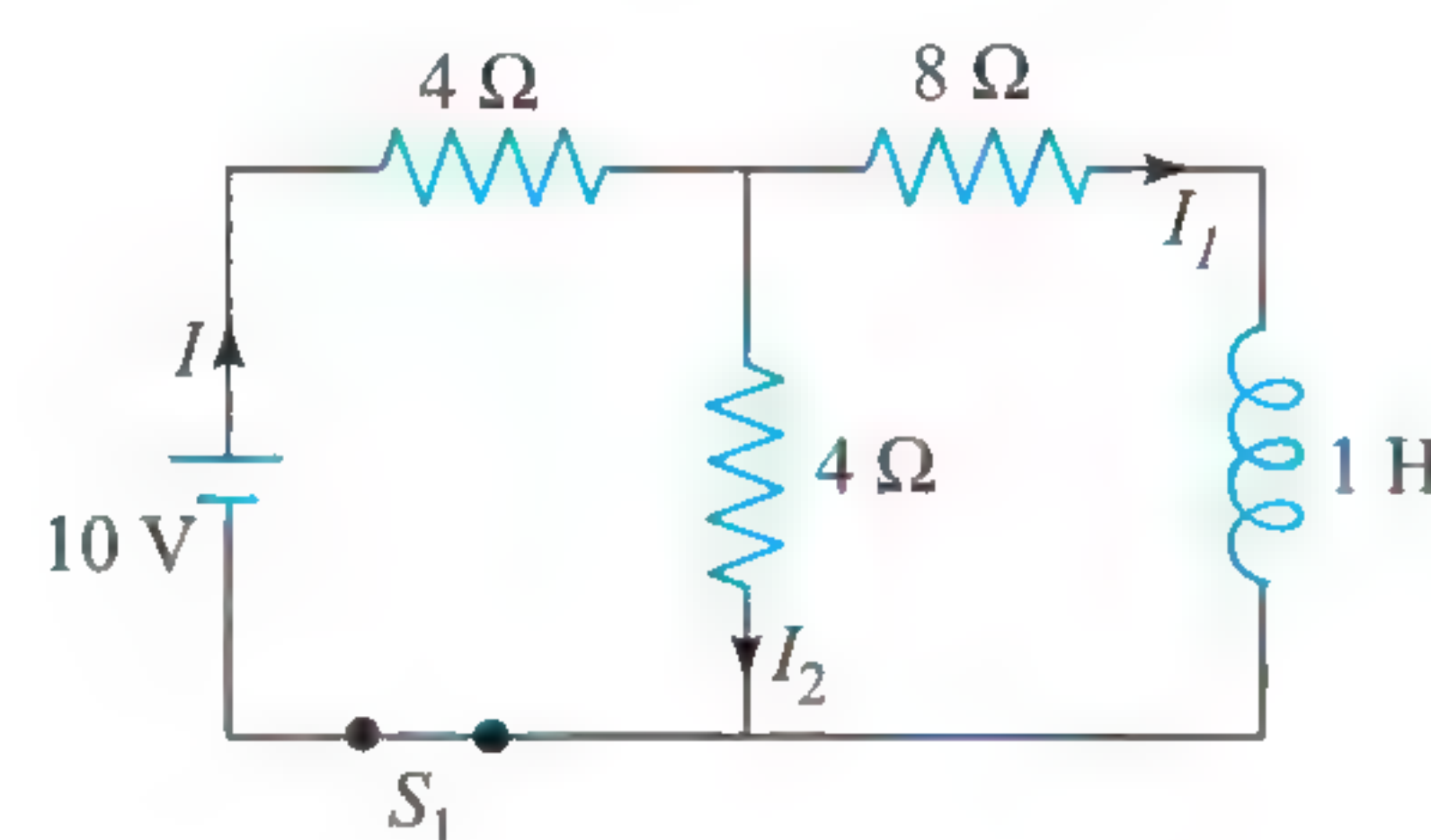
12. $10 - 4I - 8I_1 - \frac{dI_1}{dt} = 0 = 0 \quad \dots(i)$

$$I_1 + I_2 = I \quad \dots(ii)$$

$$10 - 4I - 4I_2 = 0$$

$$\Rightarrow I_2 = \left(\frac{10 - 4I}{4} \right)$$

$$I_1 + \frac{10}{4} = 2I \Rightarrow 4I = 2I_1 + 5$$



Put in equation (i), $10 - 2I_1 - 5 - 8I_1 = \frac{dI_1}{dt}$

$$dt = \frac{dI_1}{5 - 10I_1}$$

$$-t \times 10 = \ln \left(\frac{5 - 10I_1}{5} \right)$$

Current through inductor $I_1 = \frac{1}{2} (1 - e^{-10t})$

Current through switch $I = \frac{5}{4} + \frac{I_1}{2} = \frac{5}{4} + \frac{1}{4} (1 - e^{-10t})$

Exercise 5.4

1. At different times, $(U_C)_{\text{max}} = (U_L)_{\text{max}}$, so $\left[\frac{1}{2} C (\Delta V)^2 \right]_{\text{max}} = \frac{1}{2} L I_i^2$

$$\text{Then, } I_i = \sqrt{\frac{C}{L}} (\Delta V)_{\text{max}} = \sqrt{\frac{1.00 \times 10^{-6} \text{ F}}{10.0 \times 10^{-3} \text{ H}}} (40.0 \text{ V})$$

$$= 0.400 \text{ A} = 400 \text{ mA}$$

2. When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_i = \frac{\mathcal{E}}{R}$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the LC loop. We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV , in $\frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI_i^2$.

$$\begin{aligned}\text{Then, } L &= \frac{C(\Delta V)^2}{I_i^2} = \frac{C(\Delta V)^2 R^2}{\mathcal{E}^2} \\ &= \frac{(1.0 \times 10^{-6})(100)^2 (200)^2}{(50)^2} \\ &= 0.16 \text{ H} = 160 \text{ mH}\end{aligned}$$

3. At different times, the maximum energy stored in the capacitor is equal to the maximum energy stored in the inductor.

$$\left[\frac{1}{2}C(\Delta V)^2 \right]_{\max} = \frac{1}{2}LI_i^2$$

$$\text{So } (\Delta V_c)_{\max} = \sqrt{\frac{L}{C}} I_i = \sqrt{\frac{20.0 \times 10^{-3}}{0.500 \times 10^{-6}}} (0.100) = 20.0 \text{ V}$$

4. (a) The frequency of oscillation of the circuit is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.0)(1.0 \times 10^{-6})}} = \frac{500}{\pi} \text{ Hz}$$

- (b) The maximum charge on the capacitor is

$$Q = C\mathcal{E} = (1.00 \times 10^{-6})(12.0) = 12.0 \text{ } \mu\text{C}$$

- (c) To find the maximum current I_i , we equate

$$\frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_i^2$$

Then solve for I_i to obtain

$$I_i = \mathcal{E} \sqrt{\frac{C}{L}} = 12.0 \sqrt{\frac{1.00 \times 10^{-6}}{1.0}} = 12.0 \text{ mA}$$

- (d) The total energy the circuit possesses at $t = 3.00$ s and at all times is

$$U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}(1.00 \times 10^{-6})(12.0)^2 = 72.0 \text{ } \mu\text{J}$$

5. (a) The voltage across the inductor is given by

$$\mathcal{E}_L = -L \frac{di}{dt} = -L \frac{d}{dt}(Kt) = -LK$$

- (b) The current into the capacitor is $i = \frac{dq}{dt}$, so the charge that flows into the capacitor is

$$q = \int_0^t i dt = \int_0^t Kt dt = \frac{1}{2}kt^2$$

Going across the capacitor in the direction of the current, the potential drops from the positive to the negative side, so

$$\Delta V_c = \frac{-q}{C} = -\frac{kt^2}{2C}$$

- (c) When $\frac{1}{2}C(\Delta V_c)^2 \geq \frac{1}{2}LI^2$,

$$\frac{1}{2}C \left(\frac{k^2 t^4}{4C^2} \right) \geq \frac{1}{2}L(k^2 t^2)$$

Thus, the energy in the capacitor begins to exceed the energy in the inductor after $t = 2\sqrt{LC}$.

6. The total energy equals the energy in the capacitor and inductor:

$$\frac{1}{2} \frac{Q^2}{C} = \frac{1}{2C} \left(\frac{Q}{2} \right)^2 + \frac{1}{2} Li^2 \text{ so } i = \sqrt{\frac{3Q^2}{4CL}}$$

The flux through each turn of the coil is

$$\Phi_B = \frac{Li}{N} = \frac{q}{2N} \sqrt{\frac{3L}{C}}, \text{ where } N \text{ is the number of turns.}$$

7. (a) The rate at which the energy is being stored in the inductor is

$$\begin{aligned}\frac{dU_B}{dt} &= \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = Li \frac{di}{dt} \\ &= L \left(\frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \right) \left(\frac{\mathcal{E}}{R} \frac{1}{\tau_L} e^{-t/\tau_L} \right) \\ &= \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L}) e^{-t/\tau_L}.\end{aligned}$$

$$\text{Now, } \tau_L = \frac{L}{R} = \frac{2.0}{10} = 0.20 \text{ s}$$

and $\mathcal{E} = 100$ V, so the above expression yields $dU_B/dt = 2.4 \times 10^2$ W when $t = 0.10$ s.

- (b) The rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\mathcal{E}^2}{R^2} (1 - e^{-t/\tau_L})^2 R = \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L})^2.$$

At $t = 0.10$ s, this yields $P_{\text{thermal}} = 1.5 \times 10^2$ W.

- (c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9 \times 10^2 \text{ W.}$$

8. The rate at which the energy is being stored in the inductor is

$$\begin{aligned}\frac{dU_B}{dt} &= \frac{d(Li^2/2)}{dt} = Li \frac{di}{dt} \\ &= L \left(\frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \right) \left(\frac{\mathcal{E}}{R} \frac{1}{\tau_L} e^{-t/\tau_L} \right) \\ &= \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L}) e^{-t/\tau_L}\end{aligned}$$

where $\tau_L = L/R$ has been used. The rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\mathcal{E}^2}{R^2} (1 - e^{-t/\tau_L})^2 R = \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L})^2.$$

We equate this to dU_B/dt , and solve for the time:

$$\frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L})^2 = \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L}) e^{-t/\tau_L}$$

$$\Rightarrow t = \tau_L \ln 2 = 40.0 \ln 2 \text{ ms.}$$

Exercises

Single Correct Answer Type

1. (1) $U = \frac{1}{2} Li^2$

$$\text{Rate} = \frac{dU}{dt} = (Li) \left(\frac{di}{dt} \right)$$

$$\text{At } t = 0, \quad i = 0, \quad \therefore \text{rate} = 0$$

at $t = \infty$, $i = i_0$ but $\frac{di}{dt} = 0$,

Therefore, rate = 0.

2. (2) $e = E - iR$

Clearly, the graph is a straight line with negative slope.

3. (3) The mutual inductance M remains the same whether X or Y is used as the primary.

4. (2) At $t = 0$, charge on C is zero, so p.d. across C is zero, p.d. across R is also zero. Hence there is no current in R .

At $t = \infty$, current through L is maximum and constant, so p.d. across L is zero, therefore p.d. across R is zero. Hence no current in R .

5. (2) $I = \frac{I_0}{\alpha}$ at $t = t_0$

$$\text{Since } I = I_0 e^{-t_0/\tau} \Rightarrow \frac{1}{\alpha} = e^{-t_0/\tau}$$

$$\alpha = e^{t_0/\tau} \Rightarrow t_0 = \tau \log_e \alpha$$

$$\tau = \frac{t_0}{\log_e \alpha}$$

6. (3) Since, $\varepsilon = \frac{d\phi}{dt} = \frac{d}{dt}(NBA)$

$$\varepsilon = \frac{d}{dt}[NA(\mu_0 nI)]; \varepsilon = NA\mu_0 n \left(\frac{dI}{dt} \right)$$

where N is total number of turns in the coil and n is the number of turns per unit length in the solenoid.

$$\varepsilon = (300)(1.2 \times 10^{-3})(4\pi \times 10^{-7}) \times \frac{2000}{0.3} \times \frac{4}{0.25}$$

$$\varepsilon = 4.8 \times 10^{-2} \text{ V} = 48 \text{ mV}$$

7. (2) $\frac{L}{R} = 2 \times 10^{-3}$... (i)

$$\frac{L}{R+90} = 0.5 \times 10^{-5} \quad \dots \text{(ii)}$$

From (i) and (ii), on solving, we get

$$L = 60 \text{ mH and } R = 30 \Omega$$

8. (4) Induced current in $B = 0.006 \text{ A} = 6 \times 10^{-3} \text{ A}$

$$\text{Induced e.m.f. in } B = 6 \times 10^{-3} \times 4 \text{ V} = 24 \times 10^{-3} \text{ V}$$

$$\text{Now, } M \frac{dI}{dt} = 24 \times 10^{-3}$$

$$\text{or } dI = \frac{24 \times 10^{-3} \times 0.02}{3 \times 10^{-3}} \text{ A} = 0.16 \text{ A}$$

9. (2) $I_{t=\infty} = I_0$

$$I_{t=1s} = I_0 \left(1 - e^{-\frac{10}{5} \times 1} \right)$$

$$= I_0(1 - e^{-2}) = I_0 \left(1 - \frac{1}{e^2} \right)$$

$$\therefore = \frac{I_{t=\infty}}{I_{t=1s}} = \frac{I_0 e^2}{I_0(e^2 - 1)} = \frac{e^2}{e^2 - 1}$$

10. (4) $V = RI + L \frac{dI}{dt}$, at $t = 0$, $I = 0$, thus we have

$$\frac{dI}{dt} = \frac{V}{L} = \frac{1.6}{0.2} = 8 \text{ A/s}$$

11. (3) $M = \frac{\mu_0 N_1 N_2 A}{l} a$, here $l = 1 \text{ m}$

$$= 503 \times 10^{-6} \text{ H}$$

12. (1) Let I = current in one loop. The magnetic field at the centre of the other co-axial loop at a distance l from the centre of the first loop is

$$B = \frac{\mu_0}{4\pi} \frac{2I\pi a^2}{(a^2 + l^2)^{3/2}}$$

$$\text{where } p_m = I\pi a^2$$

= magnetic moment of the loop

Flux through the other loop is

$$\phi_{12} = B\pi a^2$$

$$= \frac{\mu_0}{4\pi} \frac{2\pi a^2 I}{(a^2 + l^2)^{3/2}} \pi a^2$$

$$\text{or } M = \frac{\phi_{12}}{I} = \frac{\mu_0}{4\pi} \frac{2\pi^2 a^4}{(a^2 + l^2)^{3/2}}$$

$$= \frac{\mu_0 \pi a^4}{2(a^2 + l^2)^{3/2}} = \frac{\mu_0 \pi a^4}{2l^3} \quad (a \ll l)$$

13. (2) Let l_1 is the length of wire, then

$$l_1 = 2\pi rN \Rightarrow Nr = \frac{l_1}{2\pi} \Rightarrow L = \frac{\mu_0 \pi}{l} \frac{l_1^2}{4\pi^2}$$

$$\Rightarrow l_1 = \sqrt{\frac{Ll4\pi}{\mu_0}} = \sqrt{\frac{10^{-3} \times 1 \times 4\pi}{4\pi \times 10^{-7}}} = 10^2 \text{ m} = 0.10 \text{ km}$$

14. (4) Energy spent = $\frac{1}{2} LI^2$

15. (2) $B = \mu_0 nI = 4\pi \times 10^{-7} \times 200 \times 10^2 \times 1.5 = 3.8 \times 10^{-2} \text{ T}$

$$\phi = BA = 3.8 \times 10^{-2} \times 3.14 \times 10^{-4} = 1.2 \times 10^{-5} \text{ Wb}$$

When the current in the solenoid is reversed, the change in magnetic flux,

$$d\phi = 2 \times (1.2 \times 10^{-5}) = 2.4 \times 10^{-5} \text{ Wb}$$

$$\therefore \text{ Induced e.m.f., } e = N \left(\frac{d\phi}{dt} \right)$$

$$= 100 \times \left(\frac{2.4 \times 10^{-5}}{0.05} \right) = 0.048 \text{ V}$$

16. (4) Coefficient of mutual inductance M is given by

$$|M| = \frac{e_1}{(di_2/dt)} = \frac{\phi_2}{i_1}$$

$$\therefore \phi_2 = \frac{e_1 i_1}{(di_2/dt)} = \frac{(25.0 \times 10^{-3})(3.6)}{(15)}$$

$$= 6 \times 10^{-3} = 6 \text{ mWb}$$

17. (1) $i = i_0 e^{-t/t_0}$

$$\left(-\frac{di}{dt} \right) = \frac{i_0}{t_0} e^{-t/t_0}$$

$$\left(-\frac{di}{dt} \right) = \frac{i_0}{t_0} = r \quad (\text{at } t = 0)$$

$\therefore$ The desired time is $\frac{i_0}{r}$ or t_0 .

18. (2) Magnetic field at the centre of a large coil $E \propto \frac{\mu_0 I}{2R}$.

$$\text{Magnetic flux linked with smaller coil} = \frac{\mu_0 I}{2R} \times \pi r^2$$

$$M = \frac{\phi}{I} = \frac{\mu_0 \pi r^2}{2R}$$

$$\therefore M \propto \frac{r^2}{R}$$

$$19. (3) L = \frac{\mu_0 N^2 A}{l}$$

If x is the length of the solenoid with r as radius, then

$$x = 2\pi r N, A = \pi r^2$$

$$\therefore L = \mu_0 \left(\frac{x^2}{4\pi^2 r^2} \right) \frac{\pi r^2}{l} \quad \left[\therefore N = \frac{x}{2\pi r} \right]$$

$$\therefore x = \sqrt{\frac{4\pi L l}{\mu_0}}$$

20. (2) The power dissipated in the resistor,

$$P = \frac{dW}{dt} = I^2 R$$

Since the current through resistor varies with time, we must integrate.

The total energy produced as heat in the resistor

$$W = \int_0^\infty I^2 R dt$$

The current in an RL circuit is $I = I_0 e^{-(R/L)t}$

$$W = \int_0^\infty I_0^2 e^{-(2R/L)t} R dt = \frac{I_0^2 R}{-2R/L} \left[e^{-\frac{2R}{L}t} \right]_0^\infty = \frac{1}{2} L I_0^2$$

We can integrate by substituting

Note that the total heat produced equals the energy $(1/2) L I_0^2$ originally stored in the conductor.

21. (2) The current is short circuited through inductor. In steady state, current will pass through inductor instead of 20Ω .

Then current through 30Ω is:

$$I = \frac{3}{30} = 0.1 \text{ A}$$

22. (1) For a solenoid, $L = \mu_0 N^2 \frac{A}{l}$. If x is the length of the wire and a is the area of cross-section, then

$$R = \frac{\rho x}{a} \text{ and } m = axD$$

$$Rm = \frac{\rho x}{a} axD, \quad x = \sqrt{\frac{Rm}{\rho D}}$$

$$\text{Also, } x = 2\pi r N, \quad N = \frac{x}{2\pi r} \quad \left(\therefore L = \frac{\mu_0 N^2 A}{l} \right)$$

$$\therefore L = \mu_0 \left(\frac{x}{2\pi r} \right)^2 \frac{\pi r^2}{l} = \frac{\mu_0}{4\pi l} \frac{Rm}{\rho D}$$

23. (3) $\omega_r = \text{constant}$

$$\Rightarrow LC = \text{constant}$$

$$\Rightarrow L dC + C dL = 0 \Rightarrow \frac{dL}{L} = -\frac{dC}{C} = -1\%$$

$$24. (4) L = \frac{\phi}{I}, \phi = NAB$$

$$B = \mu_0 nI$$

$$\text{where } n = \frac{N}{2\pi R}$$

$$\therefore \phi = N\pi r^2 \left(\mu_0 \frac{N}{2\pi R} I \right)$$

$$\phi = \frac{\mu_0 N^2 r^2 I}{2R}$$

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 r^2}{2R}$$

$$25. (4) E = \frac{M di}{dt} = Ma, \quad i = \frac{Ma}{R} (1 - e^{-tR/L})$$

26. (3) Flux through a closed circuit containing an inductor does not change instantaneously.

$$\therefore L \left(\frac{E}{R} \right) = \frac{L}{4} (i) \Rightarrow i = \frac{4E}{R}$$

27. (1) The current in L for steady state = $\frac{E}{R_1}$

$$\therefore \text{Energy stored in } L = \frac{1}{2} L I_0^2 = \frac{1}{2} L \frac{E^2}{R_1^2}$$

$$28. (2) |e| = B \frac{dA}{dt}$$

$$|e| = B \frac{(A_2 - A_1)}{(t_2 - t_1)}$$

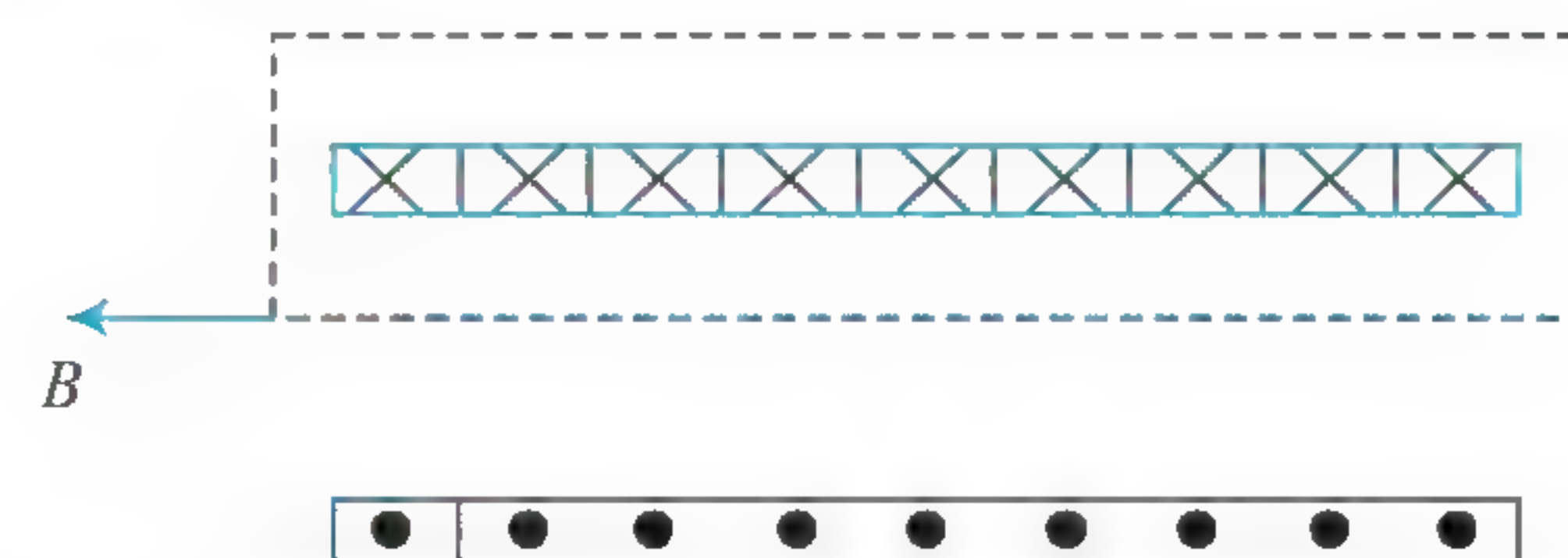
$$|e| = \frac{1 \left(\frac{\pi}{4} - 0 \right)}{(1 - 0)}$$

$$e = \frac{\pi}{4} \text{ V}$$

$$29. (1) \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I \Rightarrow B 2b = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2b} \rightarrow \text{due to one tape}$$

Net field = $2B$



Magnetic flux passing through this double tape

$$\phi = 2BA = 2B(\ell h) \Rightarrow \phi = \frac{\mu_0 I}{b} \ell h$$

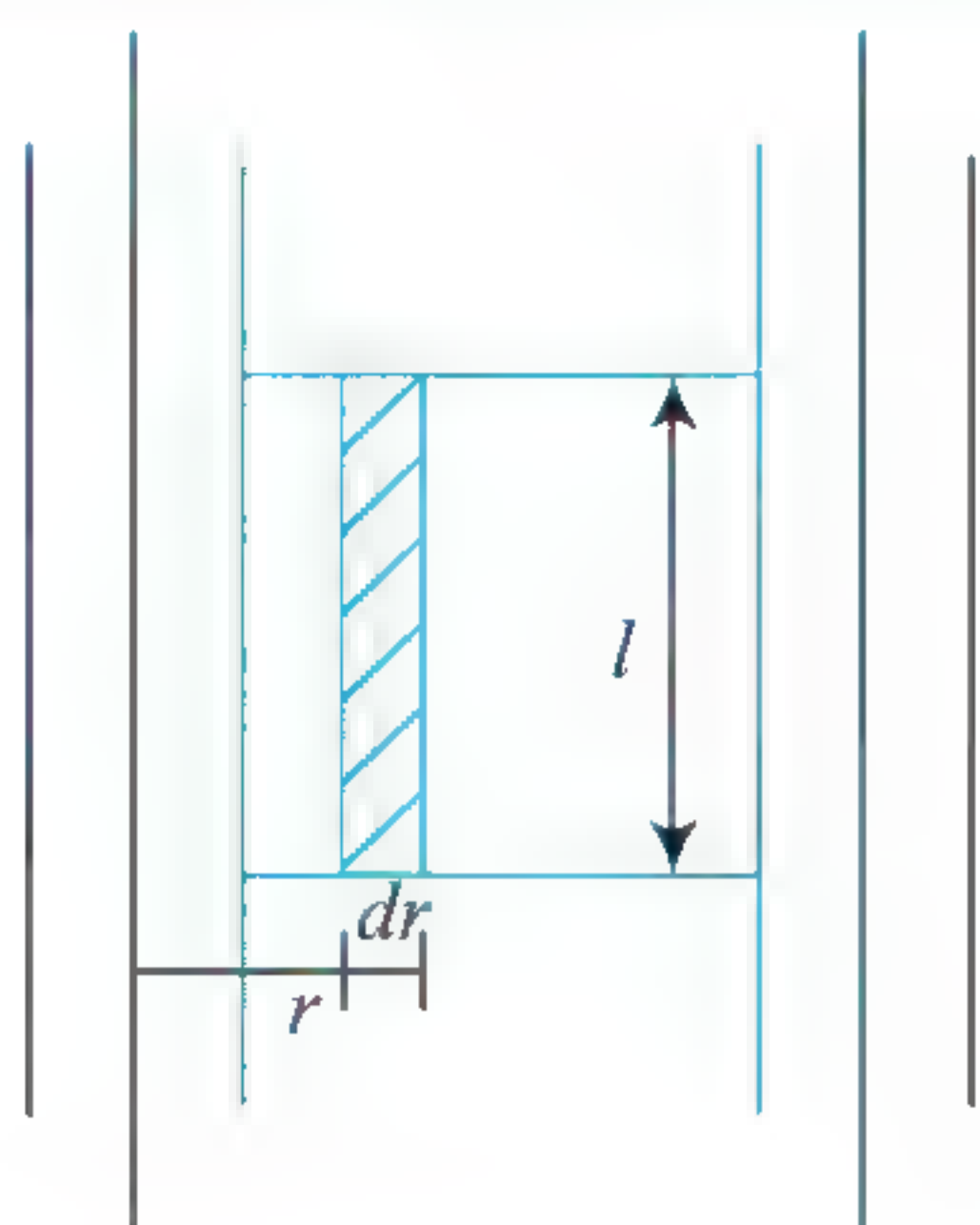
$$L = \frac{\phi}{I} = \frac{\mu_0 \ell h}{b} \Rightarrow \frac{L}{\ell} = \frac{\mu_0 h}{b}$$

30. (2) Since the wires are infinite, so the system of these two wires can be considered as a closed rectangle of infinite length and breadth equal to d . Flux through the strip:

$$\phi = \int_a^{d-a} \frac{\mu_0 I}{2\pi r} (ldr) = \frac{\mu_0 I l}{2\pi} \ln \left(\frac{d-a}{a} \right)$$

The other wire produces the same result, so the total flux through the dotted rectangle is

$$\phi_{\text{total}} = \frac{\mu_0 I l}{\pi} \ln \left(\frac{d-a}{a} \right)$$



The total inductance of length l

$$L = \frac{\phi_{\text{total}}}{I} = \frac{\mu_0 l}{\pi} \ln \left(\frac{d-a}{a} \right)$$

$$\text{Inductance per unit length} = \frac{L}{l} = \frac{\mu_0}{\pi} \ln \left(\frac{d-a}{a} \right)$$

31. (2) Let current i_1 in the straight wire be upward. Then the magnetic field due to the straight wire has magnitude $B_1 = \mu_0 i_1 / 2\pi r$ at a distance r . In accordance with right hand rule, B_1 points inward to the plane of page. We consider a differential strip of thickness dr , area $dA_2 = a dr$. Magnetic flux through area dA ,

$$d\phi_B = B_1 (a dr)$$

Total flux through the loop,

$$\begin{aligned} \phi_B &= \int B_1 a dr = \int_c^{c+b} \frac{\mu_0 i_1}{2\pi r} a dr \\ &= \frac{\mu_0 i_1 a}{2\pi} \int_c^{c+b} \frac{dr}{r} = \frac{\mu_0 i_1 a}{2\pi} \ln \left(\frac{c+b}{c} \right) \end{aligned}$$

Therefore mutual inductance,

$$M = \frac{\phi}{i_1} = \frac{\mu_0 a}{2\pi} \ln \left(1 + \frac{b}{c} \right)$$

32. (2) Inductors 5 mH and 10 mH are connected in parallel, hence equivalent inductance $L_{\text{eq}} = \frac{5 \times 10}{5+10} = \frac{50}{15} = \frac{10}{3}$ mH

$$\text{Current at steady state, } I = \frac{20}{5} = 4 \text{ A}$$

As L_1 and L_2 are in parallel,

$$I_1 = I \left(\frac{L_2}{L_1 + L_2} \right) = 4 \left[\frac{10}{10+5} \right] = \frac{8}{3} \text{ A}$$

33. (1) When energy on both is same, means energy on capacitor is half of its maximum energy.

$$\frac{q^2}{2C} = \frac{1}{2} \frac{Q^2}{2C} \Rightarrow q = \frac{Q}{\sqrt{2}}$$

$$\Rightarrow Q \cos \omega t = \frac{Q}{\sqrt{2}} \Rightarrow \cos \omega t = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4} \sqrt{LC}$$

34. (4) $E = L \frac{dI}{dt}$ or $dI = \frac{E}{L} dt$

$$\text{or } I = \frac{2}{4} t = 0.5 t \Rightarrow t = 2I = 2 \times 5 = 10 \text{ s}$$

35. (2) In steady state: $P = I^2 R$

$$\text{Energy in inductor} = \frac{1}{2} LI^2$$

After connecting x and z, the whole energy of inductor will go

$$\text{into heat so heat produced: } H = \frac{1}{2} LI^2 = \frac{1}{2} L \frac{P}{R} = \frac{1}{2} P\tau$$

$$36. (1) \Delta q = \frac{\Delta \phi}{R}$$

$$\phi_i = 0$$

$$\phi_f = \left(\frac{\mu_0}{2} \frac{i}{b} \right) (\pi a^2) = \frac{\mu_0 i a^2 \pi}{2b}$$

$$\therefore \Delta \phi = \phi_f - \phi_i = \frac{\mu_0 i a^2 \pi}{2b}$$

$$\text{So, } \Delta q = \frac{\mu_0 i a^2 \pi}{2Rb}$$

37. (3) The current at time t is given by

$$i = i_0 (1 - e^{-t/\tau})$$

$$\text{Here } i_0 = \frac{E}{R} \text{ and } \tau = \frac{L}{R}$$

$$\therefore q = \int_0^\tau i dt = \int_0^\tau i_0 (1 - e^{-t/\tau}) dt$$

$$= \frac{i_0 \tau}{e} = \frac{\left(\frac{E}{R} \right) \left(\frac{L}{R} \right)}{e} = \frac{EL}{eR^2}$$

38. (1) At $t = 0$, the branch containing L will offer infinite resistance while the branch containing the capacitor will be effectively a short circuit. Hence, $(i)_{t=0} = \frac{\mathcal{E}}{R}$. Similarly at $t = \infty$, L will offer zero resistance whereas C will be an open circuit. Hence effective resistance $= R + \frac{6 \times 3R}{6+3} = 3R$ $(i)_{t=\infty} = \frac{\mathcal{E}}{3R}$.

$$\text{The required ratio} = \frac{\mathcal{E}}{R} \times \frac{3R}{\mathcal{E}} = 3:1$$

39. (3) Induced e.m.f. $e = L \frac{dI}{dt} = A \frac{dB}{dt}$

$$\Rightarrow \int_0^I dI = \int_0^B \frac{A}{L} dB \Rightarrow I = \frac{A}{L} B$$

$$\Rightarrow I_{\text{max}} = \frac{A}{L} B_{\text{max}} = \frac{10^{-2}}{10 \times 10^{-3}} \times 0.1 = 0.1 \text{ A} = 100 \text{ mA}$$

40. (2) $E_i - ir_1 = 0 \Rightarrow i = \frac{E_1}{r_1} = \frac{3}{1} = 3 \Omega$

$$i = \frac{10}{3R+2}, \frac{10}{3R+2} = 3, R = \frac{4}{9} \Omega$$

41. (2) Use $i = \frac{E}{R} [1 - e^{-t/\tau}]$ and $U = \frac{1}{2} Li^2$

42. (3) Constancy of flux implies that $\frac{E}{R} L_1 = i(L_1 + L_2)$

$$\text{i.e., } i = \frac{EL_1}{R(L_1 + L_2)}$$

43. (3) Let i_1 be the current in the circuit before shifting

$$i_1 = \frac{E}{R} \quad \dots(i)$$

Since the flux associated with the inductors will be same just before and just after shifting, therefore

$$i_1 5L = i_2 9L$$

$$i_2 = \frac{5}{9} \frac{E}{R}$$

44. (4) At $t=0$, for the purpose of current calculation in circuit, inductor can be assumed as open and capacitor as short circuited.

45. (1) $L = \frac{\mu_0 N^2 \pi r^2}{\ell}$

Length of wire $= N 2\pi r = \text{Constant} (= C, \text{ suppose})$

$$\therefore L = \mu_0 \left(\frac{C}{2\pi r} \right)^2 \frac{\pi r^2}{\ell}$$

$$\therefore L \propto \frac{1}{\ell}$$

$\therefore$ Self-inductance will become $2L$.

46. (2) Equivalent inductance

$$L_{\text{eq}} = L + 2L = 3L, C_{\text{eq}} = C + 2C = 3C$$

$\therefore$ Frequency of oscillation

$$f = \frac{1}{2\pi \sqrt{L_{\text{eq}} C_{\text{eq}}}} = \frac{1}{6\pi \sqrt{LC}}$$

47. (3) Just before opening the switch, the current in the inductor is $\frac{E}{R}$.

$$\text{Energy stored in it} = \frac{1}{2} L \left(\frac{E}{R} \right)^2$$

This energy will dissipate in resistors R_1 and R_2 in the ratio $\frac{1}{R_1}$ and $\frac{1}{R_2}$.

48. (1) Given, $L_1 = 1 \text{ mH}$, $L_2 = 2 \text{ mH}$, $R_1 = 1 \Omega$, $R_2 = 2 \Omega$

In the first circuit,

$$L = L_1 + L_2 \text{ and } R = R_1 + R_2$$

$$\tau_1 = \frac{L}{R} = \frac{3 \text{ mH}}{3 \Omega} = 1 \text{ ms}$$

In the second circuit,

$$L = \frac{L_1 L_2}{L_1 + L_2} = \frac{2 \times 10^{-6}}{3 \times 10^{-3}} = \frac{2}{3} \times 10^{-3}$$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{2}{3} \Omega$$

$$\tau_2 = \frac{\frac{2}{3} \times 10^{-3}}{\frac{2}{3}} = 1 \text{ ms}$$

In the third circuit,

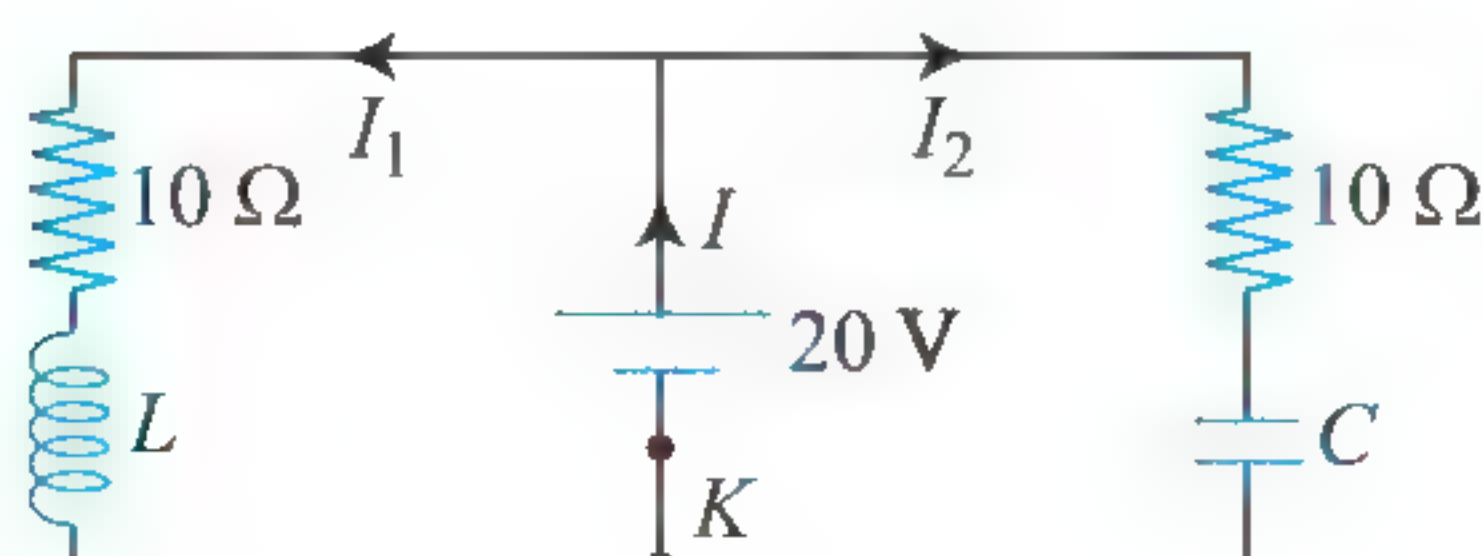
$$L = L_1 + L_2 = 3 \text{ mH}$$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{2}{3} \Omega$$

$$\tau_3 = \frac{L}{R} = \frac{3 \times 10^{-3}}{\frac{2}{3}} = \frac{9}{2} \text{ ms}$$

49. (3) Current in branches containing L and R will flow independently

$$I_1 = \frac{20}{10} \left(1 - e^{-\frac{t}{5 \times 10^{-4}}} \right) = \frac{3}{2} = 1.5 \text{ A}$$



$$I_2 = \frac{20}{10} e^{-\frac{t}{10^{-3}}} = 1.0 \text{ A}$$

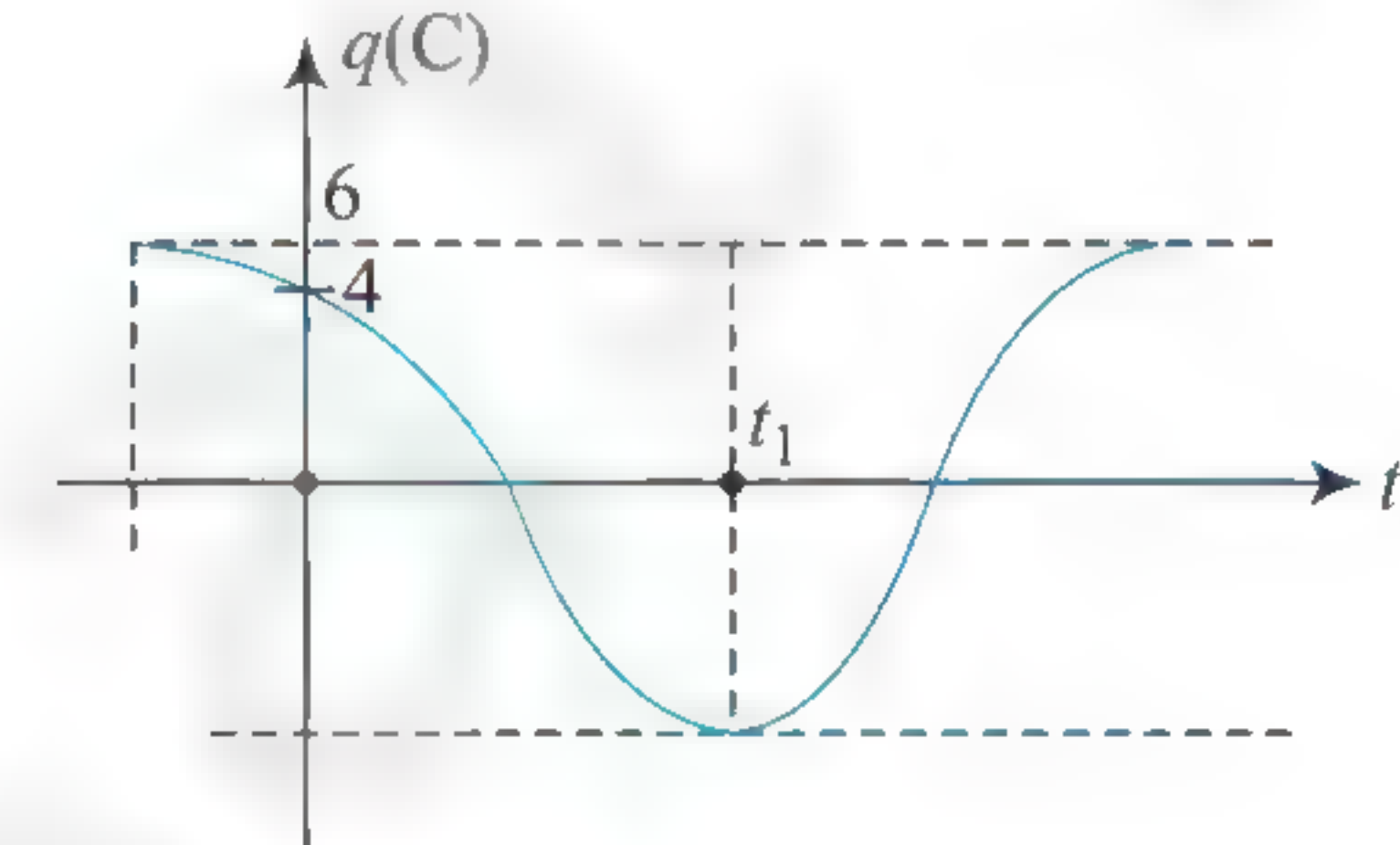
$$I = I_1 + I_2 = 2.5 \text{ A}$$

50. (1) For $t < t_0$, $E = L \frac{di}{dt} \Rightarrow i = \frac{E}{L} t$

$$\text{For } t > t_0, L \frac{di}{dt} = 0 \Rightarrow i = \text{constant}$$

51. (1) $i = \sqrt{5} \text{ A}$

$$\frac{q_m^2}{2C} = \frac{q^2}{2C} + \frac{1}{2} Li^2 \Rightarrow q_{\text{max}} = 6 \text{ C}$$



$$q = Q \cos(\omega t + \delta), \text{ at } t = 0, q = 4$$

$$\Rightarrow 4 = 6 \cos \delta \Rightarrow \cos \delta = \frac{2}{3} \Rightarrow \delta = \cos^{-1} \left(\frac{2}{3} \right)$$

Charge on the capacitor is maximum at $t = t_1$
for this $\omega t_1 + \delta = \pi$

$$\Rightarrow t_1 = \frac{1}{\omega} (\pi - \delta) = \sqrt{LC} (\pi - \delta) = 2 \left[\pi - \cos^{-1} \left(\frac{2}{3} \right) \right]$$

52. (1) $R_{\text{eq}} = \frac{5R}{6} \Rightarrow I = \frac{6E}{5R} = 1 \text{ A}$

53. (1) $\omega = \frac{1}{\sqrt{L_{\text{eq}} C_{\text{eq}}}} = \frac{1}{\sqrt{2LC/2}} = \frac{1}{\sqrt{LC}}$

54. (1) At $t = 0$, i.e., when the key is just pressed, no current exists inside the inductor. So 10Ω and 20Ω resistors are in series and a net resistance of $(10 + 20) = 30 \Omega$ exists across the circuit.

$$\text{Hence, } I_1 = \frac{2}{30} = \frac{1}{15} \text{ A}$$

As $t \rightarrow \infty$, the current in the inductor grows to attain a maximum value, i.e., the entire current passes through the inductor and no current passes through 10Ω resistor.

$$\text{Hence, } I_2 = \frac{2}{20} = \frac{1}{10} \text{ A}$$

55. (3) $U_{\text{max}} = \frac{1}{2} LI_0^2, U = \frac{U_{\text{max}}}{2}$

$$\Rightarrow \frac{1}{2} LI^2 = \frac{1}{2} \left[\frac{1}{2} LI_0^2 \right] \Rightarrow I_0^2 [1 - e^{-t/\tau}]^2 = \frac{I_0^2}{2}$$

$$\Rightarrow e^{-t/\tau} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\Rightarrow -\frac{t}{\tau} = \ln \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right) \Rightarrow t = \tau \ln \left(\frac{\sqrt{2}}{\sqrt{2}-1} \right)$$

56. (3) The magnetic field produced by the square loop is parallel to the plane of the circular loop. Hence the mutual inductance is zero.

57. (4) Let current i flows in the bigger ring, then the magnetic field on its axis

$$B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$

Flux linked with the smaller ring: $\phi = B\pi r^2$

$$\phi = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}} \pi r^2 = M i$$

$$\therefore M = \frac{\mu_0 \pi R^2 r^2}{2(R^2 + x^2)^{3/2}}$$

58. (1) Maximum mutual inductance:

$$M = \sqrt{L_1 L_2} = 200 \text{ mH}$$

59. (2) Current through the inductor before closing the switch

$$= 10/(5 + 5) = 1 \text{ A}$$

Current through the inductor after closing the switch (in steady

$$\text{state}) I = \frac{20}{5} = 4 \text{ A}$$

$$\therefore \Delta\phi = L\Delta I = 1.5 \text{ Wb}$$

60. (1) Initially no current in the inductor, so initial current

$$i_1 = \frac{15}{R_1 + R_2 + R_3} = 0.25 \text{ A}$$

In steady state, no current in R_2 and R_3 . So final current through

$$\text{battery: } i_2 = \frac{15}{R_1} = 0.5 \text{ A}$$

61. (2) Since resistance of the circuit is increasing and hence current in the circuit is decreasing, so di/dt is negative.

$$\text{Current in the circuit is given by } i = \frac{6 - L \frac{di}{dt}}{12}$$

Since (di/dt) is negative so the value of numerator will be more than 6V and hence current in the circuit at that instant will be more than 0.5 A.

62. (4) Induced emf in the coil is given by $E = -L \frac{di}{dt}$

From the graph it is clear that for time t_1 , current does not change with time, i.e., $\frac{di}{dt} = 0$. So, induced emf is zero.

Hence, graph (1) and (2) are wrong.

From time interval t_1 to t_2 , (di/dt) is negative and constant.

$$\text{So, } E = -L \frac{di}{dt} = -L \left(-\frac{di}{dt} \right) \Rightarrow E = \frac{L di}{dt}$$

Hence, induced emf is positive and constant.

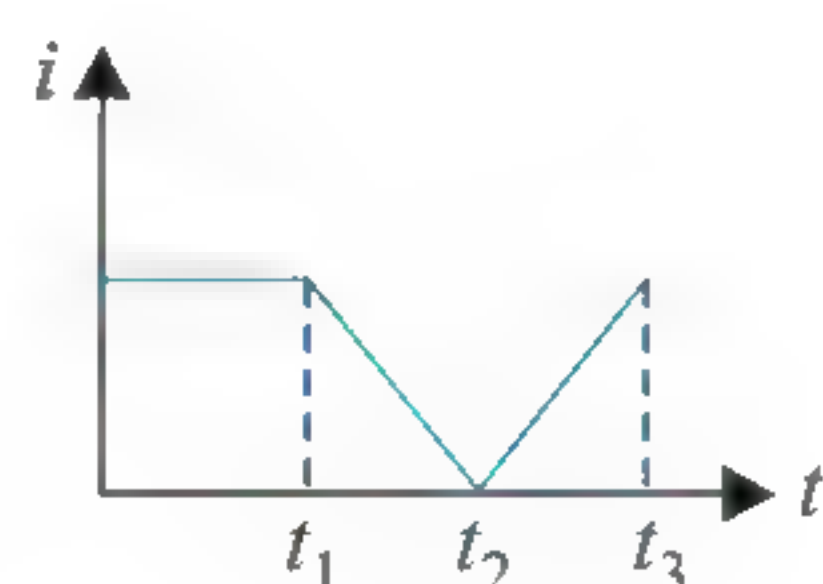
From t_2 to t_3 , (di/dt) is positive and constant.

$$\text{So, induced emf } E = -L \left(+\frac{di}{dt} \right) = -\frac{L di}{dt}$$

Hence, induced emf is negative and constant.

Hence, choice (4) is correct.

Choice (3) is wrong because in the graph the induced emf is never negative and is not constant.



63. (2) Maximum current $I_m = \frac{100}{50} = 2 \text{ A}$

Potential drop across the resistor is $2 \times 50 = 100 \text{ V}$

Initial potential difference across capacitor is zero

$$L_{eq} = (200 + 50) = 250 \text{ mH}$$

$$\text{Energy stored} = \frac{1}{2} L_{eff} I_M^2 = \frac{1}{2} \times 250 \times 10^{-3} \times 2^2 = 0.5 \text{ J}$$

$$\text{When } S_1 \text{ is opened } \frac{1}{2} L_{eff} I_M^2 = \frac{1}{2} C V_{max}^2$$

$$V_{max} = \sqrt{\frac{2 \times 0.5}{100 \times 10^{-6}}} = 100 \text{ V}$$

64. (1) See the diagram in the problem

The $(L - R)$ and $(C - R)$ sections having the same time constant are in parallel.

The current through the $R - L$ branch will be unaffected by the parallel $R - C$ branch

$$i_L(t) = \frac{E}{R} \left[1 - e^{-\frac{Rt}{L}} \right] \quad \dots(i)$$

Similarly the current in the RC branch is given by

$$i_C(t) = \frac{E}{R} e^{-\frac{t}{RC}} \quad \dots(ii)$$

Hence the current through the battery is

$$i(t) = i_L(t) + i_C(t) = \frac{E}{R} \left[1 - e^{-\frac{R}{C}t} + e^{-\frac{t}{RC}} \right] \quad \dots(iii)$$

Time constant of RL branch is $\tau_L = \frac{L}{R}$ and the time constant

of RC branch is $\tau_C = RC$.

If these are equal, $\tau_L = \tau_C = T$, equation (iii) becomes

$$i(t) = \frac{E}{R} \left[1 - e^{-\frac{t}{T}} + e^{-\frac{t}{T}} \right] = \frac{E}{R} \text{ for all values of } t > 0. \text{ Hence the result.}$$

65. (4) The inductance acts as a short-circuit. The whole of the current. The whole of the current flows through the inductance. Since no current flows through the bulb does not glow.

66. (3) Steady state current $I_0 = \frac{E}{R}$

$$= \frac{12}{12/4} = 4 \text{ A}$$

67. (2) $\phi = MI = MI_0 t$

$$\text{Hence, induced emf } e = \frac{d\phi}{dt} = MI_0$$

$$\text{Current through ring} = \frac{e}{R} = \frac{MI_0}{R}$$

68. (1) Notice that magnetic field due to left coil is towards left. Since magnetic field due to right coil is also towards left, so it increases flux associated with left coil. To oppose it, induced current due to mutual induction opposes the original current in left coil.

69. (4) Angular frequency of oscillation

$$\omega = \frac{1}{\sqrt{LC_1}} = \frac{1}{\sqrt{0.2 \times 10^{-3} \times 8 \times 10^{-6}}} = 2.5 \times 10^4 \text{ rad/s} = 25 \times 10^3 \text{ rad/s}$$

$$\frac{Q}{C_1} - L \frac{dI}{dt} = 0 \Rightarrow \frac{Q}{C_1} + L \frac{d^2 Q}{dt^2} = 0$$

The solution of this equation is $Q = Q_0 \sin(\omega t + \phi)$ where at $t = 0$, $Q = Q_0$

$$Q_0 = C_{eq} V = \frac{C_1 C_2}{C_1 + C_2} V$$

$$= \frac{8 \times 2 \times 10^{-12} \times 20}{10 \times 10^{-6}} = 32 \times 10^{-6} \text{ C}$$

$$\phi = \frac{\pi}{2}$$

$$Q = (32 \mu\text{C}) \cos(2.5 \times 10^4 \times 125 \times 10^{-6}) \approx -32 \mu\text{C}$$

70. (1) Magnetic field energy stored $= U = \frac{1}{2} Li^2$

$$32 = \frac{1}{2} L(4)^2 \quad \text{or} \quad L = 4 \text{ H}$$

Power dissipated as heat is $P = i^2 R$

$$320 = 4^2 R \quad \text{or} \quad R = 20 \Omega$$

$$\text{Time constant of circuit} = \tau = \frac{L}{R} = \frac{4}{20} = 0.2 \text{ s}$$

71. (1) Time constant of circuit $= \frac{L}{R} = \frac{50 \text{ mH}}{10} = 5 \text{ ms}$

Current at time T is given by

$$i = i_0(1 - e^{-T/\tau})$$

$$\text{For } i = \frac{i_0}{2}, \frac{i_0}{2} = i_0(1 - e^{-T/\tau})$$

$$e^{-T/\tau} = \frac{1}{2} \quad \text{or} \quad \frac{T}{\tau} = \log_e 2$$

$$T = \tau \log_2 2 = 5 \text{ ms} \times 0.693 = 3.2 \text{ ms}$$

72. (1) Current at time t is given by

$$i = i_0(1 - e^{-t/\tau}) \text{ where } i_0 = E/R$$

The charge passed through the battery during the period t to $t + dt$ is idt . Thus, the total charge passed during time 0 to τ is

$$Q = \int_0^\tau idt = i_0 \int_0^\tau (1 - e^{-t/\tau}) dt$$

$$= i_0 \left[t + \frac{e^{-t/\tau}}{-1/\tau} \right]_0^\tau = i_0 [\tau + \tau(e^{-1} - 1)]$$

$$= \frac{i_0 \tau}{e} = \frac{E}{R} \frac{\tau}{e}$$

when $t = \infty$.

Multiple Correct Answers Type

1. (3), (4)

When switch is just closed in the circuit shown, at that moment current through the circuit is zero. Hence, e.m.f. induced across inductance L will be equal to e.m.f. E of the battery.

But as the current through the circuit increases, the induced e.m.f. in the solenoid decreases. But induced e.m.f. in the solenoid is equal to $|e| = L \frac{di}{dt}$.

Since di/dt decreases as time passes, therefore the graph for induced e.m.f. e and time t will be as shown in option (3).

Hence (1) is wrong and (3) is correct.

The graph for current should be such that at initial moment current is zero and current increases with time in such a way that the rate of increase of current gradually decreases. Hence, slope of the current-time curve should decrease with time. Therefore, the graph between current and time will be as shown in option (4).

Hence (2) is wrong and (4) is correct.

2. (1), (3)

In the circuit shown in the figure in problem, 6Ω and 12Ω resistances are in parallel with each other and their parallel combination is in series with 4Ω and the inductance of 2H . Hence, equivalent resistance of these three resistances is equal to 8Ω . Therefore, the time constant for the circuit is

$$\lambda = \frac{L}{R} = \frac{2}{8} = 0.25 \text{ s}$$

Hence (1) is correct.

In steady state, no e.m.f. will be induced in the inductance. Hence current through the circuit will be equal to E/R where R is the equivalent resistance. Hence, the steady state current will be equal to $6/8 = 0.75 \text{ A}$.

Hence (3) is correct and (2) is wrong.

3. (1), (2)

The coils are in parallel, so

$$L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt} \Rightarrow \int L_1 dI_1 = \int L_2 dI_2$$

$$\Rightarrow \text{Initially } I_1 = 0, I_2 = 0 \Rightarrow C = 0$$

$$\text{So } L_1 I_1 = L_2 I_2$$

4. (1), (3)

$$V = L \frac{dI}{dt} \Rightarrow \int_0^I dI = \frac{1}{L} \int_0^t V dt$$

$$\Rightarrow I = \frac{1}{L} [\text{Area under } v-t \text{ graph from } t = 0 \text{ to } t = t]$$

$$\text{At } t = 2 \text{ s, } I = \frac{1}{2} \left[\frac{1}{2} \times 2 \times 10 \right] = 5 \text{ A}$$

For $t = 0$ to 2 s , $V = 5t$

$$I = \frac{1}{L} \int_0^t 5t dt = \frac{1}{2} \left[\frac{5t^2}{2} \right]_0^t = \frac{5t^2}{4}$$

For $t = 2$ to 4 s , $V = -5t + 20$

$$I = \frac{1}{2} \int_0^t (-5t + 20) dt \Rightarrow I = \frac{-5t^2}{4} + 10t$$

Hence the correct graph is (3).

5. (1), (2)

$$\text{At } t < 0, I_L = \frac{6}{6} = 1 \text{ A}$$

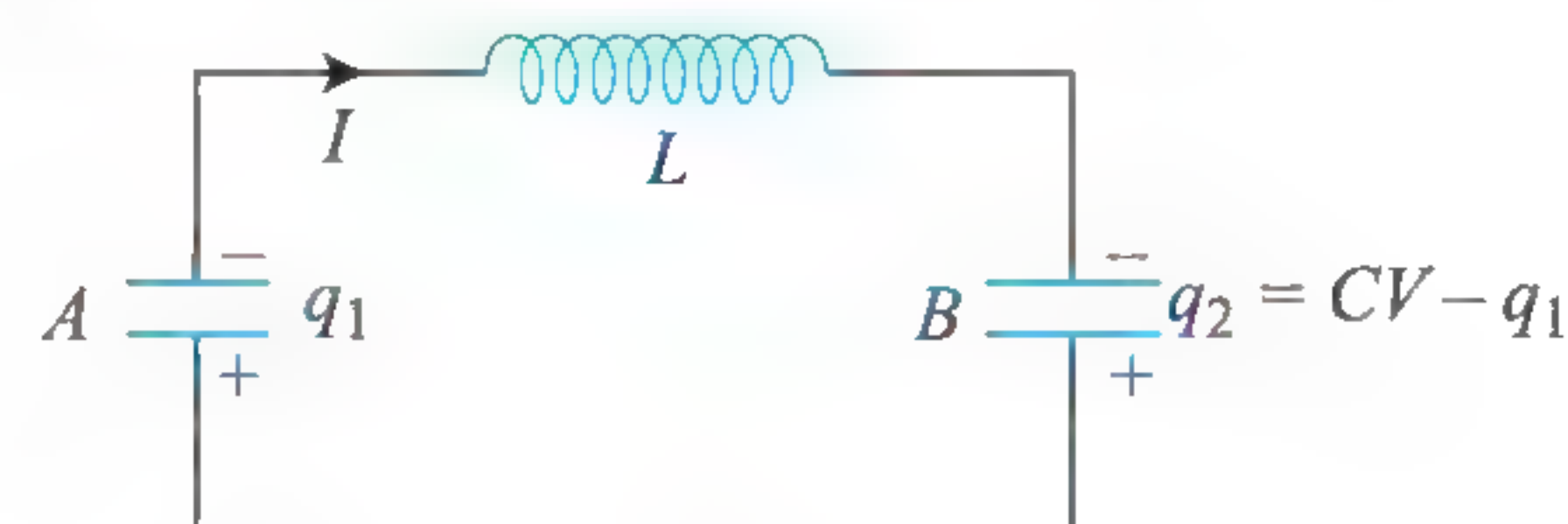
$$\text{At } t \gg 0, I_L = \frac{12}{3} = 4 \text{ A}$$

$$\therefore |\phi| = L[i_f - i_i] = 500 \times 10^{-3} \times 3 = 1.5 \text{ Wb}$$

$\therefore$ (1) and (2) are the correct choices.

6. (1), (3)

Let at any time, charge and current in the circuit wire are as shown.



$$I = \frac{dq_1}{dt}$$

Applying Kirchhoff's law:

$$\frac{CV - q_1}{C} - \frac{q_1}{C} - L \frac{dI}{dt} = 0 \Rightarrow \frac{CV - 2q_1}{LC} = L \frac{d^2 q_1}{dt^2}$$

$$\Rightarrow \frac{d^2 q_1}{dt^2} = -\frac{2}{LC} \left(q_1 - \frac{CV}{2} \right)$$

$$\Rightarrow q_1 - \frac{CV}{2} = A \sin(\omega t + \delta) \quad \dots(i)$$

where $\omega = \sqrt{\frac{2}{LC}} = 100 \pi \text{ rad/s}$

At $t = 0, q_1 = 0 \Rightarrow 0 - \frac{CV}{2} = A \sin \delta \quad \dots(ii)$

Differentiating (i), $\frac{dq_1}{dt} - 0 = A\omega \cos(\omega t + \delta)$

$$\Rightarrow I = A\omega \cos(\omega t + \delta) \quad \dots(iii)$$

At $t = 0, I = 0 \Rightarrow 0 = A\omega \cos \delta \Rightarrow \delta = \frac{\pi}{2}$

From (ii), $A = -\frac{CV}{2}$

From (i), $q_1 = \frac{CV}{2} - \frac{CV}{2} \sin\left(\omega t + \frac{\pi}{2}\right)$

$$\Rightarrow q_1 = \frac{CV}{2} (1 - \cos \omega t) \quad \dots(iv)$$

$$\Rightarrow q_1 = 50(1 - \cos 100 \pi t) \text{ mC}$$

Now $q_2 = CV - q_1 = \frac{CV}{2} (1 + \cos \omega t) \quad \dots(v)$

$$= 50(1 + \cos 100 \pi t)$$

From (ii), $I = -A\omega \sin \omega t$

Current in the circuit is maximum for $\omega t = \frac{\pi}{2}$.

And for $\omega t = \frac{\pi}{2}$, we see from (iv) and (v) that $q_1 = q_2 = \frac{CV}{2}$

7. (1), (2)

$$L \frac{di}{dt} = Bv\ell \Rightarrow \int di = \frac{B\ell}{L} \int v dt \Rightarrow i = \frac{B\ell}{L} x \quad \dots(i)$$

$$F = ma \Rightarrow -iB\ell = mv \frac{dv}{dx}$$

$$\Rightarrow -\frac{B^2 \ell^2 x}{L} = mv \frac{dv}{dx} \Rightarrow -\frac{B^2 \ell^2}{mL} \int_0^d x dx = \int_{v_0}^{v_0/2} v dv$$

$$\Rightarrow -\frac{B^2 \ell^2 d^2}{2mL} = \frac{-3v_0^2}{8}, v_0 = \frac{J}{m}$$

$$\Rightarrow d = \sqrt{\frac{3J^2 L}{4B^2 \ell^2 m}}$$

Put $x = d$ in (i), $i = \frac{B\ell}{L} \sqrt{\frac{3J^2 L}{4B^2 \ell^2 m}} = \sqrt{\frac{3J^2}{4Lm}}$

8. (1), (2), (3), (4)

At any time: $L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$

$$\Rightarrow \int_0^{i_1} L_1 di_1 = \int_0^{i_2} L_2 di_2 \Rightarrow L_1 i_1 = L_2 i_2 \Rightarrow \frac{i_1}{i_2} = \frac{L_2}{L_1} \quad \dots(i)$$

Total steady state current

$$i = E/R = i_1 + i_2 \quad \dots(ii)$$

from equation (i) and (ii)

$$i_1 = \frac{EL_2}{R(L_1 + L_2)}, i_2 = \frac{EL_1}{R(L_1 + L_2)}$$

$$U = \frac{1}{2} L_{eq} i^2 = \frac{1}{2} \left(\frac{L_1 L_2}{L_1 + L_2} \right) \frac{E^2}{R^2}$$

9. (1), (3), (4)

When the current becomes constant, the p.d. across inductor will become zero. Hence the entire p.d. will be across R .

Also energy stored in the inductor becomes constant if current through it becomes constant.

10. (1), (2), (3)

$$V = 2t \Rightarrow L(di/dt) = 2t$$

$$2(di/dt) = 2t \Rightarrow di/dt = t$$

$$i = t^2/2 \Rightarrow i - t \text{ graph is parabola}$$

$$\text{at } t = 2s, i = 2 \text{ A}$$

$$U = (1/2)Li^2 = 4 \text{ J}$$

$$dU/dt = Li(di/dt) = t^3 = 1 \text{ J/s}$$

11. (1), (2), (3)

Initially current $I = 0$, so $U = \frac{1}{2} Li^2 = 0$

$$P = ei = 0$$

But initially $\frac{di}{dt} \neq 0$, so $e = L \frac{di}{dt} \neq 0$

12. (2), (3)

Based on theory

13. (1), (3)

Time constant of the circuit is given as $\tau_L = \frac{L}{R} = \frac{2}{2} = 1s$

Half life time of the above circuit for the current growth is given as

$$t_{1/2} = (\ln 2) \tau_L = (\ln 2)s$$

Thus the given time is half life time so in this period current reduces to half which is given as

$$i = \frac{i_0}{2} = \frac{8/2}{2} = 2A$$

Rate of energy supplied by battery is given as

$$P = Ei = 8 \times 2 = 16 \text{ J/s}$$

Power dissipated as heat across resistance is given is

$$P_R = i^2 R = (2)^2 (2) = 8 \text{ J/s}$$

Potential difference across the inductor is given as

$$V_a - V_b = E - iR = 8 - 2 \times 2 = 4V$$

14. (2), (4)

Steady state current for both the circuits is same. Therefore,

$$\frac{V}{R_1} = \frac{V}{R_2} \text{ or } R_1 = R_2$$

Further $\tau_{L_1} < \tau_{L_2}$ (τ_L = time constant)

$$\frac{L_1}{R_1} < \frac{L_2}{R_2} \text{ or } L_1 < L_2 \quad (R_1 = R_2)$$

Linked Comprehension Type

For Problems 1–2

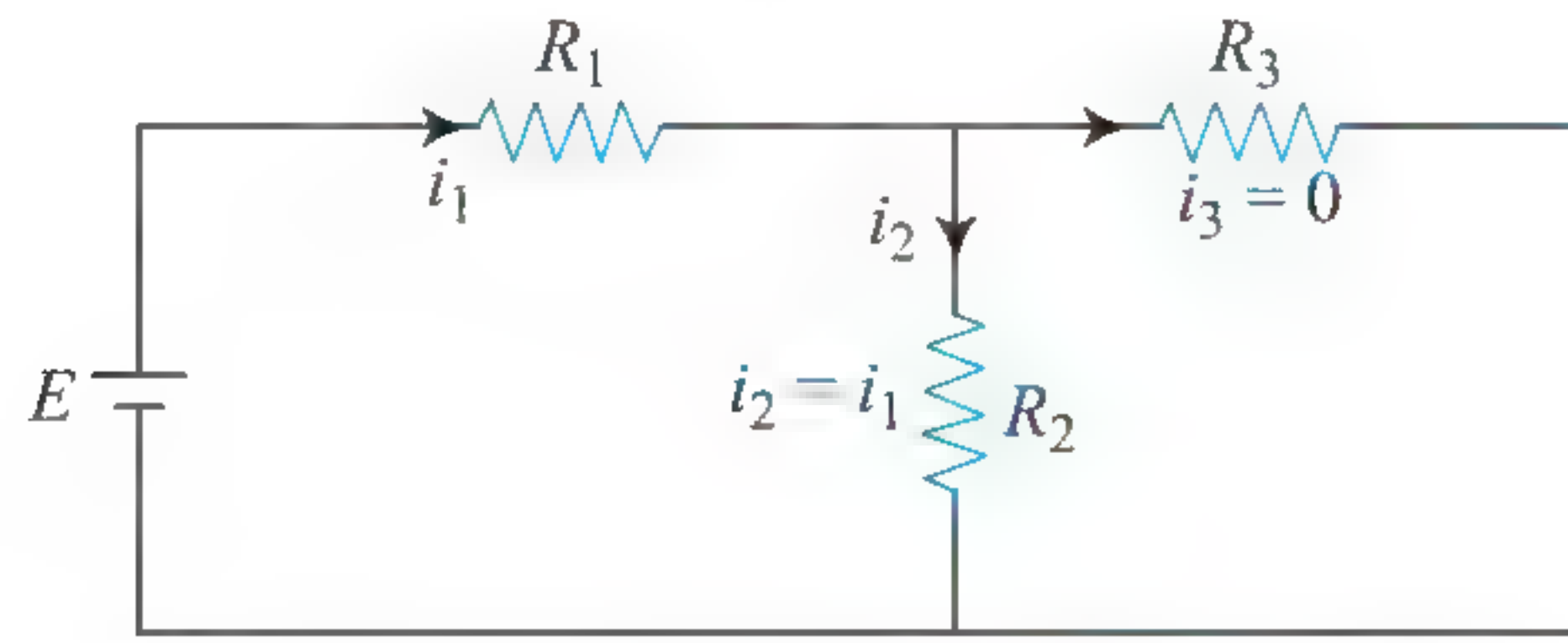
1. (2)

2. (3)

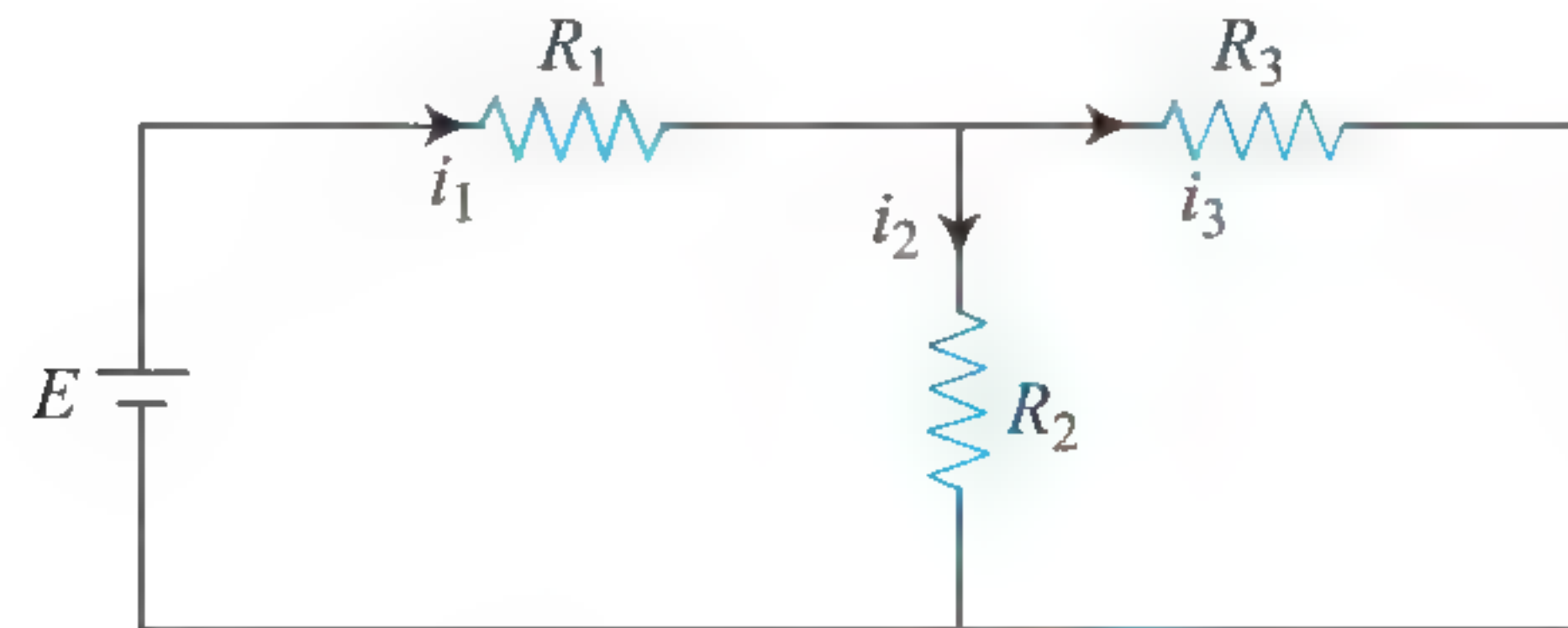
At $t = 0$, capacitor will behave like a short circuit and the inductor as open circuit but as $t \rightarrow \infty$, the nature is just opposite.

For Problems 3–5

3. (1) 4. (1) 5. (2)

 At $t = 0$, circuit can be considered as follows:


After a long time, circuit can be considered as follows:


 Immediately after closing S , $\frac{di_3}{dt} \neq 0$. Because induced e.m.f.

$$\frac{L di_3}{dt} \neq 0.$$

For Problems 6–11

 6. (2) 7. (1) 8. (3) 9. (1)
 10. (3) 11. (2)

Instantaneous current in the capacitor,

$$q = CV_C = (2)(3e^{-2t}) = 6e^{-2t} \text{ A}$$

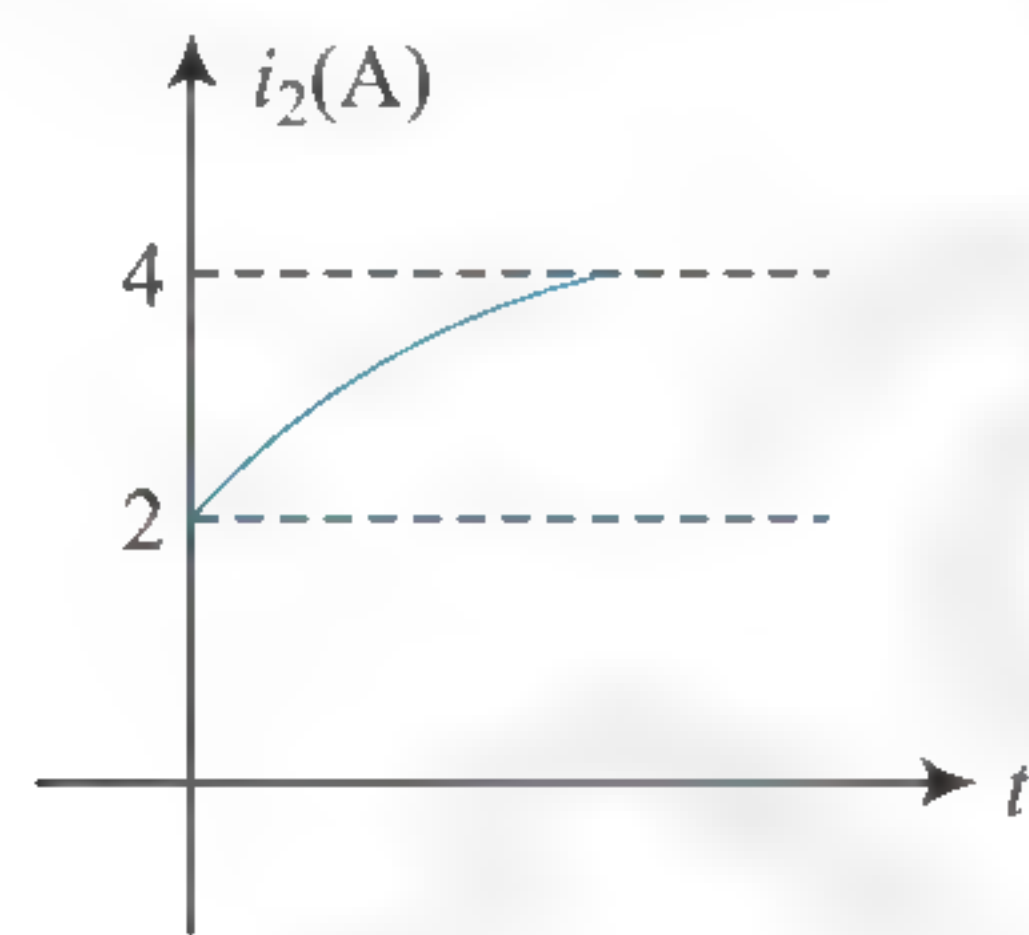
$$\text{Current, } i_C = \frac{dq}{dt} = -12e^{-2t} \text{ A}$$

 This current flows from B to O .

From KVL, we have

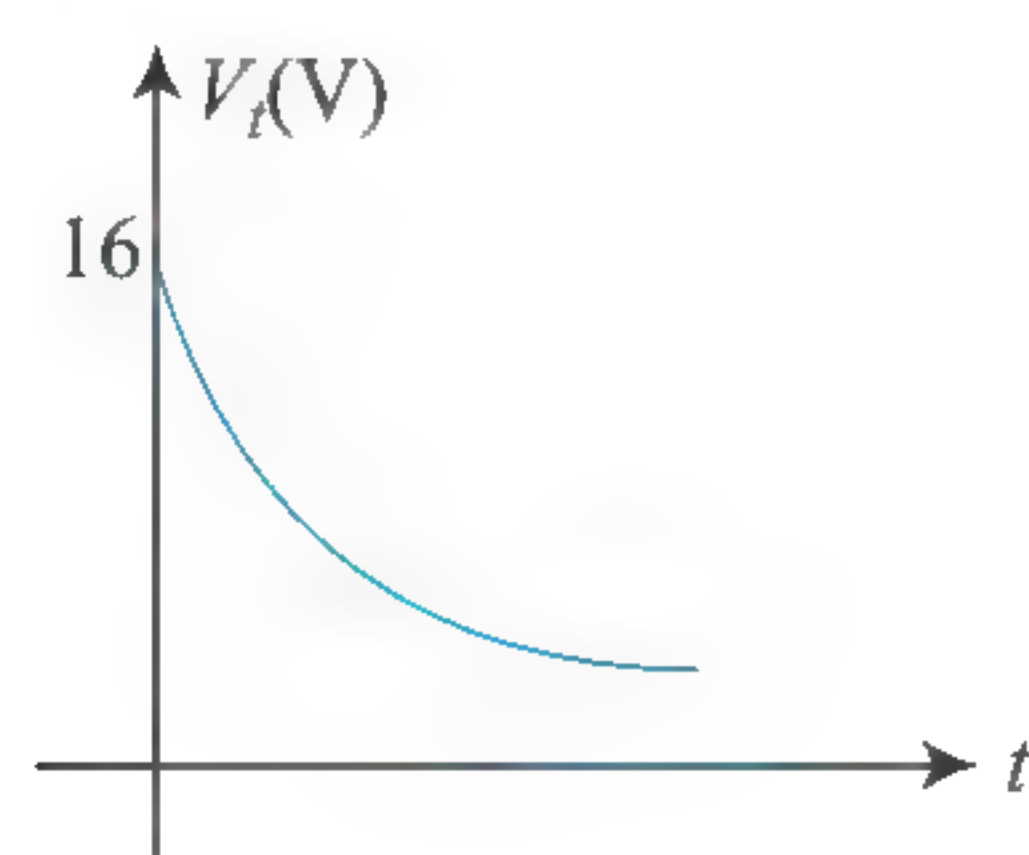
$$\begin{aligned} i_L &= i_1 + i_2 + i_C = 10e^{-2t} + 4 - 12e^{-2t} \\ &= (4 - 2e^{-2t}) \text{ A} = [2 + 2(1 - e^{-2t})] \text{ A} \end{aligned}$$

 i_L vs. time graph is as shown in figure.

 i_L increases from 2 A to 4 A exponentially.


$$\begin{aligned} V_L &= L \frac{di_L}{dt} \\ &= (4) \frac{d}{dt} (4 - 2e^{-2t}) = 16e^{-2t} \text{ V} \end{aligned}$$

 V_L decreases exponentially from 16 A to 0 as shown in figure.

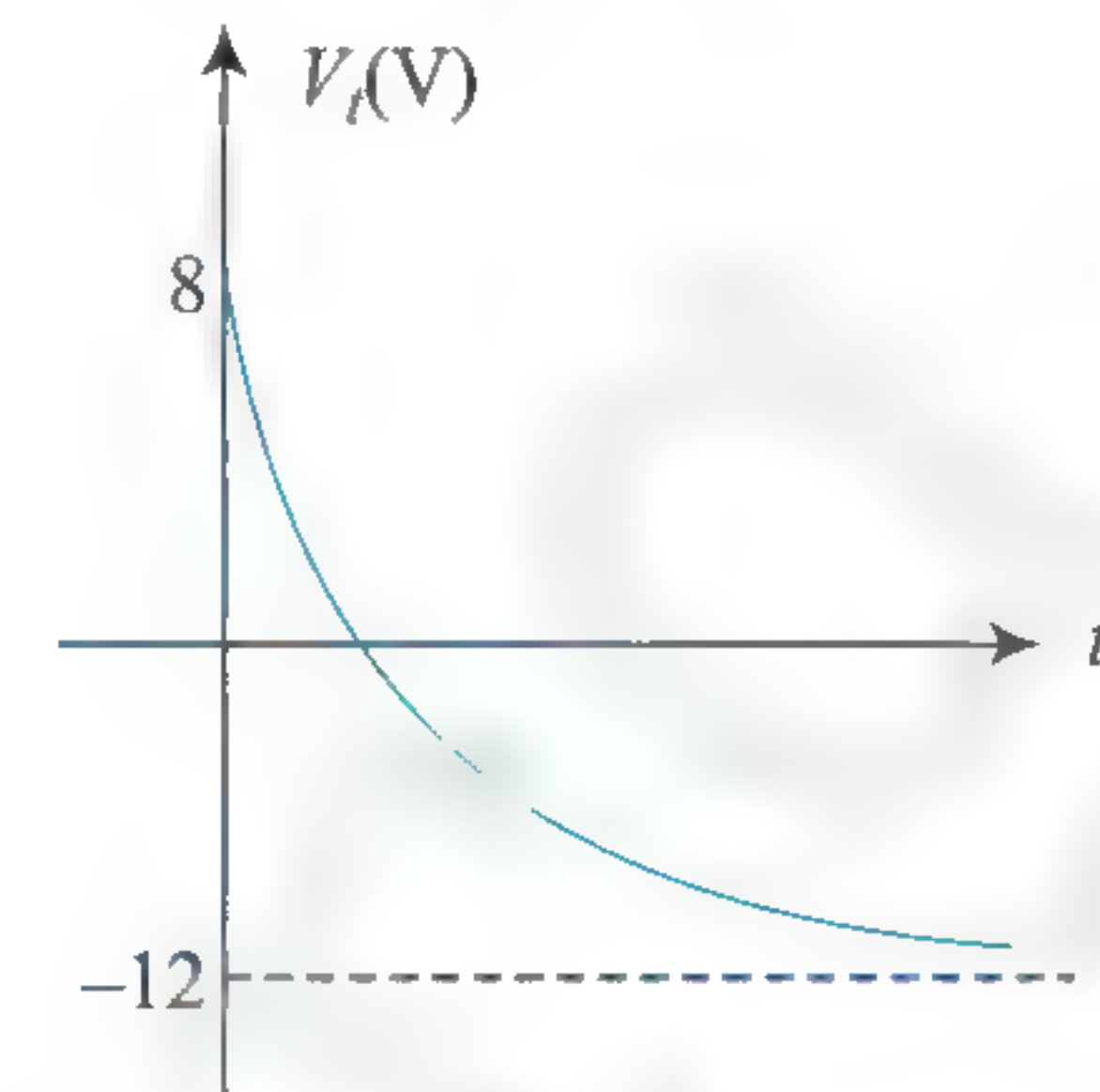
 To determine V_{AC} , we begin from A and end at C . From KVL, we have


$$V_A - i_1 R_1 + i_2 R_2 = V_C$$

$$V_A - V_C = i_1 R_1 - i_2 R_2$$

Substituting the values, we have

$$V_{AC} = (10e^{-2t})(2) - (4)(3)$$



$$V_{AC} = (20e^{-2t} - 12) \text{ V}$$

$$\text{At } t = 0, V_{AC} = 8 \text{ V}$$

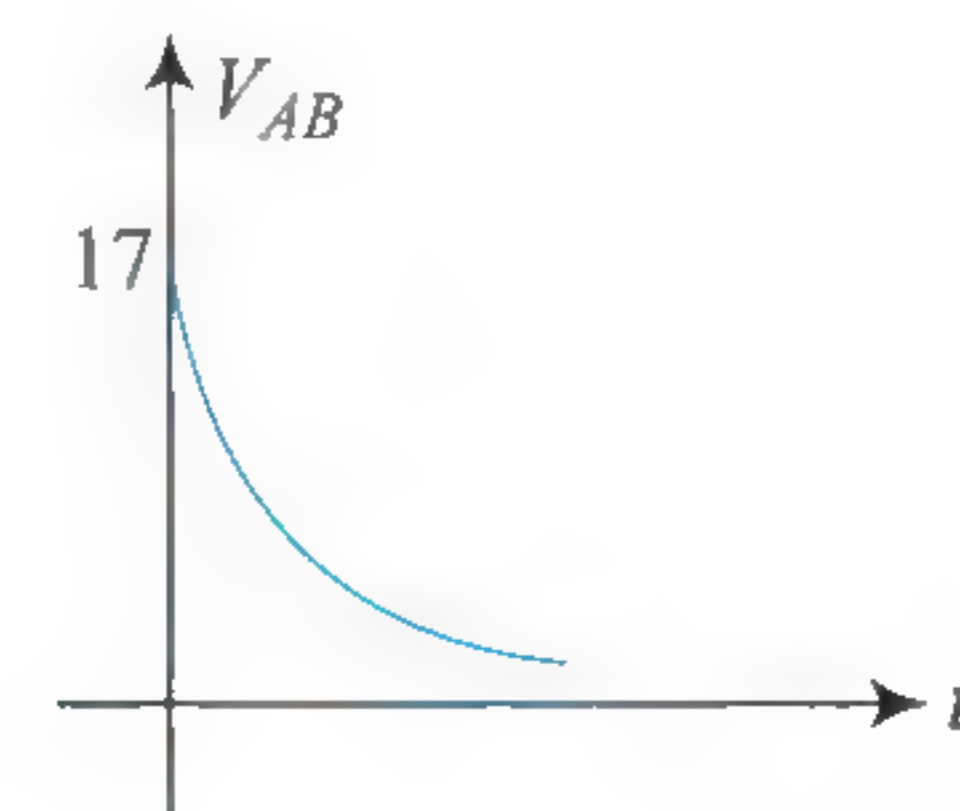
$$\text{At } t = \infty, V_{AC} = -12 \text{ V}$$

 Therefore, V_{AC} decreases exponentially from 8 V to -12 V.

 Similarly, we have from A to B

$$V_A - i_1 R_1 + V_C = V_B$$

$$V_{AB} = V_A - V_B = i_1 R_1 - V_C$$



Substituting the values, we have

$$V_{AB} = (10e^{-2t})(2) - 3e^{-2t}$$

$$V_{AB} = 17e^{-2t} \text{ V}$$

 Thus, V_{AB} decreases exponentially from 17 V to 0.

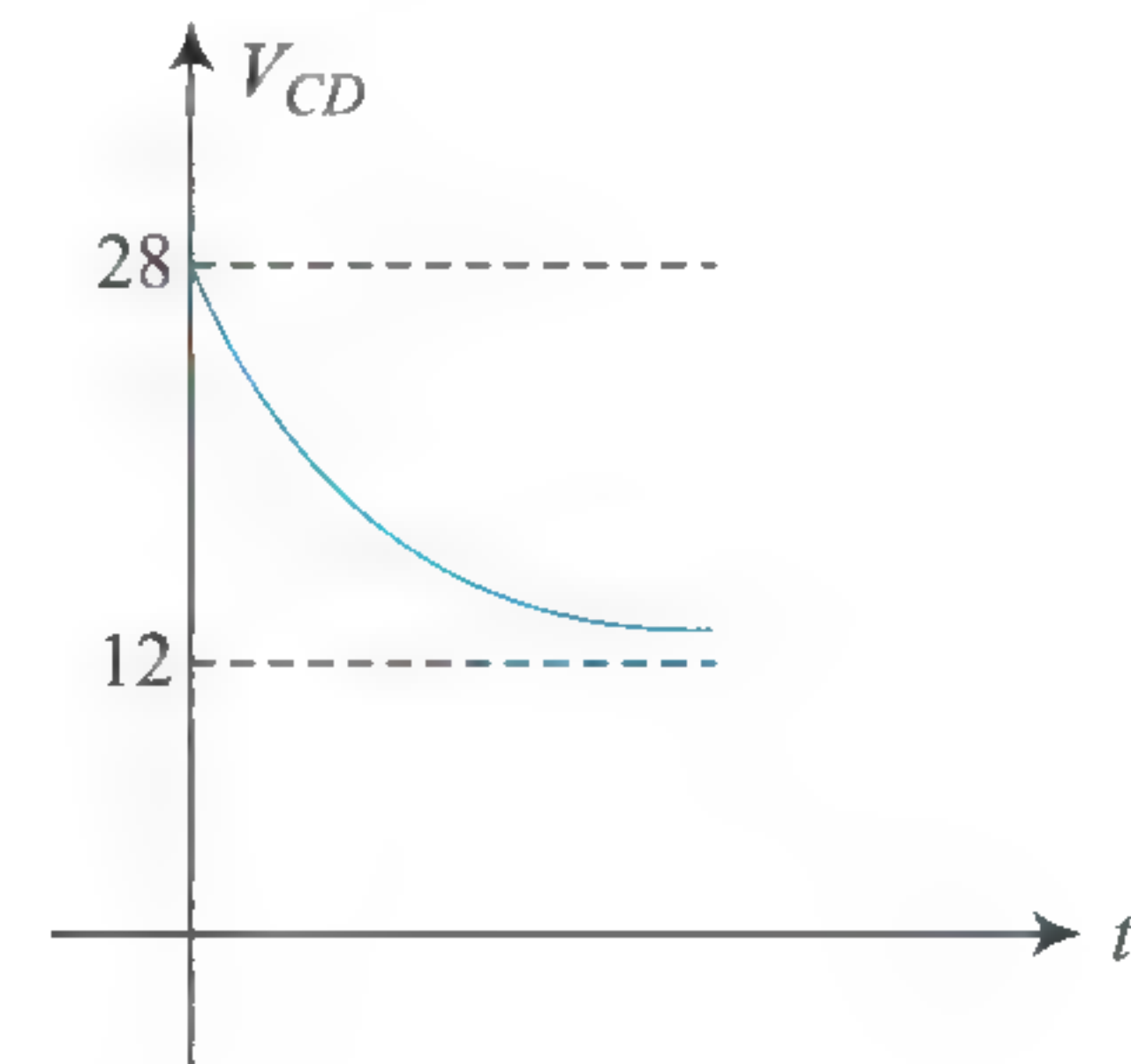
 As we move from C to D ,

$$V_C - i_2 R_2 - V_L = V_D$$

$$V_{CD} = V_C - V_D = i_2 R_2 + V_L$$

Substituting the values we have,

$$V_{CD} = (4)(3) + 16e^{-2t}$$



$$V_{CD} = (12 + 16e^{-2t}) \text{ V}$$

$$\text{At } t = 0, V_{CD} = 28 \text{ V}$$

$$\text{and at } t = \infty, V_{CD} = 12 \text{ V}$$

 i.e., V_{CD} decreases exponentially from 28 V to 12 V.

For Problems 12–13

12. (4) 13. (1)

$$12. E = \frac{r}{2} \frac{dB}{dt} = \frac{r}{2} \mu_0 n \frac{dI}{dt} \Rightarrow E = -\frac{\mu_0 n r}{2} I_{\max} \omega \sin \omega t$$

$$13. E = \frac{R^2}{2r} \frac{dB}{dt} = -\frac{R^2}{2r} \mu_0 n I_{\max} \omega \sin \omega t$$

$$= -\frac{\mu_0 n I_{\max} \omega R^2}{2r} \sin \omega t$$

For Problems 14–16

14. (2) 15. (4) 16. (1)

S_1 and S_2 are closed for 1 s.

Charge on capacitor,

$$q = CE(1 - e^{-t/RC}) = 1 - \frac{1}{e} \quad \dots(i)$$

The current in inductor,

$$I = \frac{E}{R}(1 - e^{-tR/L}) = 1 - \frac{1}{e} \quad \dots(ii)$$

Now S_1 and S_3 are opened and S_2 is closed,

It is LC circuit,

$$q = q_{\max} \sin(\omega t + \phi) \quad \dots(iii)$$

$$I = (q_{\max}) \omega \cos(\omega t + \phi) \quad \dots(iv)$$

As total energy (Magnetic + electrical) is constant

$$\frac{1}{2} LI_{\max}^2 = \frac{1}{2} \frac{q_{\max}^2}{C} = \frac{1}{2} LI^2 + \frac{1}{2} \frac{q^2}{C} \quad \dots(v)$$

$$q_{\max} = \sqrt{2} \left(1 - \frac{1}{e}\right) \quad \dots(vi)$$

$$I_{\max} = \sqrt{2} \left(1 - \frac{1}{e}\right) \quad \dots(vii)$$

From (iii) at $t = 0$, we get

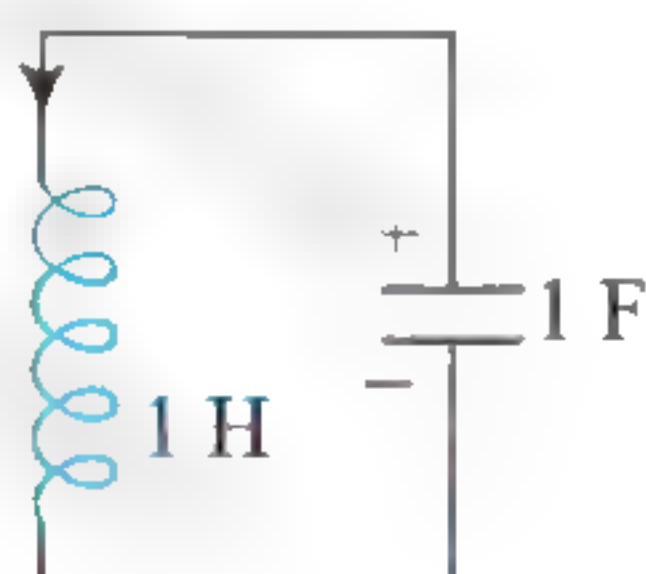
$$\left(1 - \frac{1}{e}\right) = \sqrt{2} \left(1 - \frac{1}{e}\right) \sin \phi \Rightarrow \phi = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

After closing S_2 , circuit will be as shown. Direction of current shows that charge on capacitor will be decreasing. Hence $\phi = 3\pi/4$.

$$q = \sqrt{2} \left(1 - \frac{1}{e}\right) \sin\left(\omega t + \frac{3\pi}{4}\right)$$

$$\text{where, } \omega = \frac{1}{\sqrt{LC}} = 1$$

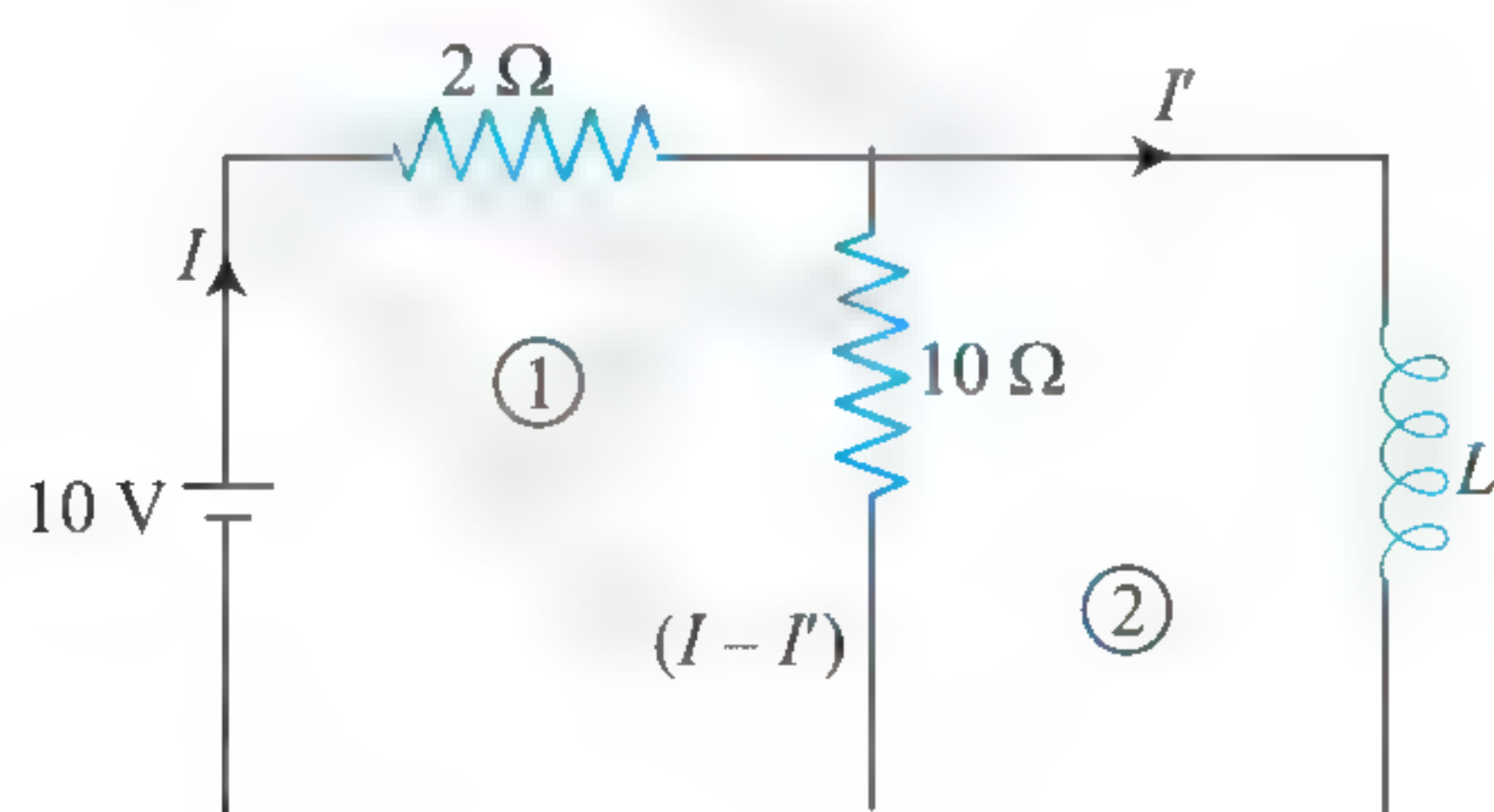
$$q = \sqrt{2} \left(1 - \frac{1}{e}\right) \sin\left(t + \frac{3\pi}{4}\right)$$



For Problems 17–19

17. (2) 18. (2) 19. (1)

From loop by applying Kirchhoff's law,



$$12I - 10I' = 10 \quad \dots(i)$$

From loop (ii)

$$-L \frac{dI'}{dt} + 10I - 10I' = 0$$

$$I - I' = \frac{L}{10} \frac{dI'}{dt} \quad \dots(ii)$$

Solving simultaneously (i) and (ii), we have

$$I' = 5 - 5e^{-\frac{5t}{3L}} \quad \dots(iii)$$

$$\text{and } I = 5 - \frac{25}{6} e^{-\frac{5t}{3L}} \quad \dots(iv)$$

$$I - I' = \frac{5}{6} e^{-\frac{1000t}{3}}$$

$$E_L = \frac{1}{2} L(I')^2; E_L = \frac{125}{2} (1 - e^{-\frac{1000t}{3}})^2 \text{ mJ}$$

Current in the inductor at $t = \infty$, $I' = 5 \text{ A}$

$$E_L(t \rightarrow \infty) = \frac{1}{2} \times 5 \times 10^{-3} (5)^2$$

$$E_L = 62.5 \text{ mJ}$$

$$E_C(t \rightarrow \infty) = \frac{1}{2} CV^2 = \frac{1}{2} \times 20 \times 10^{-6} \times 100 = 1 \text{ mJ}$$

For Problems 20–21

20. (2) 21. (2)

The equivalent inductance $L = 500 \text{ mH}$. Just before opening the switch, current through the inductors is

$$I_M = \frac{200}{100} = 2 \text{ A}$$

Potential drop across the resistor is $2 \times 100 = 200 \text{ V}$

Hence, across capacitor potential difference is zero.

So energy stored

$$U = \frac{1}{2} L(2)^2 = \frac{1}{2} \times 500 \times 10^{-3} \times 2^2 = 1 \text{ J}$$

$$\frac{1}{2} LI_M^2 = \frac{1}{2} CV_{\max}^2 = 1 \text{ J}$$

$$V_{\max} = I_M \sqrt{\frac{L}{C}} = 2 \times \sqrt{\frac{500 \times 10^{-3}}{50 \times 10^{-6}}} = 200 \text{ V}$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{500 \times 10^{-3} \times 50 \times 10^{-6}}} = 200 \text{ rad s}^{-1}$$

For Problems 22–24

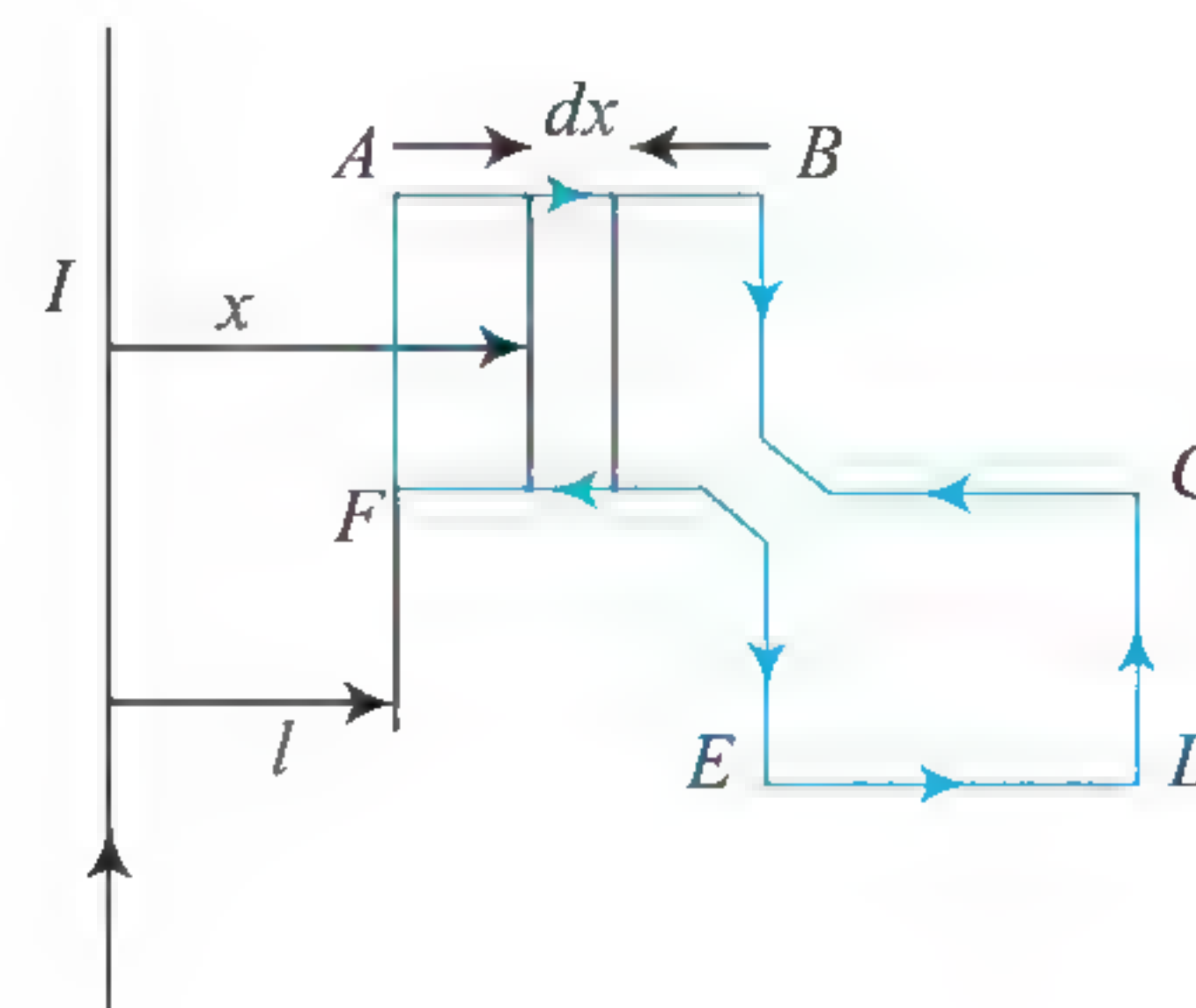
22. (1) 23. (1) 24. (2)

Consider a strip at a distance x from the wire of thickness dx .

Magnetic flux associated with this strip $\phi = B(x) adx = \frac{\mu_0 Ia}{2\pi x} dx$

$$\phi = \frac{\mu_0 Ia}{2\pi} \left[\int_l^{a+l} \frac{dx}{x} + \int_{a+l}^{2a+l} \frac{dx}{x} \right] = \frac{\mu_0 Ia}{2\pi} \ln\left(\frac{2a+l}{l}\right)$$

$$M = \frac{\phi}{I} \Rightarrow M = \frac{\mu_0 a}{2\pi} \ln\left(\frac{2a+l}{l}\right)$$



$$e = -M \frac{dl}{dt}$$

$$e = -MI_0 = -\frac{\mu_0 I_0 a}{2\pi} \ln\left(\frac{2a+l}{l}\right)$$

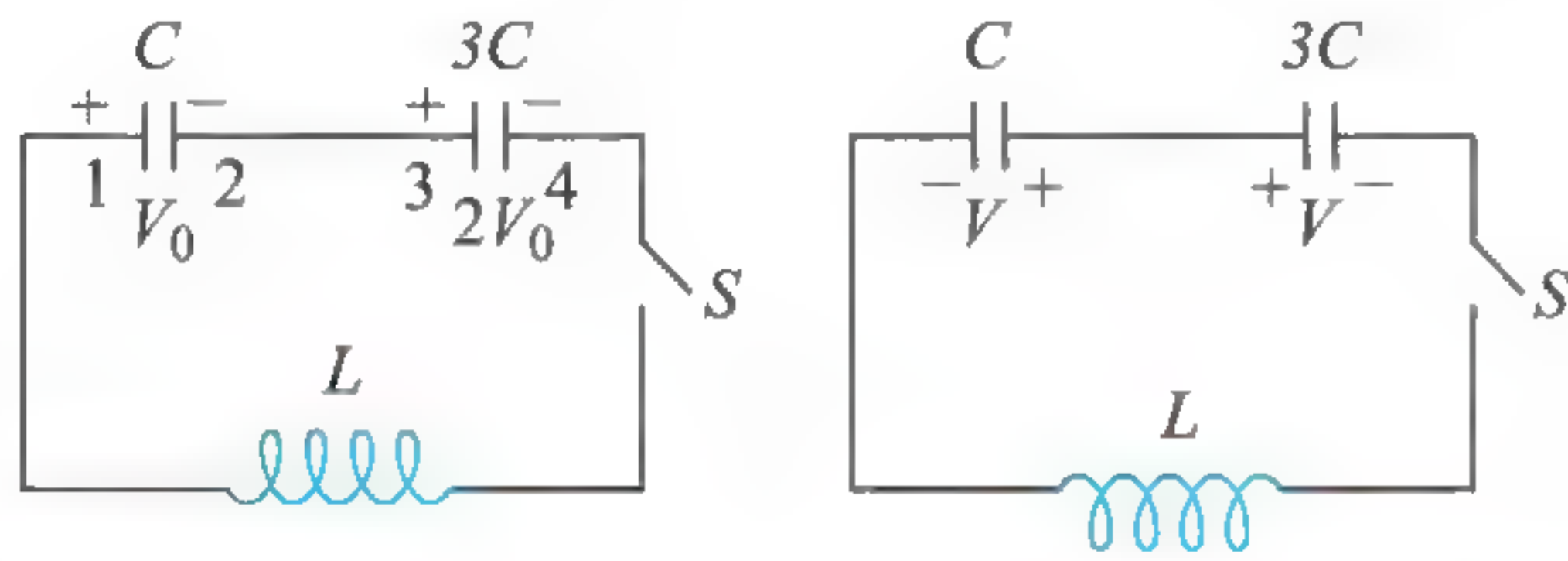
$$\text{Heat produced} = \frac{\varepsilon^2}{R} t = \frac{\left[\frac{\mu_0 I_0}{2\pi} \ln\left(\frac{2a+l}{l}\right)\right]^2}{8\lambda} at$$

For Problems 25–27

25. (1) 26. (3) 27. (3)

When current is maximum $\frac{di}{dt} = 0$

$\therefore$ e.m.f. across $L = 0$, so potential difference across the capacitor will be same.



From the law of conservation of charge on plates 2 and 3,
 $3CV + CV = 6CV_0 - CV_0$

$$\Rightarrow V = \frac{5V_0}{4}$$

Loss in energy of capacitor = energy stored in inductor

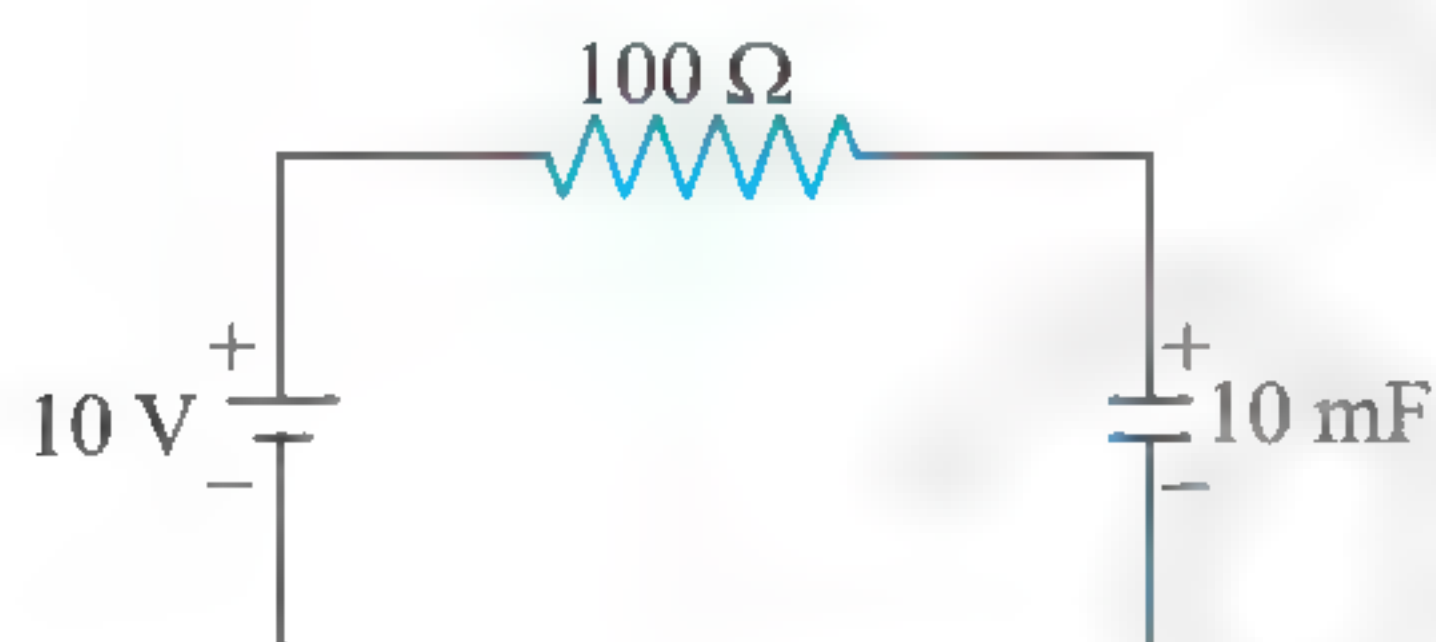
$$\Rightarrow \frac{1}{2} CV_0^2 + \frac{1}{2} 3C(2V_0)^2 - \frac{1}{2} \times 4CV^2 = \frac{1}{2} LI^2$$

$$\Rightarrow I = \frac{3}{2} V_0 \sqrt{\frac{3C}{L}}$$

For Problems 28–30

28. (2) 29. (1) 30. (4)

Initial charge = $CV_0 = Q_0$
 $= 10 \times 10^{-3} \times 5 = 50 \text{ mC}$



When capacitor is connected at position 1.

$$E - IR - \frac{q}{C} = 0$$

$$\int_0^t \frac{1}{RC} dt = \int_{Q_0}^q \frac{dq}{EC - q} \quad \text{or} \quad q = 50 [2 - e^{-t}] \text{ mC}$$

At $t = 1 \text{ s}$, $q = 50 [2 - e^{-1}]$

Voltage across the capacitor at that time

$$V = \frac{q}{C} = \frac{50[(2 - 1/e)]}{10 \times 10^{-3}} = 5 \times 10^3 [2 - (1/e)] \text{ V}$$

$$\frac{1}{2} Li^2 = \frac{1}{2} CV^2 \Rightarrow i = \left(2 - \frac{1}{e}\right) \times 10^4 \text{ A}$$

$$\text{Frequency} = \frac{1}{2\pi\sqrt{LC}} = \frac{100}{\pi} \text{ Hz}$$

For Problems 31–33

31. (2) 32. (4) 33. (3)

Initially there is no current in the inductor.

$$\text{So initially, } V_A - V_B = \left(\frac{6}{1+5}\right) \times 1 = 1 \text{ V} \quad \dots(i)$$

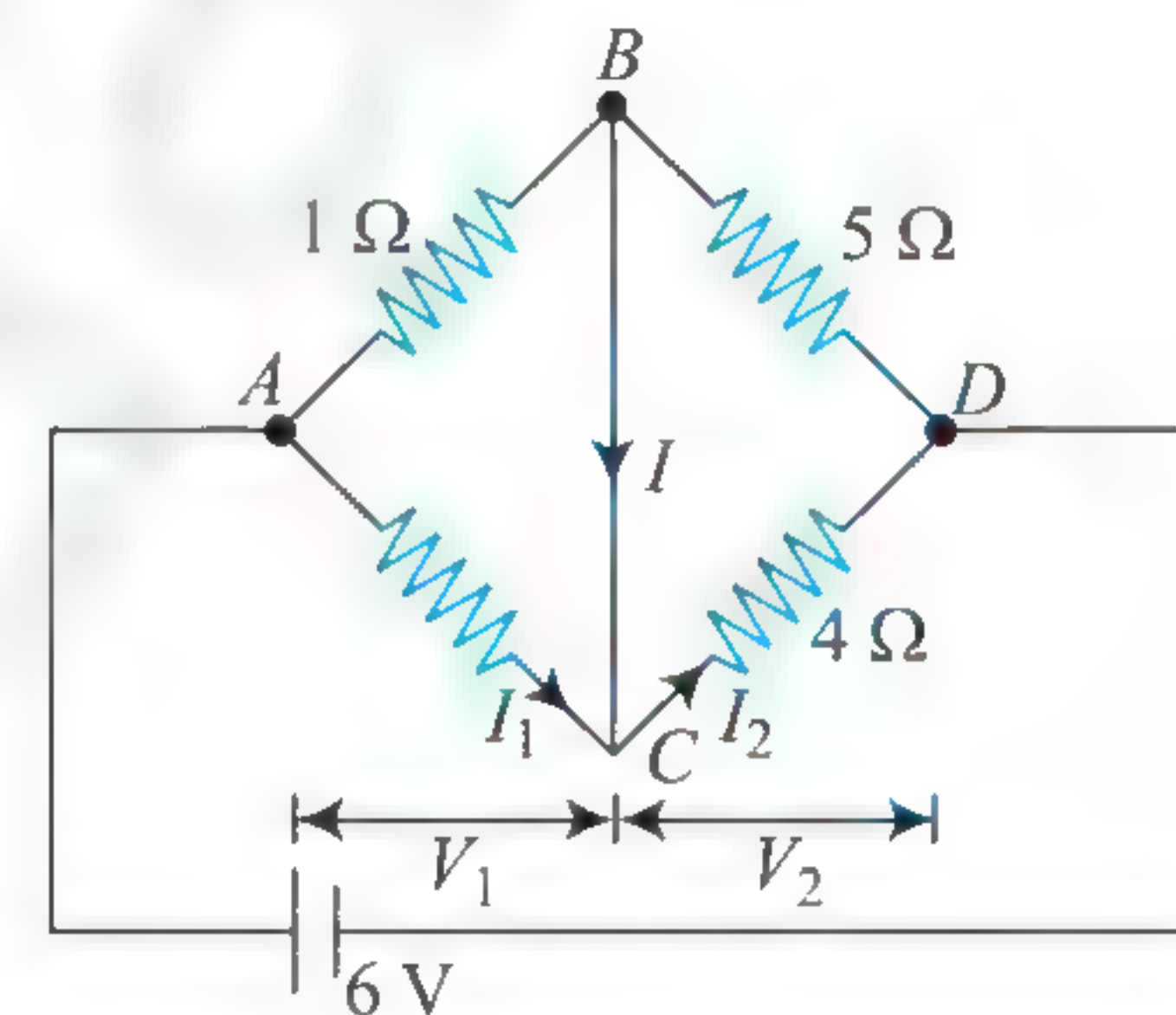
$$V_A - V_C = \left(\frac{6}{2+4}\right) \times 2 = 2 \text{ V} \quad \dots(ii)$$

From (i) and (ii), $V_B - V_C = 2 - 1 = 1 \text{ V}$

$$\Rightarrow V_B - V_C = L \frac{di}{dt}$$

$$\Rightarrow 1 = 0.1 \frac{di}{dt} \Rightarrow \frac{di}{dt} = 10 \text{ As}^{-1}$$

Current through 6Ω resistor will remain constant because it is independently connected to 6 V . After a long time, inductor will behave like a simple wire.



1Ω and 2Ω are in parallel, their equivalent is $\frac{2}{3} \Omega$.

5Ω and 4Ω are in parallel, their equivalent is $\frac{20}{9} \Omega$.

$$V_1 = \frac{\frac{2}{3} \times 6}{\frac{2}{3} + \frac{20}{9}} = \frac{18}{13} \text{ V}, V_2 = V - V_1 = 6 - \frac{18}{13} = \frac{60}{13} \text{ V}$$

$$I_1 = \frac{V_1}{2} = \frac{18}{13 \times 2} = \frac{9}{13} \text{ A}, I_2 = \frac{V_2}{4} = \frac{60}{13 \times 4} = \frac{15}{13} \text{ A}$$

$$\text{Current through inductor: } I = I_2 - I_1 = \frac{15}{13} - \frac{9}{13} = \frac{6}{13} \text{ A}$$

For Problems 34–36

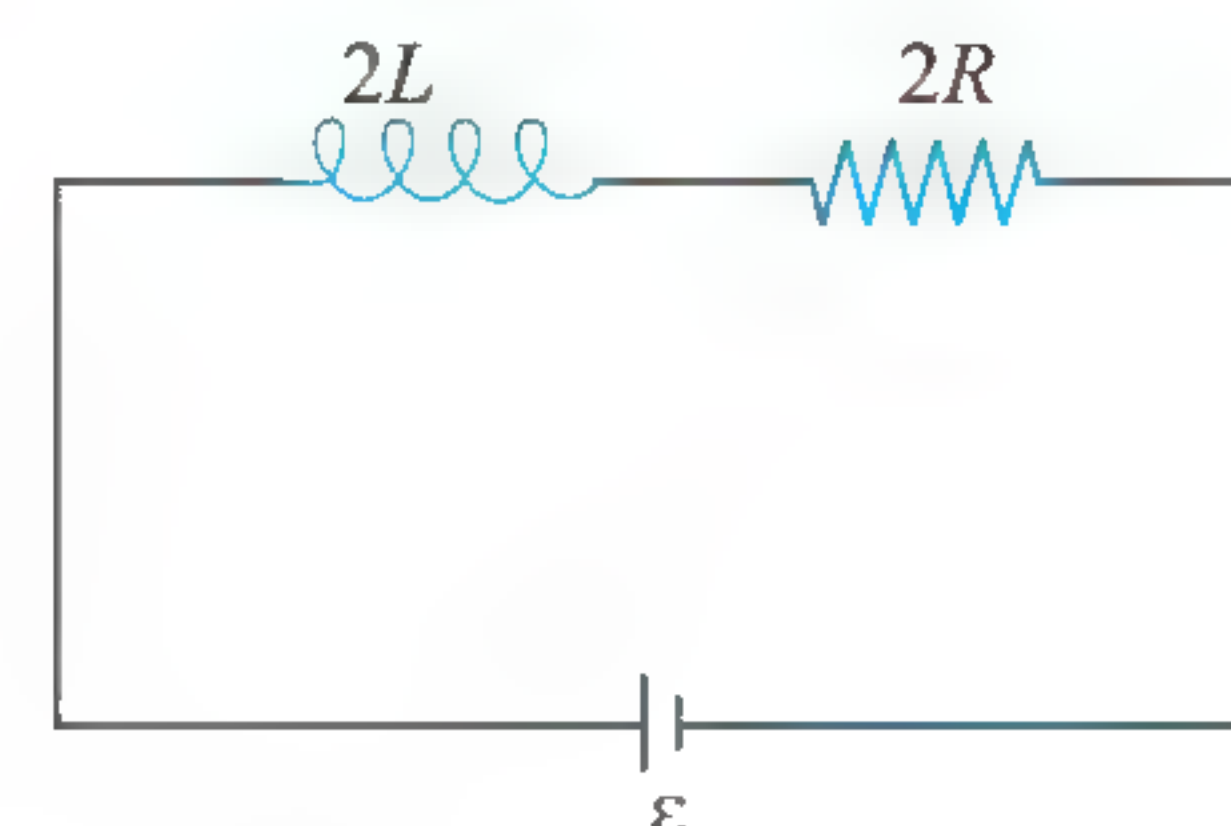
34. (2) 35. (3) 36. (2)

Since the field is increasing in inside direction in bigger loop, so the current will be induced to oppose this increasing flux. Hence in anticlockwise direction.

$$\phi = BA = B(\ell^2 + b^2)$$

$$|\varepsilon| = \left| \frac{d\phi}{dt} \right| = \frac{dB}{dt} (\ell^2 + b^2) = 0.5 \text{ V}$$

$$i = \frac{\varepsilon}{2R} \left[1 - e^{-\frac{t2R}{2L}} \right] = \frac{0.5}{20} [1 - e^{-t}]$$



$$i = \frac{1}{40} [1 - e^{-t}]$$

For Problems 37–39

37. (3) 38. (1) 39. (3)

When inductance of coil is changed abruptly, flux in it remains same, i.e. $\phi = LI = \text{constant}$. Since here inductance will increase on insertion of iron rod, so current will decrease. Now current will increase to attain the steady state value (E/R) . During this time, inductance (nL) will remain same and p.d. across A and B will also remain same. But as current will change with time so rate of production of heat will vary.

For Problems 40–42

40. (4) $L_1 = L, L_2 = \eta L, \phi = L_1 I_1 = L_2 I_2, I_1 = E/R$

$$\Rightarrow I_2 = \frac{L_1}{L_2} I_1 = I_1 / \eta$$

$$\begin{aligned} W &= U_f - U_i = \frac{1}{2} L_2 I_2^2 - \frac{1}{2} L_1 I_1^2 \\ &= \frac{1}{2} \eta L \left(\frac{I_1}{\eta} \right)^2 - \frac{1}{2} L I_1^2 = \frac{L E^2}{2 \eta R^2} [1 - \eta] \end{aligned}$$

41. (1) As current increases on pulling out the iron piece, the current will decrease to achieve its steady state value E/R again. At any time:

$$E = IR + \eta L \frac{dI}{dt} \Rightarrow \int_{I_2}^I \frac{dI}{E - IR} = \int_0^t \frac{dt}{\eta L}$$

$$\text{Solve to get: } i = \frac{E}{R} \left[1 - \left(1 - \frac{1}{\eta} \right) e^{-t/\lambda} \right]$$

$$42. (4) P = Ei = \frac{E^2}{R} \left[1 - \left(1 - \frac{1}{\eta} \right) e^{-t/\lambda} \right]$$

For Problems 43–45

43. (4) Since $V_L = \frac{L di}{dt}$

$$L di = V_L dt$$

$$\text{Integrating } L \int di = \int V_L dt$$

$$L(i_f - i_i) = \int_0^t V_L dt = \text{area under } V_L - t \text{ graph.}$$

44. (2) Form explanation on previous page

$$L(i_f - i_i) = \text{area under } V_L - t \text{ graph}$$

$$2(i_f - 0) = \frac{1}{2} \times 1 \times 8 = 4$$

$$i_f = 2A$$

45. (2) For $t \leq 1s$

$$V_L = 8t$$

$$L di = V_L dt$$

$$\int L di = \int 8t dt$$

$$Li = \frac{8t^2}{2} = 4t^2$$

$$i = 2t^2$$

So current versus time graph is a parabola.

For $1s \leq t \leq 2s$

$$V_L = 16 - 8t$$

$$\int L di = \int (16 - 8t) dt$$

$$Li = 16t - \frac{8t^2}{2}$$

$$i = 8t - 2t^2$$

So $i-t$ graph in a parabola.

So for entire time shown, $i-t$ graph in a parabola.

Matrix Match Type

1. i. $\rightarrow$ a.; ii. $\rightarrow$ b.; iii. $\rightarrow$ c.; iv. $\rightarrow$ a.

- Just after switch S is closed, flux in M starts increasing in left direction, so in N also the flux starts increasing in left direction. This will induce current in N in a direction so that the flux is in right direction. This is possible if induced current in N is from A to B .
- In this case just reverse of (i) will happen, because after closing the switch, the flux in M starts decreasing in left direction.
- After a long time of closing the switch, flux becomes constant. Hence, no current is induced.
- Just after closing S , flux starts increasing, but because M moves away, so due to this flux through N will decrease. But there will be a net increase in flux in N in left direction. This is the case similar to (i).

2. i. $\rightarrow$ c.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a., c.; iv. $\rightarrow$ a., c.

- Current in inductor when switch is open:

$$I_0 = \frac{E}{R}$$

Initially induced e.m.f. will be equal to E and finally it is zero. So, energy stored will be zero.

- Same as (i).
- Here current becomes zero suddenly.

$$\text{So, } \frac{dI}{dt} \text{ is large.}$$

Hence, induced e.m.f. $L \frac{dI}{dt}$ will be large. Finally, energy stored in inductor will be zero.

3. i. $\rightarrow$ c.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ b.

When the switch is connected with a for a long time, current in the circuit would be E/R and E/R is greater for (i) and (iii) than for (ii) and (iv).

Next, comparison is made on the basis of time constant. Shorter time constant means faster decay of the current.

4. i. $\rightarrow$ a., c.; ii. $\rightarrow$ a., c., d.; iii. $\rightarrow$ a., b.; iv. $\rightarrow$ a., c., d.

- Let $I = I_0 \sin(\omega t + \phi)$, so current leads emf by ϕ

$$\tan \phi = \frac{-X_L}{R}, \text{ so } \phi \neq 0 \Rightarrow \cos \phi \neq 1$$

ϕ is negative, so current lags emf

$$P_{av} = E_v I_v \cos \phi \neq 0$$

- $R = 0, \omega > \frac{1}{\sqrt{LC}},$

$$\tan \phi = \frac{X_C - X_L}{R} = \frac{\frac{1}{\omega C} - \omega L}{R} \approx -\infty \Rightarrow \phi = -\pi/2$$

$$\cos \phi = 0, \text{ current lags emf. } P_{av} = E_v I_v \cos \phi = 0$$

- $\tan \phi = \frac{X_C - X_L}{R} > 0 \Rightarrow \phi > 0$

$\cos \phi \neq 1$ also $P_{av} \neq 0$ because $\cos \phi \neq 0$

Current leads emf

$$\text{iv. } \tan \phi = \frac{-X_L}{0} = -\infty \Rightarrow \phi = -\pi/2$$

$$P_{av} = E_v I_v \cos \phi = 0$$

Numerical Value Type

$$1. (8) V_A - IR + E - L \frac{dI}{dt} = V_B$$

$$\Rightarrow V_A - 5 \times 1 + 8 - 5 \times 10^{-3}(-10^{-3}) = V_B$$

$$\Rightarrow V_B - V_A = 8 \text{ V}$$

$$2. (8) \text{ Potential difference across the coil is } V = L \frac{di}{dt}$$

$$\text{or } V = (1)(4) = 4 \text{ V}$$

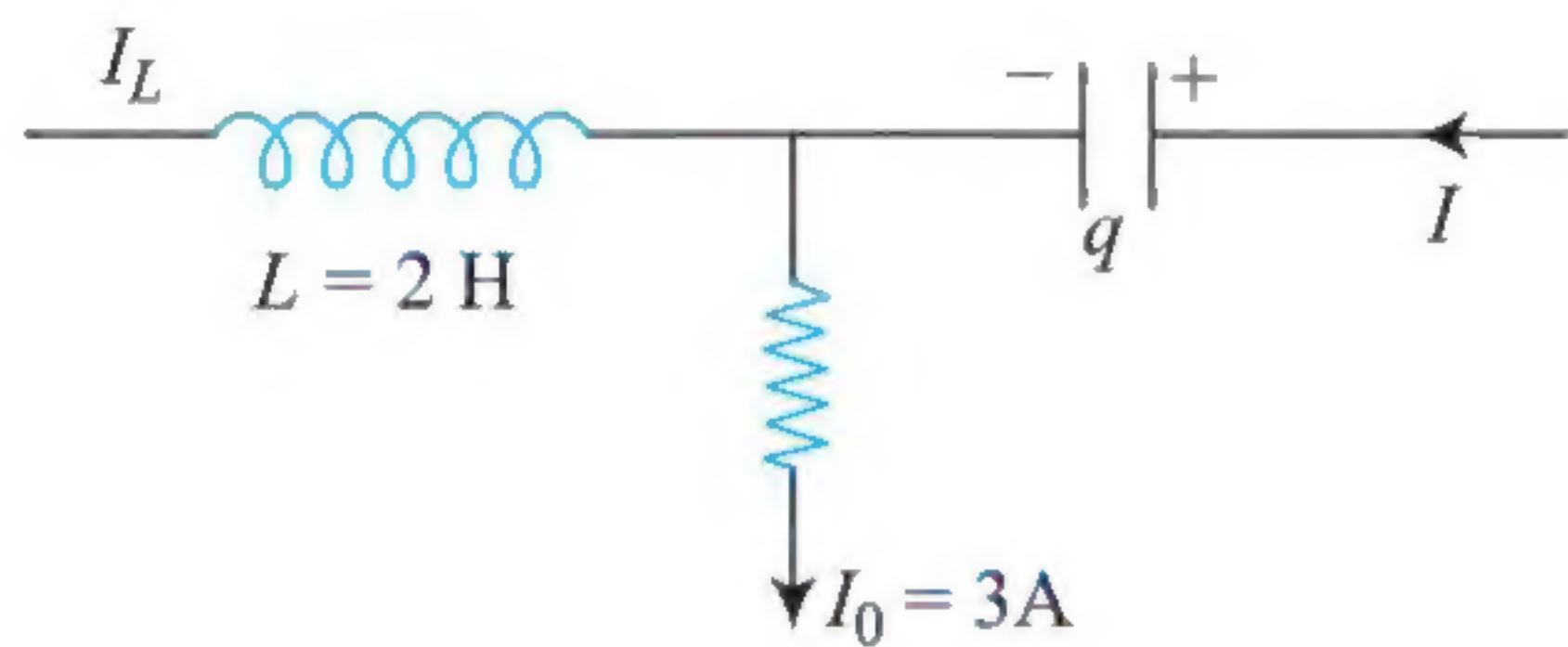
$$\text{Now energy stored per unit time} = \text{power}$$

$$= Vi = (4)(2) = 8 \text{ J/s}$$

$$3. (4) e = M \frac{dI}{dt} = M 4t = 20t$$

$$i = \frac{e}{R} = \frac{20t}{10} = 2t, \quad q = \int I dt = \int_0^2 2t dt = 4 \text{ C}$$

$$4. (4) q = CV, \quad I = \frac{dq}{dt} = C \frac{dV}{dt} = V \frac{dC}{dt}, \quad I_L = I_0 - I$$



$$\frac{dI_L}{dt} = \frac{dI_0}{dt} - \left[C \frac{d^2V}{dt^2} + \frac{dV}{dt} \frac{dC}{dt} + \frac{dV}{dt} \frac{dC}{dt} + V \frac{d^2C}{dt^2} \right]$$

$$= -1 \times 10^{-3} - \left[6 \times 10^{-3} \times \frac{1}{2} + 2(-0.5 \times 10^{-3}) + 2(-0.5 \times 10^{-3}) + 0 \right]$$

$$= -2 \times 10^{-3} \text{ A/s}$$

$$V_L = L \frac{dI_L}{dt} = 2 \times 2 \times 10^{-3} = 4 \times 10^{-3} \text{ V} = 4 \text{ mV}$$

$$5. (1) \text{ After charging, charge on capacitor} = C\varepsilon$$

Now, at $t = 0$ two circuits are formed.

$$6. (8) \text{ The mutual inductance of solenoid coil system}$$

$$\begin{aligned} M &= \mu_0 n_1 N_2 A_2 = \mu_0 n_1 N_2 \pi r_2^2 \\ &= 4\pi \times 10^{-7} \times 2 \times 10^4 \times 100 \times \pi \times (0.01)^2 \\ &= 8\pi^2 \times 10^{-5} \text{ H} \end{aligned}$$

$$\text{EMF induced in the coil: } e_2 = -M \frac{\Delta i}{\Delta t}$$

$$= -8\pi^2 \times 10^{-5} \times \left(\frac{-2-2}{0.05} \right) = 640\pi^2 \times 10^{-5} \text{ V}$$

Required charge:

$$q = i \Delta t = \frac{e}{R} \Delta t = \frac{640 \times \pi^2 \times 10^{-5}}{40\pi^2} \times 0.05 = 8 \mu\text{C}$$

$$7. (6) \text{ From energy conservation: } \frac{1}{2} CV_0^2 = \frac{1}{2} CV^2 + \frac{1}{2} LI^2$$

$$\Rightarrow \frac{1}{2} \times 2 \times 10^{-6} \times 12^2 = \frac{1}{2} \times 2 \times 10^{-6} \times 6^2 + \frac{1}{2} \times 6 \times 10^{-6} I^2$$

$$\Rightarrow I = 6 \text{ A}$$

$$8. (5) \text{ Induced EMF} = \frac{1}{2} B\omega\ell^2$$

$$\text{Maximum current: } i_0 = \frac{B\omega\ell^2}{2R}$$

Torque about the hinge P is

$$\tau = \int_0^\ell i(dx) Bx \Rightarrow \tau = \frac{1}{2} i B \ell^2$$

$$\text{Putting } i = i_0/2, \text{ we get: } \tau = \frac{B^2 \omega \ell^4}{8R} = 5 \text{ mNm}$$

$$9. (1) \text{ At } t = 0, \text{ inductor behave as open circuit so } i = \frac{10}{10} = 1 \text{ A}$$

$$10. (4) U = \frac{1}{2} Li^2$$

Energy stored per unit time

$$\frac{dU}{dt} = \frac{1}{2} L 2i \frac{di}{dt} = Li \frac{di}{dt} = 2 \times 2 \times 1 = 4 \text{ J/s}$$

$$11. (5) \text{ Induced emf} = \frac{1}{2} B\omega l^2$$

$$\text{Maximum current: } i_0 = \frac{B\omega l^2}{2R}$$

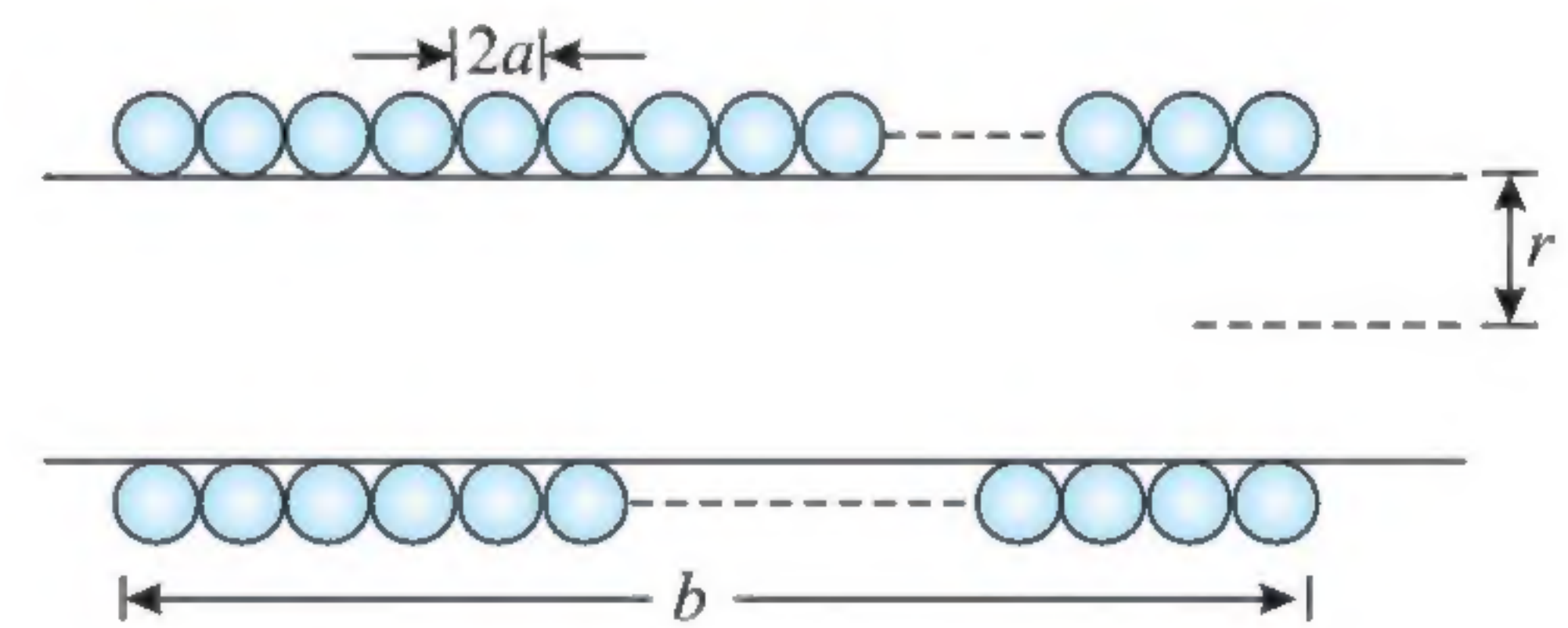
Torque about the hinge P is

$$\tau = \int_0^l i(dx) Bx dx \Rightarrow \tau = \frac{1}{2} i B l^2$$

$$\text{Put } i = i_0/2, \text{ we get: } \tau = \frac{B^2 \omega l^4}{8R} = 5 \text{ mNm}$$

$$12. (8) \text{ Length of wire } l = \frac{\text{volume } (V)}{\pi a^2}$$

Winding is as shown in the figure.



$$\text{Number of turns, } N = \frac{l}{2\pi r}$$

$$\text{Length of the helix, } b = 2a \cdot N = \frac{al}{\pi r}$$

$$\text{Number of turns per meter length } n = \frac{1}{2a}$$

$$\therefore \text{ Self-inductance, } L = \pi \mu_0 n^2 r^2 b$$

$$= \pi \mu_0 \left(\frac{1}{2a} \right)^2 \cdot r^2 \cdot \frac{al}{\pi r}$$

$$= \frac{1}{4} \mu_0 r \frac{l}{a} = \frac{1}{4} \mu_0 r \frac{V}{a(\pi a^2)}$$

$$L = \frac{\mu_0 r}{4} \frac{V}{a^3} \therefore L \propto \frac{1}{a^3}$$

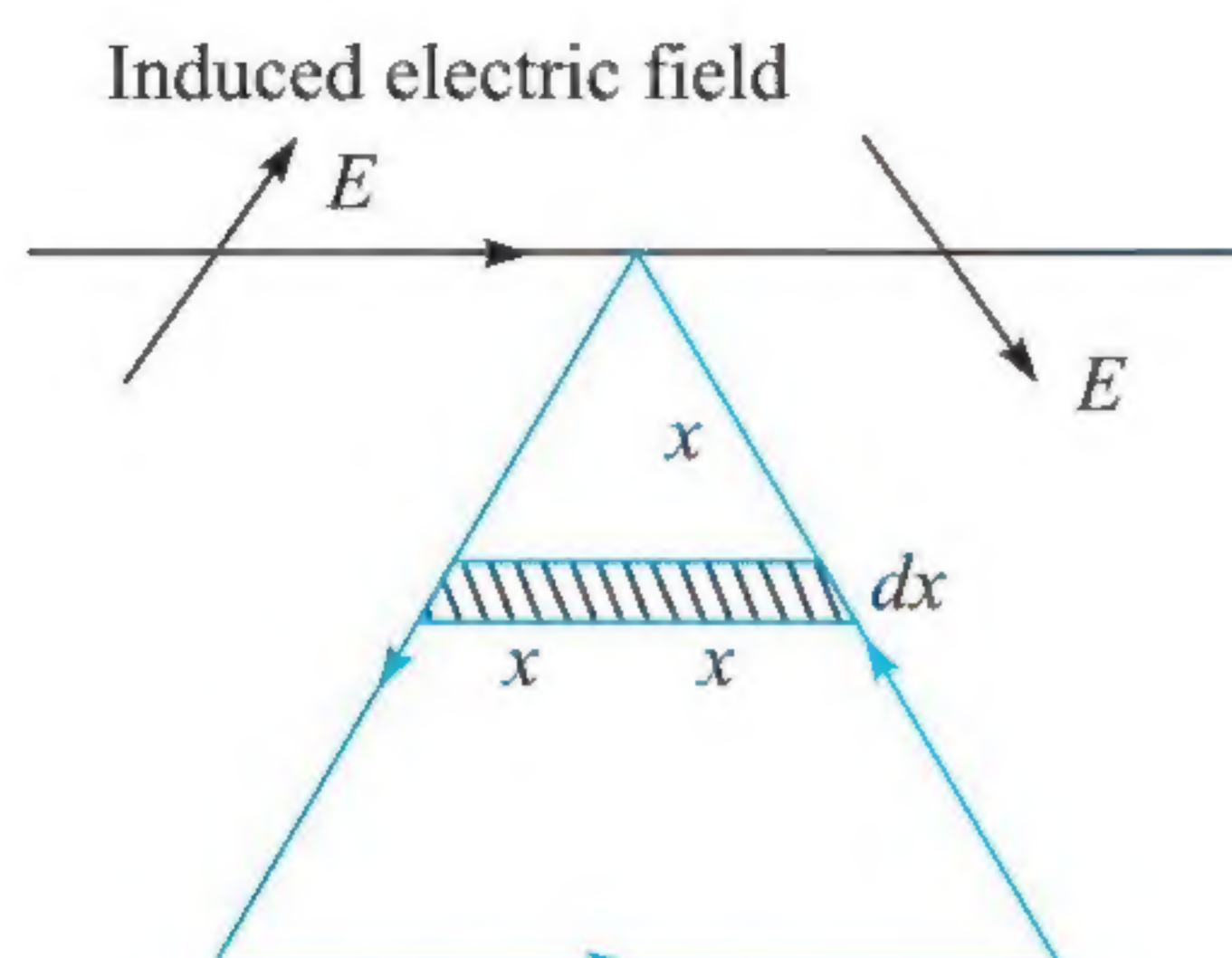
$$\therefore \frac{L_1}{L_2} = \left(\frac{2a}{a} \right)^3 = \frac{8}{1}$$

Archives

JEE Advanced

Multiple Correct Answers Type

1. (2), (4)



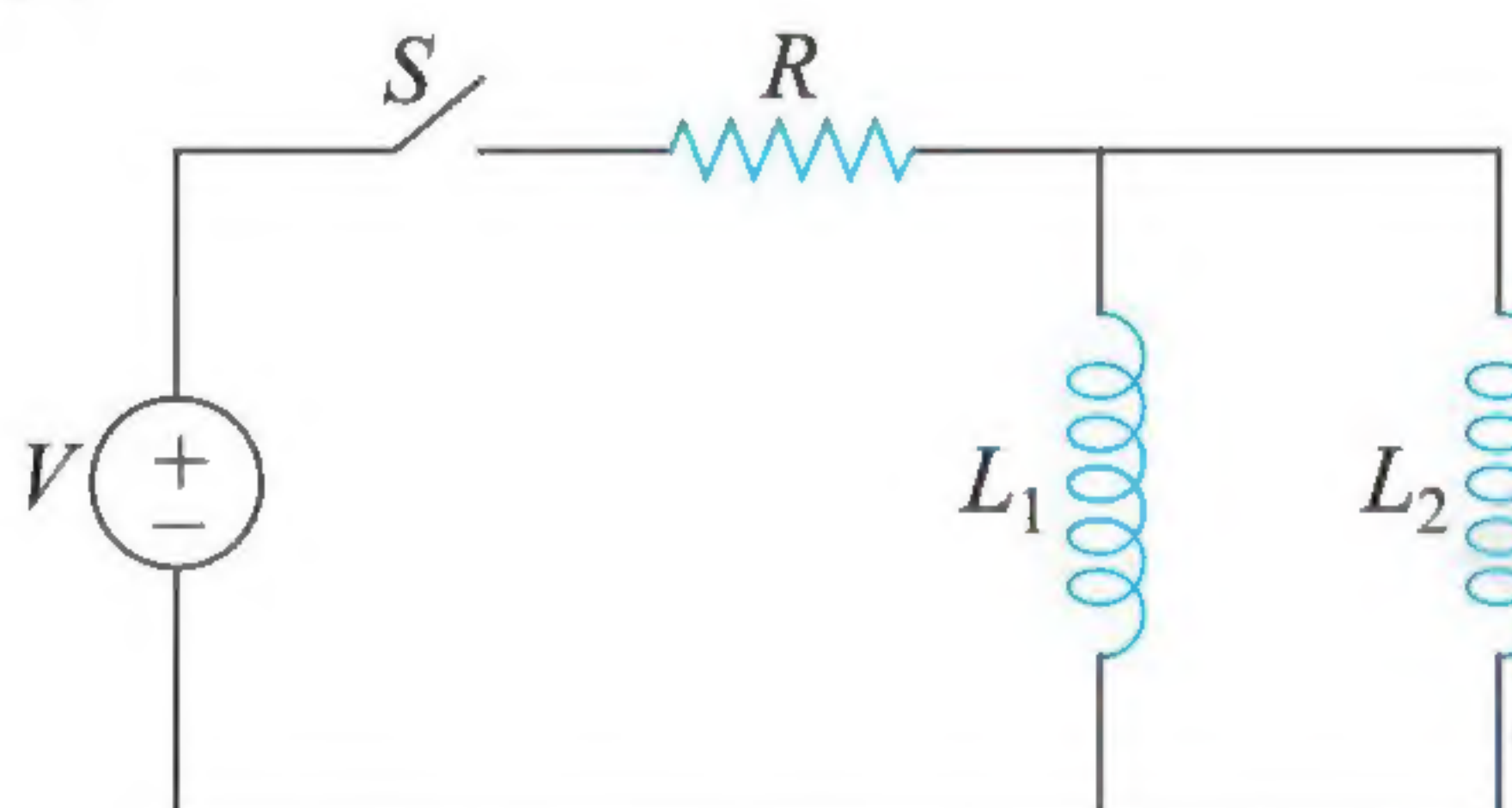
By direction of induced electric field, current in wire is in same direction of current along the hypotenuse. Flux through

$$\text{triangle if wire have current } i = \int_0^{0.1} \left(\frac{\mu_0 i}{2\pi x} \right) (2x dx) = \frac{\mu_0 i}{10\pi}$$

$$\Rightarrow \text{Mutual inductance} = \frac{\mu_0}{10\pi}$$

$$\text{Induced emf in wire} = \frac{\mu_0}{10\pi} \frac{di}{dt} = \frac{\mu_0}{10\pi} \times 10 = \frac{\mu_0}{\pi}$$

2. (1), (2), (3)



Since inductors are connected in parallel

$$V_{L_1} = V_{L_2}$$

$$L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt}$$

$$L_1 I_1 = L_2 I_2$$

$$\frac{I_1}{I_2} = \frac{L_2}{L_1}$$

$\therefore$ Option (1) is correct.

Current through resistor at any time t is given by

$$I = \frac{V}{R} (1 - e^{-\frac{RT}{L}}) \text{ where } L = \frac{L_1 L_2}{L_1 + L_2}$$

$$\text{After long time } I = \frac{V}{R}$$

$$I_1 + I_2 = I \quad \dots(i)$$

$$L_1 I_1 = L_2 I_2 \quad \dots(ii)$$

From (i) and (ii), we get

$$I_1 = \frac{V}{R} \frac{L_2}{L_1 + L_2}, \quad I_2 = \frac{V}{R} \frac{L_1}{L_1 + L_2}$$

$\therefore$ Options (2) and (3) are correct.

Value of current is zero at $t = 0$

Value of current is V/R at $t = \infty$

Hence option (4) is incorrect.

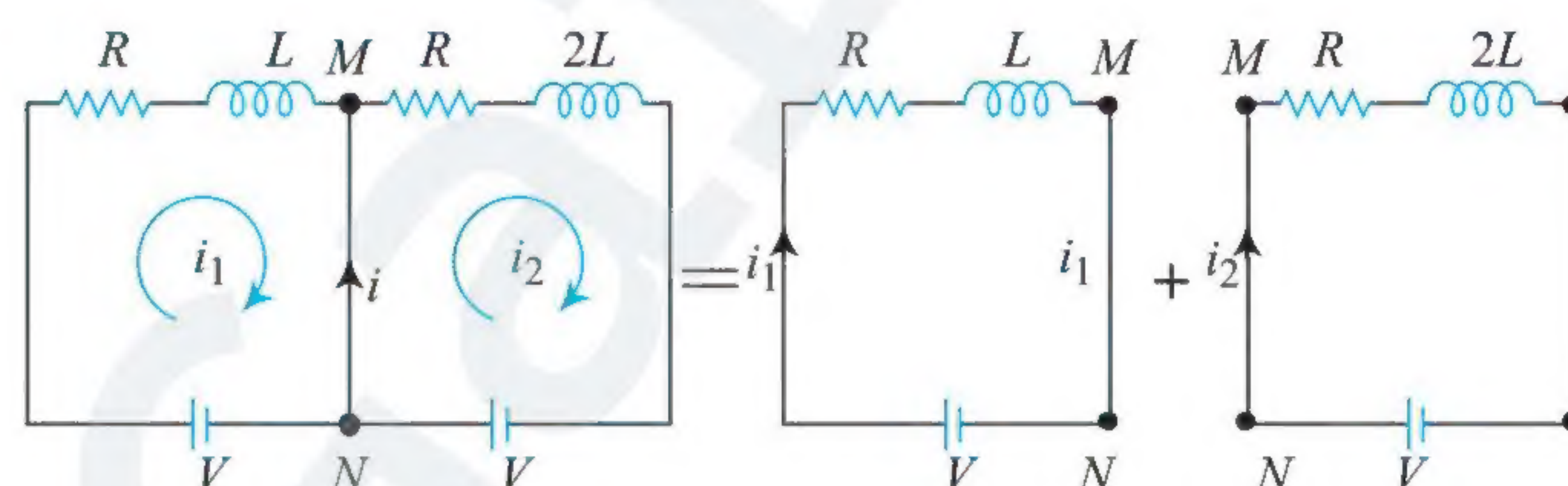
3. (2), (4)

Hence current in wire MN ; $i = i_1 - i_2$

$$\text{or } i_{\max} = (i_1 - i_2)_{\max}$$

$$i = (i_1 - i_2) = \frac{V}{R} \left[1 - e^{-\left(\frac{R}{L}\right)t} \right] - \frac{V}{R} \left[1 - e^{-\left(\frac{R}{2L}\right)t} \right]$$

$$= \frac{V}{R} \left[e^{-\left(\frac{R}{2L}\right)t} - e^{-\left(\frac{R}{L}\right)t} \right]$$



For current to be maximum, $\frac{di}{dt} = 0$

$$\frac{V}{R} \left[-\frac{R}{2L} e^{-\left(\frac{R}{2L}\right)t} - \left(-\frac{R}{L}\right) e^{-\left(\frac{R}{L}\right)t} \right] = 0$$

$$\text{or } e^{-\left(\frac{R}{L}\right)t} = \frac{1}{2} e^{-\left(\frac{R}{2L}\right)t}$$

$$e^{-\left(\frac{R}{2L}\right)t} = \frac{1}{2} \Rightarrow \left(\frac{R}{2L}\right)t = \ln 2$$

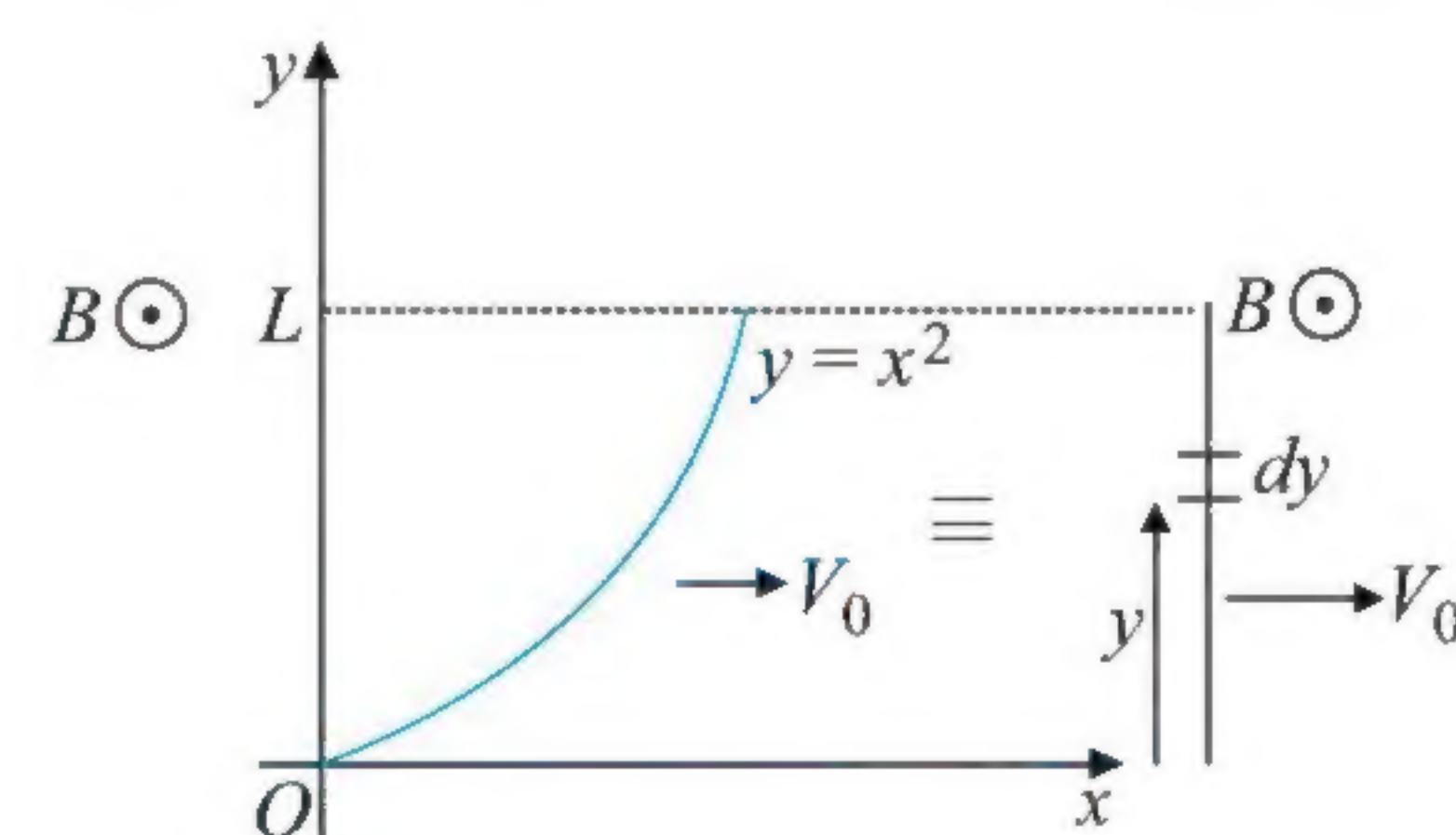
or $t = \frac{2L}{R} \ln 2 \rightarrow$ It is the time when i is maximum

$$i_{\max} = \frac{V}{R} \left[e^{-\frac{R}{2L} \left(\frac{2L}{R} \ln 2\right)} - e^{-\left(\frac{R}{L}\right) \left(\frac{2L}{R} \ln 2\right)} \right] = \frac{V}{R} [e^{-\ln 2} - e^{-\ln 4}]$$

$$|i_{\max}| = \frac{V}{R} \left[\frac{1}{2} - \frac{1}{4} \right] = \frac{V}{4R}$$

4. (1),(2),(4)

For calculating the motional emf across the length of the wire, let us project wire such that $(\vec{B}, \vec{V}, \vec{l})$ becomes mutually orthogonal.



Potential difference developed across elemental length by

$$d\phi = V_0 B_0 \left(1 + \left(\frac{y}{L} \right)^\beta \right) \cdot dy$$

$$\Rightarrow d\phi = \int d\phi = \int_0^L V_0 B_0 \left(1 + \frac{y^\beta}{L^\beta} \right) \cdot dy$$

$$\Delta\phi = V_0 B_0 \left[L + \frac{L^{\beta+1}}{(\beta+1)L^\beta} \right]$$

$$\Delta\phi = V_0 B_0 \left[L + \frac{L}{\beta+1} \right]$$

$$\therefore |\Delta\phi| = B_0 V_0 \left(1 + \frac{1}{\beta+1} \right) \cdot L$$

$$|\Delta\phi| \propto L \therefore \text{Option (2) is correct.}$$

If $\beta = 0$

$$\Delta\phi = V_0 B_0 [L + L]$$

$$\Delta\phi = 2V_0 B_0 L \Rightarrow \text{Option (3) is incorrect.}$$

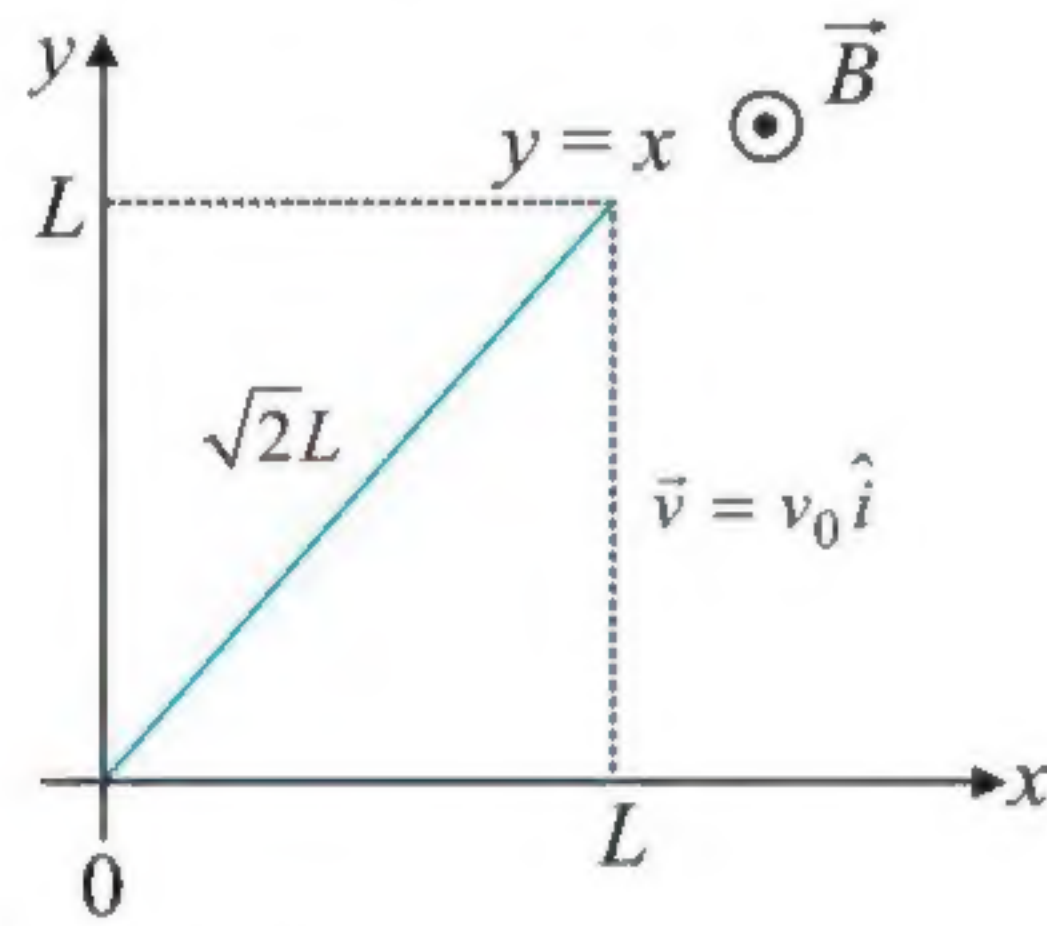
If $\beta = 2$

$$\Delta\phi = V_0 B_0 \left[L + \frac{L}{3} \right]$$

$$\Delta\phi = \frac{4}{3} V_0 B_0 L \Rightarrow \text{Option (4) is correct.}$$

$\Delta\phi$ will be same if the wire is replaced by the straight wire of length $\sqrt{2}L$ and $y = x$

$\therefore$ Range of y remains same



$\therefore$ Option (1) is correct.

Numerical Value Type

1. (6) Flux through circular ring

$$\phi = \left(\frac{\mu_0 I}{L} \right) \pi r^2 \Rightarrow \phi = \frac{\mu_0}{L} \pi r^2 I_0 \cos 300t$$

$$\text{Current in the ring } i = \frac{1}{R} \frac{d\phi}{dt}$$

$$i = \frac{\mu_0 \pi r^2 I_0}{RL} \cdot \sin 300t \times 300$$

$$= \mu_0 I_0 \sin 300t \left[\frac{\pi r^2 \cdot 300}{RL} \right]$$

$$M = I \cdot \pi r^2 = \mu_0 I_0 \sin 300t \left[\frac{\pi^2 r^4 \cdot 300}{RL} \right]$$

Taking $\pi^2 = 10$, we get

$$M = 6\mu_0 I_0 \sin 300t$$

thus $N = 6$

$$2. (7) B = \frac{\mu_0 i R^2}{2(R^2 \times X^2)^{3/2}}$$

$$B = \frac{\mu_0 i R^2}{2(R^2 \times 3R^2)^{3/2}} = \frac{\mu_0 i R^2}{2(4R^2)^{3/2}} = \frac{\mu_0 i R^2}{2 \cdot 2^3 R} = \frac{\mu_0 i}{16R}$$

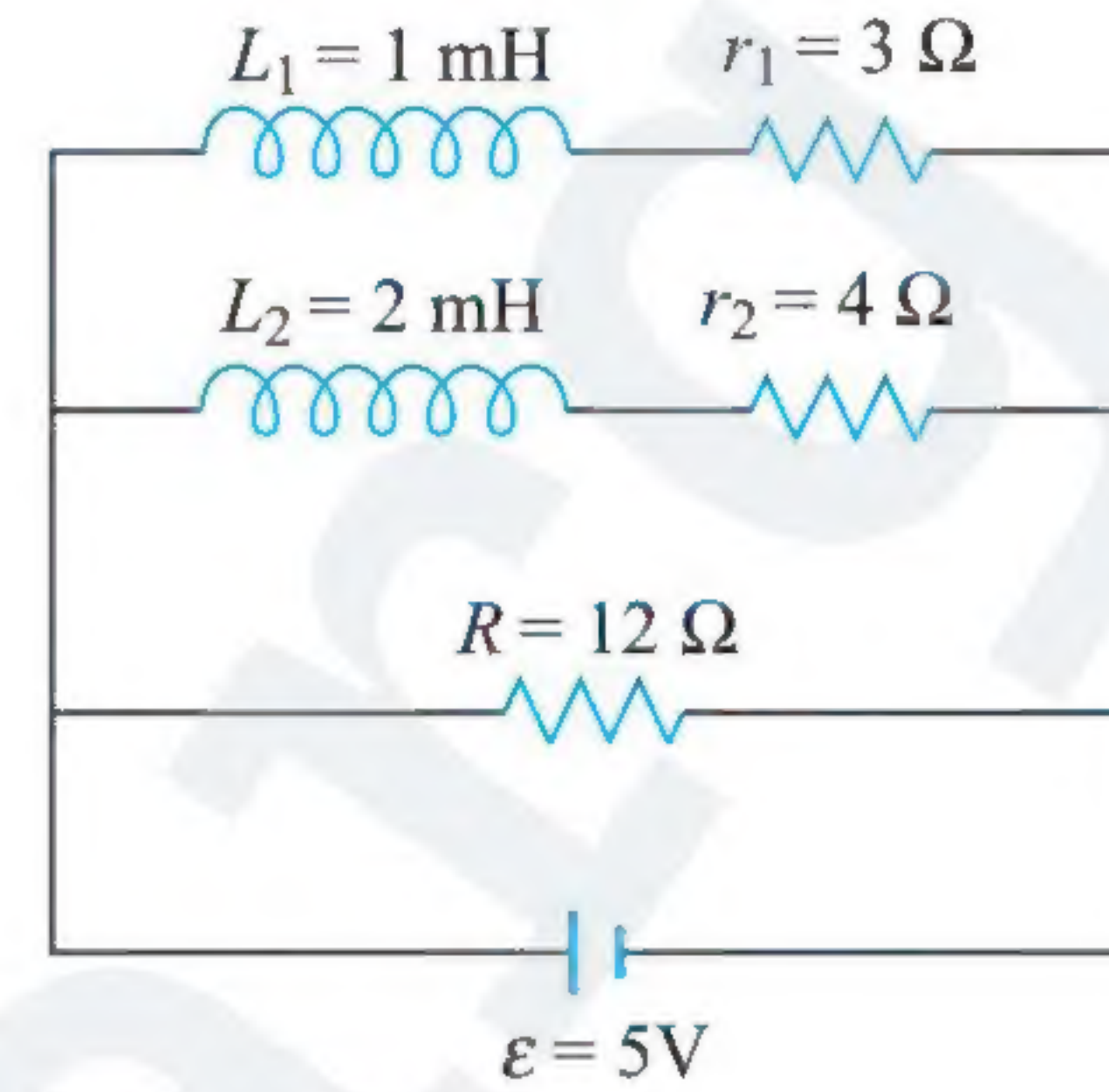
$$\phi = 2 \frac{\mu_0 i}{16R} a^2 \frac{1}{\sqrt{2}} = \frac{\mu_0 i a^2}{8\sqrt{2}R}$$

$$M = \frac{\phi}{i}$$

$$M = \frac{\mu_0 a^2}{2^{7/2} R} = \frac{\mu_0 a^2}{2^{p/2} R}$$

$$p = 7$$

3. (8)



$$I_{\max} = \frac{\epsilon}{R} = \frac{5}{12} \text{ A (Initially at } t = 0)$$

$$I_{\min} = \frac{\epsilon}{R_{\text{eq}}} = \epsilon \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R} \right) \text{ (finally in steady state)}$$

$$= 5 \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{12} \right) = \frac{10}{3} \text{ A}$$

$$\frac{I_{\max}}{I_{\min}} = 8$$

4. (0.63)

Since velocity of PQ is constant. So emf developed across it remains constant,

$$\epsilon = BLv, \text{ where } L = \text{length of wire } PQ$$

Current at any time t is given by

$$i = \frac{\epsilon}{R} \left(1 - e^{-\frac{Rt}{L}} \right)$$

$$i = \frac{BLv}{R} \left(1 - e^{-\frac{Rt}{L}} \right)$$

$$= 1 \times \left(\frac{10}{100} \right) \times \left(\frac{1}{100} \right) \times \frac{1}{1} \left(1 - e^{-\frac{1 \times 10^{-3}}{1 \times 10^{-3}}} \right)$$

$$= \frac{1}{1000} (1 - e^{-1}) = \frac{1}{1000} \times (1 - 0.37)$$

$$i = 0.63 \times 10^{-3} \text{ A} \Rightarrow x = 0.63$$

5. (55.00)

The e.m.f. induced in coil-1,

$$e_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} \quad \dots(i)$$

The e.m.f. induced in coil-2,

$$e_2 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} \quad \dots(ii)$$

Total work done by both the batteries in time dt ,

$$dW = dW_1 + dW_2 = e_1 I_1 dt + e_2 I_2 dt \quad \dots(iii)$$

From (i), (ii) and (iii)

$$dW = L_1 I_1 dI_1 + L_2 I_2 dI_2 + M I_1 dI_2 + M I_2 dI_1$$

$$dW = L_1 I_1 dI_1 + L_2 I_2 dI_2 + M d(I_1 I_2) \quad \dots(\text{iv})$$

Integrating Eq. (iv)

$$W = \int dW = L_1 \int_0^{I_1} I_1 dI_1 + L_2 \int_0^{I_2} I_2 dI_2 + M \int d(I_1 I_2) \quad \dots(\text{v})$$

$$W = \frac{L_1 I_1^2}{2} + \frac{L_2 I_2^2}{2} + M I_1 I_2$$

$$\text{Here, } I_1 = \frac{5}{5} = 1 \text{ A; } I_2 = \frac{20}{10} = 2 \text{ A}$$

$$W = \frac{10 \times 10^{-3} \times 1}{2} + \frac{20 \times 10^{-3} \times 4}{2} + 5 \times 10^{-3} \times 1 \times 2$$

$$W = 55 \times 10^{-3} \text{ J} = 55 \text{ mJ}$$